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Lacunary I-Convergent Sequences

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Abstract. In this article we introduce the concepts of lacunary I- convergent sequences. We investigate its different properties like solid, symmetric, convergence free etc.

1. Introduction

Throughout this article w, c, c_0 ℓ_∞ denote the spaces of *all, convergent, null* and *bounded* sequences respectively.

The notion of I-convergence was studied at initial stage by Kostyrko, Salat and Wilczynski [6]. Later on it was studied by Demirci [1], Salat, Tripathy and Ziman ([8],[9]), Tripathy and Hazarika ([14], [15], [16], [17]), Tripathy and Mahanta [19], Tripathy and Sarma [21] and many others.

A *lacunary sequence* is an increasing integer sequence $\theta = (k_r)$ such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$, as $r \to \infty$. The intervals determined by θ will be defined by $J_r = (k_{r-1}, k_r]$ and the ratio $\frac{k_r}{k_{r-1}}$ will be defined by ϕ_r.

Freedman, Sember and Raphael [3] defined the space N_θ in the following way:

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For any lacunary sequence $\theta = (k_r)$,

$$N_\theta = \left\{(x_k) : \lim_{r\to\infty} h_r^{-1} \sum_{k\in I_r} |x_k - L| = 0, \text{ for some } L\right\}.$$

The space N_θ is a BK space with the norm

$$||(x_k)||_\theta = \sup_r h_r^{-1} \sum_{k\in I_r} |x_k|.$$

N_θ^0 denotes the subset of those sequences in N_θ for which $L = 0$ and we have $(N_\theta^0, ||.||_\theta)$ is also a BK-space.

The notion of Lacunary convergence has been investigated by Colak [2], Fridy and Orhan ([4], [5]), Tripathy and Baruah [12], Tripathy and Et [13], Tripathy and Mahanta [18] and many others in the recent past.

2. Definition and notations

Definition 2.1. Let X be a non-empty set. Then a family of sets $I \subseteq 2^X$ (power sets of X) is said to be an *ideal* if I is additive i.e. $A, B \in I \Rightarrow A \cup B \in I$ and hereditary i.e. $A \in I$, $B \subseteq A \Rightarrow B \in I$.

Definition 2.2. A non-empty family of sets $\Im \subseteq 2^X$ is called a *filter* on X if and only if $\phi \notin \Im$, for each $A, B \in \Im \Rightarrow A \cap B \in \Im$ and for each $A \in \Im$ and $B \supseteq A \Rightarrow B \in \Im$.

Definition 2.3. An ideal $I \subseteq 2^X$ is called *non-trivial* if $I \neq 2^X$.

Definition 2.4. A non-trivial ideal $I \subseteq 2^X$ is called *admissible* if and only if $I \supset \{\{x\} : x \in X\}$.

Definition 2.5. A non-trivial ideal I is a *maximal* if there cannot exists any non-trivial ideal $J \neq I$ containing I as a subset.

For each ideal I there is a filter $\Im(I)$ corresponding to I i.e. $\Im(I) = \{K \subseteq N : K^c \in I\}$, where $K^c = N - K$.

Definition 2.6. A sequence (x_k) of complex numbers is said to be *I-convergent* to the number L if for every $\varepsilon > 0$, $\{k \in N : |x_k - L| \geq \varepsilon\} \in I$. We write $I - \lim x_k = L$.

Definition 2.7. A sequence (x_k) of complex terms is said to be *I-null* for which $L = 0$, we write $I - \lim x_k = 0$.

Definition 2.8. A sequence (x_k) of complex numbers is said to be *I-Cauchy* if for every $\varepsilon > 0$ there exists a number $m = m(\varepsilon)$ such that $\{k \in N : |x_k - x_m| \geq \varepsilon\} \in I$.

Definition 2.9. A sequence (x_k) of complex numbers is said to be *I-bounded* if there exists $M > 0$ such that $\{k \in N : |x_k| > M\} \in I$.

The usual convergence is a particular case of I-convergence. In this case $I = I_f$ (the ideal of all finite subsets of N).

Definition 2.10. A subset A of N is said to have *asymptotic density* or *density* $\delta(A)$ if $\delta(A) = \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \chi_A(k)$ exists, where χ_A is the characteristic function of E.

Definition 2.11 A complex sequence (x_k) is said to be *statistically convergent* to L if for every $\varepsilon > 0$, $\delta(\{k \in N : |x_k - L| \geq \varepsilon\}) = 0$. We write $stat - \lim x_k = L$.

The statistical convergence is a particular case of I-convergence. In this case $I = I_\delta$ (the ideal of all subsets of N of zero asymptotic density).

The notion of statistical convergence was investigated from sequence space point of view and linked with summability theory by Rath and Tripathy [7], Tripathy ([10], [11]), Tripathya and Baruah [12], Tripathy and Sarma [20], Tripathy and Sen ([22], [23]) and many others.

Definition 2.12. Let $A \subset N$ and $d_n(A) = \frac{1}{s_n}\sum_{k=1}^{n} \frac{\chi_A(k)}{k}$ for $n \in N$ where $s_n = \sum_{k=1}^{n} \frac{1}{k}$. If $\lim_{n\to\infty} d_n(A)$ exists, then it is called as the *logarithmic density* of A. $I_d = \{A \subset N : d(A) = 0\}$ is an ideal of N.

Definition 2.13. Let $T = (t_{nk})$ be a *regular non-negative matrix.* For $A \subset N$, define $d_T^{(n)}(A) = \sum_{k=1}^{\infty} t_{nk}\chi_A(k)$, for all $n \in N$. If $\lim_{n\to\infty} d_T^{(n)}(A) = d_T(A)$ exists, then $d_T(A)$ is called as $T-$*density* of A. Clearly $I_{d_T} = \{E \subset N : d_T(A) = 0\}$ is an ideal of N.

Note 1: I_δ and I_d are particular cases of I_{d_T}.

(i) Asymptotic density, for

$$t_{nk} = \begin{cases} \frac{1}{n} \text{ if } n \leq k; \\ 0, \text{ otherwise} \end{cases}$$

(ii) Logarithmic density, for

$$t_{nk} = \begin{cases} \frac{k^{-1}}{s_n} \text{ if } n \leq k; \\ 0, \text{ otherwise} \end{cases}$$

Definition 2.14. Let $A \subset N$ be defined as $A(t+1, t+s) = card\{n \in A : t+1 \leq n \leq t+s\}$ for $t \geq 0$ and $s \geq 1$. Put $\beta_s = \liminf_{t\to\infty} A(t+1, t+s)$, $\beta^s = \limsup_{t\to\infty} A(t+1, t+s)$. If both $\underline{u}(A) = \lim_{s\to\infty} \frac{\beta_s}{s}$ and $\overline{u}(A) = \lim_{s\to\infty} \frac{\beta^s}{s}$ exist and if $\underline{u}(A) = \overline{u}(A)(= u(A))$, then $u(A)$ is called the *uniform density* of the subset of A. Clearly $I_u = \{A \subset N : u(A) = 0\}$ is a non-trivial ideal and I_u-convergence is said to be *uniform statistical* *convergence.*

Definition 2.15. A sequence space E is said to be *solid* (or *normal*) if $(\alpha_k x_k) \in E$, whenever $(x_k) \in E$ and for all sequence (α_k) of scalars with $|\alpha_k| \leq 1$,for all $k \in N$.

Definition 2.16. A sequence space E is said to be *symmetric* if $(x_k) \in E$ implies $(x_{\pi(k)}) \in E$, where π is a permutation of N.

Definition 2.17. A sequence space E is said to be *sequence algebra* if $(x_k) \star (y_k) = (x_k y_k) \in E$ whenever $(x_k), (y_k) \in E$.

Definition 2.18. A sequence space E is said to be *convergence free* if $(y_k) \in E$ whenever $(x_k) \in E$ and $x_k = 0$ implies $y_k = 0$.

Definition 2.19. Let $K = \{k_1 < k_2 < ...\} \subseteq N$ and E be a sequence space. A K-*step space* of E is a sequence space $\lambda_K^E = \{(x_{k_n}) \in w : (k_n) \in E\}$.

A canonical preimage of a sequence $\{(x_{k_n})\} \in \lambda_K^E$ is a sequence $\{y_n\} \in w$ defined by

$$y_n = \begin{cases} x_n, & \text{if } n \in K; \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.20. A canonical preimage of a step space λ_K^E is a set of canonical preimages of all elements in λ_K^E, i.e. y is in canonical preimage of λ_K^E if and only if y is canonical preimage of some $x \in \lambda_K^E$.

Definition 2.21. A sequence space E is said to be *monotone* if it contains the canonical preimages of its step spaces.

Definition 2.22. Let $\theta = (k_r)$ be lacunary sequence. Then a sequence (x_k) is said to be *lacunary I-convergent* if for every $\varepsilon > 0$ such that

$$\left\{ r \in N : h_r^{-1} \sum_{k \in I_r} |x_k - L| \geq \varepsilon \right\} \in I.$$

We write $I_\theta - \lim\ x_k = L$.

Definition 2.23. Let $\theta = (k_r)$ be lacunary sequence. Then a sequence (x_k) is said to be *lacunary I-null* if for every $\varepsilon > 0$ such that

$$\left\{ r \in N : h_r^{-1} \sum_{k \in I_r} |x_k| \geq \varepsilon \right\} \in I.$$

We write $I_\theta - \lim x_k = 0$.

Definition 2.24. Let $\theta = (k_r)$ be lacunary sequence. Then a sequence (x_k) is said to be *lacunary I-Cauchy* if there exists a Subsequence $(x_{k'}(r))$ of (x_k) such that $k'(r) \in J_r$ for each $r \in N$, $\lim\limits_{r \to \infty} x_{k'}(r) = L$ and for every $\varepsilon > 0$

$$\left\{r \in N : h_r^{-1} \sum_{k \in J_r} |x_k - x_{k'(r)}| \geq \varepsilon\right\} \in I.$$

Definition 2.25. A lacunary sequence $\theta' = (k'(r))$ is said to be a *lacunary refinement* of the lacunary sequence $\theta = (k_r)$ if $(k_r) \subset (k'(r))$.

Throughout the article ℓ_∞^I, c^I, c_0^I, c_θ^I and $(c_0^I)_\theta$ represents *I-bounded*, *I-convergent*, *I-null*, *lacunary I-convergent* and *lacunary I-null* sequence spaces respectively.

The following result is well known.

Lemma 2.1. *Every solid space is monotone.*

3. Main results

Theorem 3.1. *A sequence (x_k) is I_θ-convergent if and only if it is an I_θ-Cauchy sequence.*

Proof. Let $(x_k) \in c_\theta^I$ be a sequence. Then there exists $L \in C$ such that $I_\theta - \lim\ x_k = L$.

Write $H_{(i)} = \left\{i \in N : |x_k - L| < \frac{1}{i}\right\}$, for each $i \in N$.

Hence for each $i \in N$, $H_{(i)} \supseteq H_{(i+1)}$ and $\left\{r \in N : \frac{|H_{(r)} \cap J_r|}{h_r} \geq \frac{1}{r}\right\} \in I$.

We choose k_1 such that $r \geq k_1$, implies $\left\{r \in N : \frac{|H_{(i)} \cap J_r|}{h_r} < 1\right\} \notin I$.

Next we choose $k_2 > k_1$ such that $r \geq k_2$ implies

$$\left\{r \in N : \frac{|H_{(2)} \cap J_r|}{h_r} < 1\right\} \notin I.$$

Proceeding in this way inductively we can choose $k_{p+1} > k_p$ such that $r > k_{p+1}$ implies $J_r \cap H(p+1) \neq \emptyset$. Further for all r satisfying $k_p \leq r < k_{p+1}$, choose $k'(r) \in J_r \cap H_{(p)}$ such that

$$|x_{k'_r} - L| < \frac{1}{p}.$$

This implies $\lim\limits_{r \to \infty} x_{k'_r} = L$.

Therefore, for every $\varepsilon > 0$, we have

$$\left\{r \in N : h_r^{-1} \sum_{k,\ k'(r) \in J_r} |x_k - x_{k'(r)}| \geq \varepsilon\right\} \subseteq \left\{r \in N : h_r^{-1} \sum_{k \in J_r} |x_k - L| \geq \frac{\varepsilon}{2}\right\}$$

$$\cup\left\{r \in N : h_r^{-1} \sum_{k'_r \in J_r} |x_{k'_r} - L| \geq \tfrac{\varepsilon}{2}\right\}.$$

$$i.e. \ \left\{r \in N : h_r^{-1} \sum_{k,\ k'(r) \in J_r} |x_k - x_{k'_r}| \geq \varepsilon\right\} \in I.$$

Then (x_k) is a I_θ-Cauchy sequence.

Conversely suppose (x_k) is a I_θ-Cauchy sequence. Then for every $\varepsilon > 0$, we have

$$\left\{r \in N : h_r^{-1} \sum_{k \in J_r} |x_k - L| \geq \varepsilon\right\} \subseteq \left\{r \in N : h_r^{-1} \sum_{k,\ k'(r) \in J_r} |x_k - x_{k'(r)}| \geq \tfrac{\varepsilon}{2}\right\}$$
$$\cup\left\{r \in N : h_r^{-1} \sum_{k'(r) \in J_r} |x_{k'(r)} - L| \geq \tfrac{\varepsilon}{2}\right\}.$$

It follows that (x_k) is a I_θ-convergent sequence. □

Theorem 3.2. *If θ' is a lacunary refinement of a lacunary sequence θ and $(x_k) \in c^I_{\theta'}$, then $(x_k) \in c^I_\theta$.*

Proof. Suppose that for each J_r of θ contains the points $(k'_{r,\ t})_{t=1}^{\eta(r)}$ of θ' such that

$$k_{r-1} < k'_{r,\ 1} < k'_{r,2} < ... < k'_{r,\ \eta(r)} = k_r, \text{ where } J'_{r,\ t} = (k'_{r,\ t-1}, k'_{r,\ t}].$$

Since $k_r \subseteq (k'(r))$, so $r, \eta(r) \geq 1$.
Let $(J^*_j)_{j=1}^\infty$ be the sequence of intervals $(J'_{r,\ t})$ ordered by increasing right end points. Since $(x_k) \in c^I_{\theta'}$, then for each $\varepsilon > 0$,

$$\left\{j \in N : (h^*_j)^{-1} \sum_{J^*_j \subset J_r} |x_k - L| \geq \varepsilon\right\} \in I.$$

Also since $h_r = k_r - k_{r-1}$, so $h'_{r,\ t} = k'_{r,\ t} - k'_{r,\ t-1}$.
For each $\varepsilon > 0$, we have

$$\left\{r \in N : h_r^{-1} \sum_{k \in J_r} |x_k - L| \geq \varepsilon\right\}$$
$$\subseteq \left\{r \in N : h_r^{-1} \sum_{k \in J_r} \left\{j \in N : (h^*_j)^{-1} \sum_{\substack{J^*_j \subset J_r \\ k \in J^*_j}} |x_k - L| \geq \varepsilon\right\}\right\}.$$

Therefore $\left\{r \in N : h_r^{-1} \sum_{k \in J_r} |x_k - L| \geq \varepsilon\right\} \in I.$

Hence $(x_k) \in c^I_\theta$. □

Note 3.1. If θ_1 and θ_2 are lacunary sequences then the union and intersection of θ_1 and θ_2 need not be a lacunary sequence. For example consider $\theta_1 = (3^r - 1)$ and $\theta_2 = (3^r)$.

Theorem 3.3. *Let ψ be a set of lacunary sequences*
(a) *If ψ is closed under arbitrary union, then $c^I_\mu = \bigcap\limits_{\theta\in\psi} c^I_\theta$, where $\mu = \bigcup\limits_{\theta\in\psi} \theta$;*
(b) *If ψ is closed under arbitrary intersection , then $c^I_\nu = \bigcup\limits_{\theta\in\psi} c^I_\theta$, where $\nu = \bigcap\limits_{\theta\in\psi} \theta$;*
(c) *If ψ is closed under union and intersection , then $c^I_\mu \subseteq c^I_\theta \subseteq c^I_\nu$.*

Proof. (a) By hypothesis we have $\mu \in \psi$ which is a refinement of each $\theta \in \psi$. Then from theorem 3.2, we have if $(x_k) \in c^I_\mu$ implies that $(x_k) \in c^I_\theta$.
Thus for each $\theta \in \psi$, we get $c^I_\mu \subseteq c^I_\theta$. The reverse inclusion is obvious. Hence $c^I_\mu = \bigcap\limits_{\theta\in\psi} c^I_\theta$.

(b) By part (a) and theorem 3.2, we have $c^I_\nu = \bigcup\limits_{\theta\in\psi} c^I_\theta$.

(c) By part (a) and (b) we get $c^I_\mu \subseteq c^I_\theta \subseteq c^I_\nu$. □

Result 3.1. *Let $\theta = (k_r)$ be a lacunary sequence. Then the spaces c^I_θ and $(c^I_0)_\theta$ are solid and monotone.*

Proof. Let (α_k) be a sequence of scalars with $|\alpha_k| \le 1$, for all $k \in N$. Then we have the space $(c^I_0)_\theta$ is solid by the following relation

$$\left\{ r \in N : h_r^{-1} \sum_{k\in J_r} |\alpha_k x_k| \ge \varepsilon \right\} \subseteq \left\{ r \in N : h_r^{-1} \sum_{k\in J_r} |x_k| \ge \varepsilon \right\}.$$

The space $(c^I_0)_\theta$ is monotone by Lemma 2.1.
The other part of the result follows in a similar way. □

Result 3.2. *Let $\theta = (k_r)$ be lacunary sequence. Then the spaces c^I_θ and $(c^I_0)_\theta$ are not symmetric in general.*

Proof of the result follows from the following example.

Example 3.1. For $I = I_c$. Let $T = \{t : t = i^2 \text{ or } j^3, i, j \in N\}$, then $\sum\limits_{t\in T} t^{-1} < \infty$.
Let $\theta = (3^r)$, and let (x_k) defined as follows:
$x_k = 2$, for $k = j^2; j \in N$.
$x_k = k$, otherwise.
Then $(x_k) \in c^I_\theta$.
Let (y_k) be a rearrangement of (x_k), defined by

$(y_k) = (x_1, x_2, x_4, x_3, x_9, x_5, x_{16}, x_6, ...)$.
Then $(y_k) \notin c^I_\theta$.
Hence c^I_θ is not symmetric.

Similarly the other result follows. □

Result 3.3. *Let θ be a lacunary sequence. Then the spaces c_θ^I and $(c_0^I)_\theta$ are not convergence free in general.*

Proof of the result follows from the following example.

Example 3.2. Let $\theta = (3^r)$ and let (x_k) and (y_k) be two sequences defined by

$x_k = \frac{1}{k}$, for all $k \in N$

and

$y_k = k^2$, for all $k \in N$.

Then (x_k), (y_k) belongs to c_θ^I, but $x_k = 0$ does not imply $y_k = 0$, for all $k \in N$.
Hence c_θ^I is not convergence free.
The result gets in same way. □

Result 3.4. *Let θ be a lacunary sequence. Then the spaces c_θ^I and $(c_0^I)_\theta$ are not sequence algebra in general.*

Proof of the result follows from the following example.

Example 3.3. For $I = I_d$. Let $\theta = (3^r)$ and let (x_k) and (y_k) be two sequences as

$$x_k = \begin{cases} 1 + \frac{1}{k^2}, & \text{if } k \textit{ is even}; \\ 1, & \text{otherwise} \end{cases}$$

and

$$y_k = \begin{cases} \frac{1}{k^2}, & \text{if } k \textit{ is odd}; \\ 0, & \text{otherwise} \end{cases}$$

Then (x_k) (y_k) belongs to c_θ^I, but $(x_k).(y_k)$ does not belongs to c_θ^I.
Hence c_θ^I is not a sequence algebra.
The other result gets in same way. □

Conclusions. In this article we have investigated the notion of lacunary convergence from I-convergence of sequences point of view. Still there are a lot to be investigated on sequence spaces applying the notion of I-convergence. The workers will apply the techniques used in this article for further investigations on I-convergence.

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