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음향결정 구조의 레벨셋 기반 위상 및 형상 최적설계

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Level Set based Topological Shape Optimization of Phononic Crystals

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Abstract

A topology optimization method for phononic crystals is developed for the design of sound barriers, using the level set approach. Given a frequency and an incident wave to the phononic crystals, an optimal shape of periodic inclusions is found by minimizing the norm of transmittance. In a sound field including scattering bodies, an acoustic wave can be refracted on the obstacle boundaries, which enables to control acoustic performance by taking the shape of inclusions as the design variables. In this research, we consider a layered structure which is composed of inclusions arranged periodically in horizontal direction while finite inclusions are distributed in vertical direction. Due to the periodicity of inclusions, a unit cell can be considered to analyze the wave propagation together with proper boundary conditions which are imposed on the left and right edges of the unit cell using the Bloch theorem. The boundary conditions for the lower and the upper boundaries of unit cell are described by impedance matrices, which represent the transmission of waves between the layered structure and the semi-infinite external media. A level set method is employed to describe the topology and the shape of inclusions. In the level set method, the initial domain is kept fixed and its boundary is represented by an implicit moving boundary embedded in the level set function, which facilitates to handle complicated topological shape changes. Through several numerical examples, the applicability of the proposed method is demonstrated.

Keywords : *topological shape optimization, level set method, noise barrier performance, Bloch theorem, acoustic wave transmission, adjoint variable method*

1. Introduction

Periodic materials designed for acoustic or elastic wave propagation are called phononic crystals. The significance of these materials results from the band structure with band gaps. Utilization of this property makes it possible to control the wave transmission in the material. Since the capability of such a structure strongly depends on the shape and arrangement of inclusions characterizing the periodicity, it is worthwhile to explore an effective layout. Many researchers have attempted to develop acoustic barriers by utilizing the band structure(Qjan *et al*., 2008; Vasseur *et al*., 1994). The shape optimization method for the periodically distributed inclusion can be a useful tool for this task(Håkansson *et al*., 2004). Sigmund and Jensen(2003) tried to find optimal shape of inclusions for maximization of the band gap using topology optimization methods.Recently, He *et al*.(2007) performed the level set method incorporating topological derivatives into shape derivatives to obtain layout as well as shape of photonic crystals.

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Although periodicity plays an important role in the insulation of waves, a barrier given by an array of scatters has a finite thickness. Therefore, it is valuable to know how to evaluate the transmitting waves through a barrier with finite thickness in the context of the shape design. Yamada *et al*.(2010) developed a level set-based topology optimization method for layers of photonic crystals imbedded in an infinite medium. In order to describe the radiation condition in semi-infinite domains locating on both sides of the barrier, they replaced the half-planes to finite energy absorbing regions. This approximation needs excessive discretization of these regions to add to the unit layer. Since the optimization analysis involves a number of wave transmission analyses, the extension of the domain size results in the increase of computational cost. Abe *et al*. (2010) proposed a transmitting boundary for semi-infinite wave fields synchronized with the periodicity of a combined layer, which is applicable not only to homogeneous media but also to periodic structures. This is given by an impedance matrix composed of propagating wave modes. The transmitting boundaries are to be coupled with the lower and upper sides of the periodic layer.

In this research, a topological shape optimization method of phononic crystals for sound barrier is developed using level set methods. The purpose of design optimization is to get an optimal shape of periodic inclusions by minimizing the norm of the transmittance at given frequencies and an incident angle of waves attacking the phononic crystals. By deriving impedance matrix to consider transmittance of acoustic energy, we can define energy transmittance instead of band-gap concept as an objective function, and an optimal solution is guaranteed since energy measure is dealt. In this research, a layered structure is considered which is composed by inclusions arranged periodically in the horizontal direction while finite inclusions are distributed in the vertical direction. The distributions of inclusions are already determined and the shape of inclusion is considered as design to control acoustic waves. The periodic boundary conditions are imposed on the left and right edges of the unit cell in a layer by using the Bloch theorem. The upper and lower ends which are for the transmission of waves between the layered structure and the semi-infinite external media are described by impedance matrices(Abe *et al*., 2010). Once the impedance matrices have been obtained, one can analyze the wave propagation without any additional degrees of freedom. Moreover, since the impedance matrix is independent of the structure of the insulating wall, the reconstruction of the matrices is not required during the optimization process.

To describe complex shape of inclusions, the level set method is employed. In the level set method, the initial domain is kept fixed and its boundary is represented by an implicit moving boundary(IMB) embedded in the level set function, which facilitates to handle complicated topological shape changes. Furthermore, a material interpolation for scatter and outside of scatters using level set function makes it easy to deal boundary conditions between two different medium without difficulties in phononic crystal structures.

2. Wave transmission analysis

2.1 Acoustic analysis for incident wave

Let us consider a periodic layer consisting of acoustic scatters(inclusions) as shown in Fig.1. The scattering layer rests horizontally on an infinite acoustic field. The periodic length of the arrange-

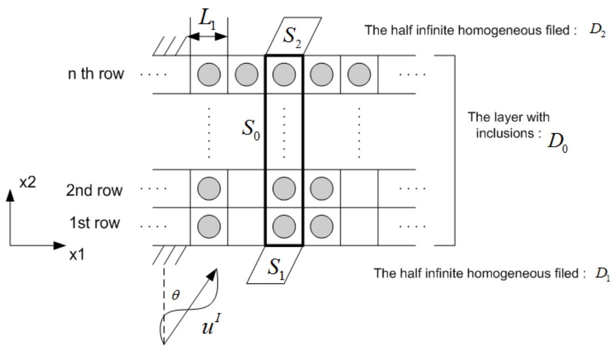


Fig. 1 Acoustic barrier in infinite acoustic field

ment is L_1. The barrier is subjected to anincident wave u^I which propagates to upwards in the lower half-infinite region D_1.

The governing equation of the acoustic wave is given by the following Helmholz equation:

$$\beta\nabla^2 u(\mathbf{x}) + \omega^2 \rho u(\mathbf{x}) = 0 \text{ for } \mathbf{x} \in \Omega \tag{1}$$

where u is the sound pressure, Ω is the acoustic medium, β and ρ are adiabatic bulk modulus, material densities, respectively. The adiabatic bulk modulus is defined as $\beta = \rho c^2$ for c the speed of sound and the circular frequency ω. The boundary value on the obstacle is given by:

$$\frac{\partial u}{\partial n} = 0 \text{ on } \Gamma \tag{2}$$

where n stands for the outward normal and Γ is the boundary of the scatteres. For a unit cell of the layer, S_0, the finite element equation is

$$\left[\hat{K}\right]\{u\} = \{f\} \tag{3}$$

where $\{u\}$ is the nodal vector of the sound pressure, $\{f\}$ is the flux vector, $[\hat{K}] = [K] - \omega^2[M]$, $[K]$ and $[M]$ are stiffness and mass matrices degenerated based on the Bloch' theorem:

$$u_R = e^{ik_1L_1} u_L,\ \ f_R = -e^{ik_1L_1} f_L \tag{4}$$

where $(\ )_L$ and $(\ )_R$ are sub-vectors consisting of nodes on the left and right sides of the unit cell, respectively. k_1 is the horizontal component of a wave vector. In terms of sub-matrices of $[\hat{K}]$, Equation (3) can be rewritten as

$$\begin{bmatrix} \hat{K}_{BB} & \hat{K}_{BT} & \hat{K}_{BM} & \hat{K}_{BL} \\ \hat{K}_{TB} & \hat{K}_{TT} & \hat{K}_{TM} & \hat{K}_{TL} \\ \hat{K}_{MB} & \hat{K}_{MT} & \hat{K}_{MM} & \hat{K}_{ML} \\ \hat{K}_{LB} & \hat{K}_{LT} & \hat{K}_{LM} & \hat{K}_{LL} \end{bmatrix} \begin{Bmatrix} u_B \\ u_T \\ u_M \\ u_L \end{Bmatrix} = \begin{Bmatrix} f_B \\ f_T \\ 0 \\ 0 \end{Bmatrix} \tag{5}$$

where $(\ )_B$ and $(\ )_T$ stand for sub-vectors corresponding to the bottom and the top of the unit cell. $(\ )_M$ indicates the rest component vector. In this problem, we assume that there is no sound source in the domain. In Equation (5), $\{f_B\}$ and $\{f_T\}$ are unknown sub-vectors. In order to evaluate these unknowns, the compatibility and equilibrium conditions at the interfaces between S_0 and D_1, and S_0 and D_2 can be considered :

$$\{F_2\} + \{f_B\} = 0,\ \{u_2\} = \{u_B\} \tag{6}$$

$$\{F_1\} + \{f_T\} = 0,\ \{u_1\} = \{u_T\} \tag{7}$$

where $()_1$ and $()_2$ are sub-vectors on the interfaces of D_1 and D_2. Consequently, $\{f_B\}$ and $\{f_T\}$ can be replaced by $\{F_1\}$ and $\{F_2\}$, respectively.The solution in the lower half-plane D_1 represented by a unit cell S_1 is composed of the incident and reflected waves as shown in Fig. 1. The acoustic wave and flux $\{u_1\}$ and $\{F_1\}$ of D_1 can be decomposed into two terms (Fig. 2(a)) as

$$\{u_1\} = \{u^I\} + \{u^R\},\ \{F_1\} = \{F_D{}^I\} + \{F_D{}^R\} \tag{8}$$

where the first terms on the right hand side are vectors relevant to the incident wave, while the second terms correspond to the reflected wave. The internal flux for the incident wave $\{F_D^I\}$ can be evaluated by the relation between the nodal acoustic pressure u_1 and the nodal flux F_1 at the interface. Since the incident wave is propagating to upwards, this relation can be described for an upper semi-infinite field D_1 complementing to the lower semi-infinite field D_1 (Fig. 2(a)) by

$$[K_{1U}]\{u_1\} = \{F_{1U}\} \tag{9}$$

where $[K_{1U}]$ is an impedance matrix for upper semi-infinite D_1. Using the Equation (9), we can obtain upward transmitting u^I waves (Fig. 2(b)) such as $\{F_U^I\} = [K_{1U}]\{u^I\}$. And using the relation of $\{F_D^I\} = -\{F_U^I\}$, the flux of the incident wave is expressed as

$$\{F_D^I\} = -[K_{1U}]\{u^I\} \tag{10}$$

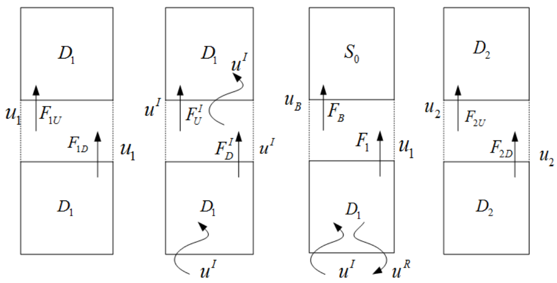


Fig. 2 Acoustic barrier in infinite acoustic field

where $\{F_U^I\}$ stands for the nodal internal flux acting on the bottom boundary of the upper semi-infinite D_1, excited by the incident wave. Since the reflected wave from the boundary of D_1 travels to downwards (Fig. 2(b) and (c)), it can be described by

$$\{F_D^R\}=[K_{1D}]\{u^R\} \tag{11}$$

where $[K_{1D}]$ is an impedance matrix for the top boundary of lower semi-infinite D_1.

In a similar manner, the transmitted wave which propagates in the upper semi-infinite D_2 can be represented by

$$\{F_2\}=[K_{2U}]\{u_2\} \tag{12}$$

where $[K_{2U}]$ is an impedance matrix for the bottom boundary of upper semi-infinite D_2(Fig. 2(d)). Substituting Equations (10) and (11) into Equation (8) and recalling that $u^R=u_1-u^I$, we can obtain the relation:

$$\{F_1\}=[K_{1D}]\{u_1\}-[K_{1U}+K_{1D}]\{u^I\} \tag{13}$$

Using Equations (6), (7), (12) and (13), we can express Equation (5) in terms of the impedance matrices and incident acoustic wave such as

$$\begin{bmatrix} \hat{K}_{BB}+K_{1D} & \hat{K}_{BT} & \hat{K}_{BM} & \hat{K}_{BL} \\ \hat{K}_{TB} & \hat{K}_{TT}+K_{2U} & \hat{K}_{TM} & \hat{K}_{TL} \\ \hat{K}_{MB} & \hat{K}_{MT} & \hat{K}_{MM} & \hat{K}_{ML} \\ \hat{K}_{LB} & \hat{K}_{LT} & \hat{K}_{LM} & \hat{K}_{LL} \end{bmatrix} \begin{Bmatrix} u_B \\ u_T \\ u_M \\ u_L \end{Bmatrix} = \begin{Bmatrix} [K_{1U}+K_{1D}]\{u^I\} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{14}$$

By solving Equation (14), we can obtain the acoustic wave field in sound barrier layer D_0, and analyze the acoustic wave transmittance through this barrier. We must note that additional degrees of freedom are not needed for the calculating the incident flux.

2.2 Derivation of impedance matrices

In the following, as an example, we will describe the outline of the derivation of the impedance matrix $[K_{2U}]$. Let us consider wave propagation in a unit cell S_2, which is attached to the top boundary of S_0 on the upper half-infinite region D_2. Although, in this paper, a homogeneous acoustic field is assumed, impedance matrix developed by Abe *et al.*(2010) is applicable to periodic media. The solution in the unit cell can be expressed by a linear combination of modes given by the eigenvalue problem:

$$[\tilde{K}]\{\varphi_i\}=\omega_i^2[\tilde{M}]\{\varphi_i\} \tag{15}$$

where $[\tilde{K}]$ and $[\tilde{M}]$ are the stiffness and mass matrices of the unit cell S_2, respectively, and the boundary value on the bottom and top sides of the unit cell S_2 is prescribed by the Neumann boundary condition, while the Bloch's periodicity condition is imposed on the left and right sides for wave number k_i. ω_i is the ith eigen-frequency and ϕ_i is the eigen-mode. The acoustic pressure $\{u\}$ can be given by the linear combination of the eigen-modes as

$$u_i=\sum_i \alpha_i \varphi_i \tag{16}$$

where α_i is the weighting factor. Substituting Equation (16) into Equation (15) gives:

$$\sum_i \alpha_i\left[\tilde{K}-\omega^2\tilde{M}\right]\phi_i=\{f\} \tag{17}$$

Due to the orthogonal property of the mode vectors, Equation (17) yields the relation:

$$\alpha_i m_i(\omega_i^2-\omega^2)=[\phi_i^*]^T\{f\}, m_i=\frac{k_i}{\omega_i^2} \tag{18}$$

where ( )* indicates conjugation values of ( ). From Equation (18), α_i is given by

$$\alpha_i=\frac{1}{m_i(\omega_i^2-\omega^2)}[\phi_i^*]^T\{f\} \tag{19}$$

Substitution of Equation (19) into Equation (17) leads to

$$\begin{Bmatrix} u_B \\ u_T \end{Bmatrix}=\sum_i \frac{1}{m_i(\omega_i^2-\omega^2)}[\varepsilon_i][\varepsilon_i^*]^T\begin{Bmatrix} f_B \\ f_T \end{Bmatrix}=[H]\begin{Bmatrix} f_B \\ f_T \end{Bmatrix} \tag{20}$$

where ϵ_i is composed of nodal components of the bottom and top sides of S_2. Rearranging Equation (20) with respect to nodes on the bottom and top sides, we obtain the transfer matrix $[G]$ as

$$[G]\begin{Bmatrix} u_B \\ f_B \end{Bmatrix}=\begin{Bmatrix} u_T \\ -f_T \end{Bmatrix},$$

$$[G]=\begin{bmatrix} H_{TT}H_{BT}^{-1} & -H_{TT}H_{BT}^{-1}H_{BB}+H_{TB} \\ -H_{BT}^{-1} & H_{BT}^{-1}H_{BB} \end{bmatrix} \tag{21}$$

Using the Bloch theorem between top and bottom of the unit cell S_2 such as

$$\begin{Bmatrix} u_T \\ -f_T \end{Bmatrix}=e^{-i\mathbf{k}\cdot\mathbf{d}_2}\begin{Bmatrix} u_B \\ f_B \end{Bmatrix}=e^{-ih_2}\begin{Bmatrix} u_B \\ f_B \end{Bmatrix} \tag{22}$$

where the wave number component vector is given as $\boldsymbol{k}=h_i\boldsymbol{b}_i$, and h_i is is the wave number relevant to $\boldsymbol{b}_i$, and $\boldsymbol{b}_i$ is basis of the reciprocal lattice. Here, we set the top and bottom of unit cell S_2 is perpendicular to $\boldsymbol{b}_2$. Therefore, Equation (21) can be

$$[G]\begin{Bmatrix} u_B \\ f_B \end{Bmatrix}=\lambda\begin{Bmatrix} u_B \\ f_B \end{Bmatrix}(\lambda=e^{-ih_2}) \tag{23}$$

where, in general, the coefficient h_2 is given by a complex number. The half of the eigen-modes travels to upwards, while the other half travels to downwards. Extracting the former modes and arranging them, we obtain the impedance matrix as

$$K_U=\left[f_1\cdots f_{n/2}\right]\left[u_1\cdots u_{n/2}\right]^{-1} \tag{24}$$

where n is the total number of eigen-modes. Here, the component of n_i and f_i for $i=1,2,\cdots,n/2$ is corresponding to the degrees of freedom of bottom components of unit cell S_2. Other impedance matrix $[K_D]$ can be derived in the same way as $[K_U]$.

2.3 Energy transmittance

The norm of energy transmittance is defined as the energy ratio of the transmitting wave to the incident wave, i.e.

$$E_r=\frac{\overline{E}_T}{\overline{E}_I} \tag{25}$$

Using the impedance matrices, the time averages of energy for incident and transmition of sound are expressed as

$$\overline{E}_I=\frac{\omega}{2}\mathrm{Im}(\left[u^{I*}\right]^T[K_{1D}]\{u^I\}),$$

$$\overline{E}_T=\frac{\omega}{2}\mathrm{Im}(\left[u_T^*\right]^T[K_{2U}]\{u_T\}) \tag{26}$$

where Im( ) is the imaginary part of the a complex number.

3. Level set method

Let $\Omega\subset R^d$ be a bounded open domain with a smooth boundary Γ. Imagine the boundary Γ of the domain moves in the direction normal to its boundary with a given speed V_n. At time $\tau=0$, assume the existence of a zero level set function $\phi(\boldsymbol{x},0)$ that is Lipschitz continuous and defined on Ω_I, satisfying

$$\phi(\mathbf{x},\tau=0)\begin{cases} +\zeta(\mathbf{x},\Gamma) & \mathbf{x}\in\Omega \\ =0 & \mathbf{x}\in\Gamma \\ -\zeta(\mathbf{x},\Gamma) & \mathbf{x}\in\Omega_I\setminus\overline{\Omega} \end{cases} \tag{27}$$

where $\zeta(x,\Gamma)$ is a distance function from a point x to the boundary Γ. Γ_I represents an initial reference boundary. Taking the material derivative of level set function with respect to a perturbation parameter τ leads to the "Hamilton-Jacobi Equation" as

$$\frac{\partial\phi}{\partial\tau}=V_n|\nabla\phi|,\ \left.\frac{\partial\phi}{\partial n}\right|_{\Gamma_I}=0 \tag{28}$$

Note that given a normal velocity field V_n attempts to solve the first order partial differential equation leads to the optimal implicit boundary of structures. This velocity is obtained from the design sensitivity analysis(DSA) to meet optimum conditions.

4. Shape design sensitivity analysis

In this section, we derive an analytical design sensitivity of performance function with respect to the shape change through level set function ϕ using adjoint variable method(AVM). If we set the τ amount of perturbed level set function in the direction of $\varphi(x)$, $\phi_\tau(x)$ is defined as

$$\phi_\tau(\mathbf{x})=\phi(\mathbf{x})+\tau\varphi(\mathbf{x}) \tag{29}$$

If we only consider the sound barrier layer S_0 as design domain in Fig. 1, the Equation (3) in perturbed design can be written as

$$\left[\hat{K}(\phi_\tau)\right]\{u\}=\{f\} \tag{30}$$

The objective is to find the optimal scatter's shape to minimize the norm of sound energy transmittance defined in Equation (25) under scatter's volume $V_{scatter}$ is less than allowed volume V_{allow}, that is, the optimization formulation is

$$Minimize: E_r=\int_\Omega J(\phi)d\Omega=\frac{\omega}{2\bar{E}_I}\mathrm{Im}\left(\{u^*\}^T\left[\bar{K}\right]\{u\}\right) \tag{31}$$

such that $V_{scatter}<V_{allow}$.

The Lagrangian equation for Equation (31) can be composed as

$$L=\int_{\Omega_I}J(\phi)d\Omega+\xi\left(\int_{\Omega_I}H(\phi)d\Omega-V_{allow}\right) \tag{32}$$

where ξ is the positive Lagrangian multiplier and $H(\phi)$ is Heaviside function. The shape variation with respect to τ Equation (30) can be

$$\left.\frac{dE_r}{d\tau}\right|_{\tau\to0}=\frac{\partial E_r}{\partial\phi}\varphi=\frac{\omega}{2\bar{E}_I}\mathrm{Im}\left(\{u^*\}^T\left[\bar{K}\right]\left\{\frac{\partial u}{\partial\phi}\varphi\right\}+\left\{\frac{\partial u^*}{\partial\phi}\varphi\right\}^T\left[\bar{K}\right]\{u\}\right)+\xi\int_{\Omega_I}\delta(\phi)\varphi d\Omega \tag{33}$$

To delete implicit dependent terms in Equation (33), we can compose the adjoint equations as

$$\begin{aligned}\{\lambda_1\}^T\left[\hat{K}\right]\{\delta u\}&=\{u^*\}^T\left[\bar{K}\right]\{\delta u\}\\ \{\lambda_2\}^T\left[\hat{K}^*\right]\{\delta u^*\}&=\{u\}^T\left[\bar{K}\right]^T\{\delta u^*\}\end{aligned} \tag{34}$$

where λ_1 and λ_2 are adjoint variables, and δu and δu^* are the virtual adjoint variables which are included in the same function space with $(\partial u/\partial\psi)\varphi$ and $(\partial u^*/\partial\psi)\varphi$, respectively. If we replace δu and δu^* to $(\partial u/\partial\psi)\varphi$ and $(\partial u^*/\partial\psi)\varphi$ in Equations (33), the Equation (33) can be written

$$\frac{\partial E_r}{\partial\phi}\varphi=\frac{\omega}{2\bar{E}_I}\mathrm{Im}\left(\{\lambda_1\}^T\left[\hat{K}\right]\left\{\frac{\partial u}{\partial\phi}\varphi\right\}+\left\{\frac{\partial u^*}{\partial\phi}\varphi\right\}^T\left[\hat{K}^*\right]^T\{\lambda_2\}\right)+\xi\int_{\Omega_I}\delta(\phi)\varphi d\Omega \tag{35}$$

where $\delta(\phi)$ is dirac-Delta function. The design variation of Equation (30) is

$$\left[\frac{\partial\hat{K}}{\partial\phi}\varphi\right]\{u\}+\left[\hat{K}\right]\left\{\frac{\partial u}{\partial\phi}\varphi\right\}=0 \tag{36}$$

Inserting Equation (36) into (35) gives

$$\frac{\partial E_r}{\partial\phi}\varphi=\frac{\omega}{2\bar{E}_I}\mathrm{Im}\left(-\{\lambda_1\}^T\left[\frac{\partial\hat{K}}{\partial\phi}\varphi\right]\{u\}-\{\lambda_2\}^T\left[\frac{\partial\hat{K}^*}{\partial\phi}\varphi\right]\{u^*\}\right)+\xi\int_{\Omega_I}\delta(\phi)\varphi d\Omega=\int_{\Omega_I}\left(\frac{\partial J}{\partial\phi}+\xi\delta(\phi)\right)\varphi d\Omega \tag{37}$$

To decrease the performance measures, the design velocity for Hamilton Jacobi equation can be determined as

$$V_n = \varphi = -\left(\frac{\partial J}{\partial \phi} + \xi \delta(\phi)\right) \tag{38}$$

The Lagrangian multiplier ξ is determined from the Kuhn-Tucker optimality condition.

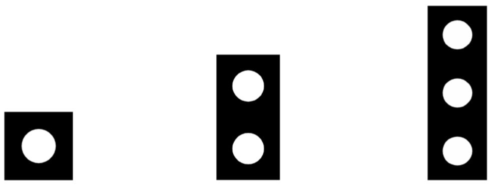
5. Numerical examples

5.1 Response analysis

This example is to show that the sound barrier model using the level set function is well working. The scatterer in the sound barrier can be represented by using the level set function as given in Fig. 3. Lets the material 1 is for the scatterer region, and material 2 is for the other region in sound barrier. Each material property is summarized in Table 1. The material interpolation is performed as

$$\rho(\mathbf{x}) = \rho_1 H(\phi) + \rho_2 \{1 - H(\phi)\} \ ,$$
$$\beta(\mathbf{x}) = \beta_1 H(\phi) + \beta_2 \{1 - H(\phi)\} \tag{39}$$

For the two kinds of wave which propagate to the lower boundary of sound barrier with the angle of 0° and 45°, respectively, three kinds of layered sound barrier models as shown in Fig. 3 are tested.

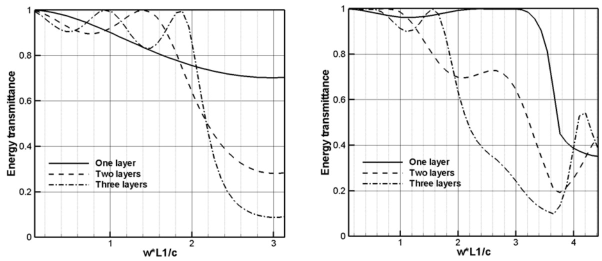


(a) One layer (b) Two layers (c) Three layers

Fig. 3 Periodic sound barrier models

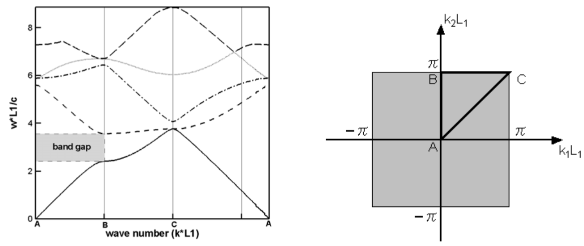
Table 1 Material properties

		Density	Adiabatic bulk modulus
D_1 and D_2		$\rho_2 = 1$	$\beta_2 = 1$
D_0	Scatter(inclusions)	$\rho_1 = 1\times10^{-4}$	$\beta_1 = 1\times10^{-4}$
	Outside of scatter	$\rho_2 = 1$	$\beta_2 = 1$



(a) 0° incident wave (b) 45° incident wave

Fig. 4 Transmittance



(a) Dispersion curve (b) The first Brillouin zone

Fig. 5 Band gap for upper and lower half-infinites

Fig. 4 shows the results of the energy transmittance for each incident angle. Fig. 5 shows the dispersion curves(Fig. 5(a)) of the infinite periodic domain corresponding to the layer with respect to the First Brillioun zone(Fig. 5(b)). Since there exists band gap between w=2.5 and 3.5 along the A-B which corresponds to the incident wave of 0° angle, the sound transmittance is reduced. However, for the case of 45° incident wave, three sound transmittances are not reduced on the same frequency region, since there is no band gap region. These facts show that the suggested methodology is working very well in the analysis for sound barrier.

5.2 Design sensitivity analysis

The purpose of this example is to verify the derived topological sensitivity expressions. The verification model is the sound barrier with one scatterer layer given in Fig. 3(a). The same model size and material properties are used with the example of section 4.1, and the 45° incident wave and frequency w=2.33 are considered. In the level set method, the design variable is the level set function $\phi(x)$ at each position. To verify the accuracy of the developed

Table 2 Verification of design sensitivity

Evaluated position	ΔE_r (a)	$(\partial E_r / \partial \phi)\delta\phi$ (b)	(b)/(a)×100
0°	8.079115E-06	8.073801E-06	100.0658
90°	-1.643022E-05	-1.641498E-05	100.0928
180°	-2.169959E-05	-2.168293E-05	100.0768
270°	7.777268E-06	7.772083E-06	100.0667

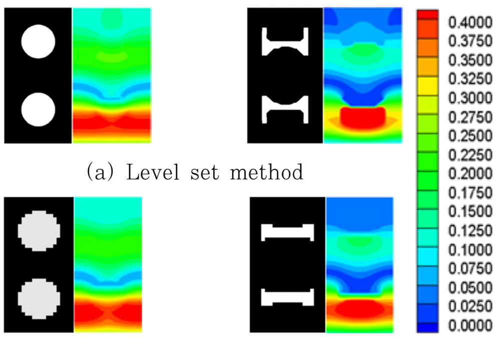
analytical DSA methods using AVM, the variations of the energy transmittance $\delta E_r = (\partial E_r/\partial\phi)\delta\phi$ are compared with those from the forward finite difference method ($\triangle E_r = E_r(\phi+\triangle\phi) - E_r(\phi)$), where the amounts of design variation $\delta\phi$ and the design perturbation $\triangle\phi$ are all 0.001. For sensitivity evaluation, about the scatterer center, 0°, 90°, 180°, and 270° scatterer boundary positions are selected, and each angle is evaluated in the direction of counter clockwise from the right ends of the circular boundary. In Table 2, the analytical sensitivity results of AVM (b) show very good agreements with the finite difference results (a).

5.3 Shape optimization

5.3.1 Two layered periodic sound barrier

This example is for the comparison of level set based optimization with SIMP method about two scatters sound barrier model.

Initial shape Optimal shape



(a) Level set method

(b) SIMP

Fig. 6 Scatterer's shape and acoustic pressure

Table 3 Comparison of optimization results

Methods	level set(a)	SIMP(b)	(a)/(b)
objectives	2.020958E-02	2.516300E-02	0.80
volume fraction	4.983686E-01	4.960320E-01	1.00

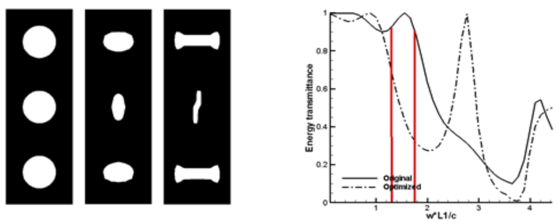
There is a volume constraint for scatter such that volume of ones is less than 50% of the original volume. Fig. 6 shows the optimization results of the level set based and the SIMP optimization for 0° incident wave. As shown in Table 3, the level set based optimization results are smaller than the SIMP method by 20%. This is because the optimal shape of scatterer using level set result blocks the acoustic energy more effectively than the one using SIMP. Since the SIMP method has a limit to represent boundary distinctly as shown in Fig. 6(b), the level set based optimization is much reliable.

5.3.2 Three layered periodic sound barrier

This example is to show that our method is applicable to reduce the energy transmittance at a certain frequency region. The volume constraint is 50% of the original one, and the optimization is performed to minimize the energy transmittanc at $1.33 < w < 1.77$. Totally, 5 energy transmittances are picked up in this frequency region to evaluate performance measure, and the performance measure is given by a weighted summation of these values as

$$J(\phi) = \sum_{i=1}^{N} E_r^i(w_i)\Lambda_i \tag{40}$$

where E_r^i is the energy transmattiance corresponding to i-th non dimensional frequecy w_i, and Λ_i is the weight. Here, $\Lambda_i = 1$ for all the cases $i = 1, 2, \cdots, 5$. The optimization results and history are shown in Fig. 7 for 45° incident waves. Tables 4 shows the optimization results at each frequency level for given incident waves. As shown these



(a) Optimization history (b) Energy transmittance

Fig. 7 Optimization result for 45° incident wave

Table 4 Minimization energy transmittance(45°)

	Initial (a)	optimal (b)	reduction rate (b)/(a)
Total	9.61867E-01	4.29063E-01	0.45
w=1.33	9.96727E-01	6.15329E-01	0.62
w=1.44	9.83458E-01	4.95780E-01	0.50
w=1.55	9.63190E-01	4.03329E-01	0.42
w=1.66	9.41515E-01	3.37357E-01	0.36
w=1.77	9.24446E-01	2.93519E-01	0.32

tables all energy transmittances at given frequency levels are decreased.

5. Conclusions

In this research, we performed level set based topological shape optimization for sound barrier with scatters. The derived shape design sensitivity is verified comparing with finite difference methods. Using this sensitivity analysis, the level set based shape optimization was performed. By comparing these results with the SIMP based topology optimization, it was proved that the developed method is well working. The level set method gives better results than SIMP since level set function improves the geometric exactness of scatterers. The objective function defined as energy transmittance guarantees optimum value since it is expressed as an energy quantity. In addition, it is possible to control energy transmittance at a certain frequency range where we want.

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요 지

본 논문에서는 레벨셋 방법을 이용하여, 소음을 차단하기 위한 음향 구조물의 형상 최적설계를 수행하였다. 형상 최적설계의 목적은 특정한 각도와 각속도로 입사되는 입사파에 대해서 음향 투과율(acoustic transmittance)이 최소가 되도록 음향 결정의 형상(inclusion shape)을 결정하는 것이다. 음향 결정 구조에서는 음향이 흩어져 있는 결정 구조에 의해서 굴절되기 때문에 결정 모양을 조정함으로써, 음향 거동을 제어할 수 있다. 본 연구에서는 음향 구조물로 결정이 수평방향으로는 주기적으로 무한히 분포하고 수직방향으로는 유한한 층간 구조를 가지고 있는 소음 방어벽(Noise barrier)을 고려한다. 주기적 구조물을 고려하기 때문에 결정의 좌와 우에 Bloch 이론을 적용해 주기적 경계조건을 부과하였고, 소음 방어벽 위와 아래에는 임피던스 행렬(impedance matrix)를 이용하여, 무한 균질 영역과 소음 방어벽 사이의 음파 투과를 모사하였다. 결정의 위상과 형상변화를 묘사하기 위해서 레벨셋 방법(level set method)을 사용하였다. 레벨셋 방법에서는 초기 영역을 고정시킨 상태에서, 레벨셋으로 표현되는 임시적 경계(implicit moving boundary)를 변화시킴으로써 복잡한 형상을 다룰 수 있다. 몇몇 수치적 예제를 통해, 제시된 방법의 적용성을 검증하였다.

핵심용어 : 위상 및 형상 최적설계, 레벨셋 방법, 소음벽 거동, Bloch 이론, 음향파 투과율, 보조변수 방법