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Somewhat fuzzy irresolute continuous mappings

Young Bin Im[1,*], Joo Sung Lee[2] and Yung Duk Cho[1]

[1] **Faculty of General Education, Dongguk University, Seoul 100-715, Korea**
[2] **Department of Mathematics, Dongguk University, Seoul 100-715, Korea**

Abstract

We define and characterize a somewhat fuzzy irresolute continuous mapping and a somewhat fuzzy irresolute semiopen mapping on a fuzzy topological space. Besides, some interesting properties of those mappings are given.

Key Words: somewhat fuzzy irresolute continuous mapping, somewhat fuzzy irresolute semiopen mapping.

1. Introduction

The class of somewhat continuous mappings was first introduced by Gentry and others [2]. Later, the concept of "somewhat" in classical topology has been extended to fuzzy topological spaces. In fact, somewhat fuzzy continuous mappings and somewhat fuzzy semicontinuous mappings on fuzzy topological spaces were introduced and studied by G. Thangaraj and G. Balasubramanian in [5] and [6] respectively.

Meanwhile, the concept of fuzzy irresolute continuous mapping on fuzzy topological space was introduced and studied by M. N. Mukherjee and S. P. Shina in [3].

In this paper, the concepts of somewhat fuzzy irresolute continuous mappings and somewhat fuzzy irresolute semiopen mappings on a fuzzy topological space are introduced and we characterize those mappings. Besides, some interesting properties of those mappings are also given.

2. Preliminaries

Throughout this paper, we denote μ^c with the complement of the fuzzy set μ on a nonempty set X, which is defined by $\mu^c(x) = (1-\mu)(x) = 1-\mu(x)$ for all $x \in X$. If μ is a fuzzy set on a nonempty set X and if ν is a fuzzy set on a nonempty set Y, then $\mu \times \nu$ is a fuzzy set on $X \times Y$, defined by $(\mu \times \nu)(x, y) = \min(\mu(x), \nu(y))$ for every $(x, y) \in X \times Y$. Let $f : X \to Y$ be a mapping and let μ be a fuzzy set on X. Then $f(\mu)$ is a fuzzy set on Y defined by

$$f(\mu)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu(x) \text{ if } f^{-1}(y) \neq \emptyset,\ y \in Y \\ 0 \quad \text{otherwise.} \end{cases}$$

Let ν be a fuzzy set on Y. Then $f^{-1}(\nu)$ is a fuzzy set on X, defined by $f^{-1}(\nu)(x) = \nu(f(x))$ for each $x \in X$. The graph $g : X \to X \times Y$ of f is defined by $g(x) = (x, f(x))$ for each $x \in X$. Then $g^{-1}(\mu \times \nu) = \mu \wedge f^{-1}(\nu)$. The product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of mappings $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ is defined by $(f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2))$ for each $(x_1, x_2) \in X_1 \times X_2$ [1].

Now let X and Y be fuzzy topological spaces. We denote Intμ and Clμ with the interior and with the closure of the fuzzy set μ on a fuzzy topological space X respectively. Then (i) $1 -$ Cl$\mu =$ Int$(1-\mu)$ and (ii) Cl$(1-\mu) = 1 -$ Intμ.

We say that a fuzzy topological space X is *product related* to a fuzzy topological space Y if, for a fuzzy set μ on X and ν on Y, $\gamma \ngeq \mu$ and $\delta \ngeq \nu$ (in which case $(\gamma^c \times 1) \vee (1 \times \delta^c) \geq (\mu \times \nu)$) where γ is fuzzy open set on X and δ is a fuzzy open set on Y, then there exists a fuzzy open set γ_1 on X and a fuzzy open set δ_1 on Y such that $\gamma_1^c \geq \mu$ or $\delta_1^c \geq \nu$ and $(\gamma_1^c \times 1) \vee (1 \times \delta_1^c) = (\gamma^c \times 1) \times (1 \times \delta^c)$.

A mapping $f : X \to Y$ is called *fuzzy continuous* if $f^{-1}(\nu)$ is a fuzzy open set on X for any fuzzy open set ν on Y. And a mapping $f : X \to Y$ is called *fuzzy open* if $f(\mu)$ is a fuzzy open set on Y for any fuzzy open set μ on X.

A fuzzy set μ on a fuzzy topological space X is called *fuzzy semiopen* if $\mu \leq$ Cl Intμ and μ is called *fuzzy semiclosed* if μ^c is a fuzzy semiopen set on X.

A mapping $f : X \to Y$ is called *fuzzy semicontinuous* if $f^{-1}(\nu)$ is a fuzzy semiopen set on X for any fuzzy

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*Corresponding Author: Young Bin Im (philpen@dongguk.edu)

open set ν on Y. And a mapping $f : X \to Y$ is called *fuzzy semiopen* if $f(\mu)$ is a fuzzy semiopen set on Y for any fuzzy open set μ on X [1].

A mapping $f : X \to Y$ is called *fuzzy irresolute continuous* if $f^{-1}(\nu)$ is a fuzzy semiopen set on X for any fuzzy semiopen set ν on Y and a mapping $f : X \to Y$ is called *fuzzy irresolute semiopen* if $f(\mu)$ is a fuzzy semiopen set on Y for any fuzzy semiopen set μ on X. It is clear that every fuzzy irresolute continuous mapping is a fuzzy semicontinuous mapping. And every fuzzy irresolute semiopen mapping is a fuzzy semiopen mapping from the above definitions. But the converses are not true in general [3].

A fuzzy set μ on a fuzzy topological space X is called *fuzzy dense* if there exists no fuzzy closed set ν such that $\mu < \nu < 1$.

A mapping $f : X \to Y$ is called *somewhat fuzzy continuous* if there exists a fuzzy open set $\mu \neq 0_X$ on X such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any fuzzy open set ν on Y . It is clear that every fuzzy continuous mapping is a somewhat fuzzy continuous mapping. But the converse is not true in general.

A mapping $f : X \to Y$ is called *somewhat fuzzy open* if there exists a fuzzy open set $\nu \neq 0_Y$ on Y such that $\nu \leq f(\mu) \neq 0_Y$ for any fuzzy open set μ on X. Note that every fuzzy open mapping is a somewhat fuzzy open mapping but the converse is not true in general [5].

A fuzzy set μ on a fuzzy topological space X is called *fuzzy semidense* if there exists no fuzzy semiclosed set ν such that $\mu < \nu < 1$.

A mapping $f : X \to Y$ is called *somewhat fuzzy semicontinuous* if there exists a fuzzy semiopen set $\mu \neq 0_X$ on X such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any fuzzy open set ν on Y. It is clear that every fuzzy continuous mapping is a somewhat fuzzy semicontinuous mapping. But the converse is not true in general.

A mapping $f : X \to Y$ is called *somewhat fuzzy semiopen* if there exists a fuzzy semiopen set $\nu \neq 0_Y$ on Y such that $\nu \leq f(\mu) \neq 0_Y$ for any fuzzy open set μ on X. Every fuzzy open mapping is a somewhat fuzzy semiopen mapping but the converse is not true in general [6].

3. Somewhat fuzzy irresolute continuous mappings

In this section, we introduce a somewhat fuzzy irresolute continuous mapping and a somewhat fuzzy irresolute semiopen mapping which are stronger than a somewhat fuzzy semicontinuous mapping and a somewhat fuzzy semiopen mapping respectively. And we characterize a somewhat fuzzy irresolute continuous mapping and a somewhat fuzzy irresolute open mapping.

Definition 3.1. A mapping $f : X \to Y$ is called somewhat fuzzy irresolute continuous if there exists a fuzzy semiopen set $\mu \neq 0_X$ on X such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any fuzzy semiopen set $\nu \neq 0_Y$ on Y.

It is clear that every fuzzy irresolute continuous mapping is a somewhat fuzzy irresolute continuous mapping. And every somewhat fuzzy irresolute continuous mapping is a fuzzy semicontinuous mapping. Also, every fuzzy semicontinuous mapping is a somewhat fuzzy semicontinuous mapping from the above definitions. But the converses are not true in general as the following examples show.

Example 3.2. Let μ_1, μ_2 and μ_3 be fuzzy sets on $X = \{a, b, c\}$ and let ν_1, ν_2 and ν_3 be fuzzy sets on $Y = \{x, y, z\}$ with

$$\mu_1(a) = 0.1, \mu_1(b) = 0.1, \mu_1(c) = 0.1,$$
$$\mu_2(a) = 0.2, \mu_2(b) = 0.2, \mu_2(c) = 0.2,$$
$$\mu_3(a) = 0.5, \mu_3(b) = 0.5, \mu_3(c) = 0.5 \text{ and}$$
$$\nu_1(x) = 0.3, \nu_1(y) = 0.0, \nu_1(z) = 0.3,$$
$$\nu_2(x) = 0.5, \nu_2(y) = 0.5, \nu_2(z) = 0.5,$$
$$\nu_3(x) = 0.5, \nu_3(y) = 0.2, \nu_3(z) = 0.5.$$

Let $\tau = \{0_X, \mu_1, \mu_1^c, 1_X\}$ be fuzzy topology on X and let $\tau^* = \{0_Y, \nu_1, \nu_2, 1_Y\}$ be fuzzy topology on Y. Consider the mapping $f : (X, \tau) \to (Y, \tau^*)$ defined by $f(a) = y$, $f(b) = y$ and $f(c) = y$. Then we have $f^{-1}(\nu_1) = 0_X$, $\mu_1 \leq f^{-1}(\nu_2) = \mu_3$ and $\mu_1 \leq f^{-1}(\nu_3) = \mu_2$. Since μ_1 is a fuzzy semiopen set on (X, τ), f is somewhat fuzzy irresolute continuous. But $f^{-1}(\nu_2) = \mu_3$ and $f^{-1}(\nu_3) = \mu_2$ are not fuzzy semiopen sets on (X, τ). Hence f is not a fuzzy irresolute continuous mapping. □

Example 3.3. Let μ_1, μ_2 and μ_3 be fuzzy sets on $X = \{a, b, c\}$ and let ν_1, ν_2 and ν_3 be fuzzy sets on $Y = \{x, y, z\}$ with

$$\mu_1(a) = 0.2, \mu_1(b) = 0.2, \mu_1(c) = 0.2,$$
$$\mu_2(a) = 0.3, \mu_2(b) = 0.3, \mu_2(c) = 0.3,$$
$$\mu_3(a) = 0.5, \mu_3(b) = 0.5, \mu_3(c) = 0.5 \text{ and}$$
$$\nu_1(x) = 0.4, \nu_1(y) = 0.0, \nu_1(z) = 0.4,$$
$$\nu_2(x) = 0.5, \nu_2(y) = 0.5, \nu_2(z) = 0.5,$$
$$\nu_3(x) = 0.5, \nu_3(y) = 0.2, \nu_3(z) = 0.5.$$

Let $\tau = \{0_X, \mu_2, \mu_1^c, 1_X\}$ be fuzzy topology on X and let $\tau^* = \{0_Y, \nu_1, \nu_2, 1_Y\}$ be fuzzy topology on Y. Consider the mapping $f : (X, \tau) \to (Y, \tau^*)$ defined by $f(a) = y$, $f(b) = y$ and $f(c) = y$. Since $f^{-1}(\nu_1) = 0_X$ and $f^{-1}(\nu_2) = \mu_3$ are fuzzy semiopen sets on (X, τ), f is fuzzy semicontinuous. But the inverse image $f^{-1}(\nu_3) = \mu_1$ of a fuzzy semiopen set ν_3 on (Y, τ^*) is not fuzzy semiopen on (X, τ). Hence f is not a fuzzy somewhat irresolute continuous mapping. □

Example 3.4. Let μ_1, μ_2 and μ_3 be fuzzy sets on $X = \{a, b, c\}$ with

$$\mu_1(a) = 0.1, \mu_1(b) = 0.1, \mu_1(c) = 0.1,$$
$$\mu_2(a) = 0.3, \mu_2(b) = 0.3, \mu_2(c) = 0.3 \text{ and}$$
$$\mu_3(a) = 0.8, \mu_3(b) = 0.8, \mu_3(c) = 0.8$$

and let $\tau = \{0_X, \mu_1, \mu_2, 1_X\}$ and $\tau^* = \{0_X, \mu_3, 1_X\}$be fuzzy topologies on X. Consider the identity mapping $i_X : (X, \tau) \to (X, \tau^*)$. We have $\mu_2^c \le i_X^{-1}(\mu_3) = \mu_3$. Since μ_2^c is a fuzzy semiopen set on (X, τ), i_X is somewhat fuzzy semicontinuous. But $i_X^{-1}(\mu_3) = \mu_3$ is not a fuzzy semiopen set on (X, τ). Hence i_X is not a fuzzy semicontinuous mapping. □

Theorem 3.5. Let $f : X \to Y$ be a mapping. Then the following are equivalent:

(1) f is somewhat fuzzy irresolute continuous.

(2) If ν is a fuzzy semiclosed set of Y such that $f^{-1}(\nu) \ne 1_X$, then there exists a fuzzy semiclosed set $\mu \ne 1_X$ of X such that $f^{-1}(\nu) \le \mu$.

(3) If μ is a fuzzy semidense set on X, then $f(\mu)$ is a fuzzy semidense set on Y.

Proof. (1) implies (2): Let ν be a fuzzy semiclosed set on Y such that $f^{-1}(\nu) \ne 1_X$. Then ν^c is a fuzzy semiopen set on Y and $f^{-1}(\nu^c) = (f^{-1}(\nu))^c \ne 0_X$. Since f is somewhat fuzzy irresolute continuous, there exists a fuzzy semiopen set $\lambda \ne 0_X$ on X such that $\lambda \le f^{-1}(\nu^c)$. Let $\mu = \lambda^c$. Then $\mu \ne 1_X$ is fuzzy semiclosed such that $f^{-1}(\nu) = 1 - f^{-1}(\nu^c) \le 1 - \lambda = \lambda^c = \mu$.

(2) implies (3): Let μ be a fuzzy semidense set on X and suppose that $f(\mu)$ is not fuzzy semidense on Y. Then there exists a fuzzy semiclosed set ν on Y such that $f(\mu) < \nu < 1$. Since $\nu < 1$ and $f^{-1}(\nu) \ne 1_X$, there exists a fuzzy semiclosed set $\delta \ne 1_X$ such that $\mu \le f^{-1}(f(\mu)) < f^{-1}(\nu) \le \delta$. This contradicts to the assumption that μ is a fuzzy semidense set on X. Hence $f(\mu)$ is a fuzzy semidense set on Y.

(3) implies (1): Let $\nu \ne 0_Y$ be a fuzzy semiopen set on Y and let $f^{-1}(\nu) \ne 0_X$. Suppose that there exists no fuzzy semiopen $\mu \ne 0_X$ on X such that $\mu \le f^{-1}(\nu)$. Then $(f^{-1}(\nu))^c$ is a fuzzy set on X such that there is no fuzzy semiclosed set δ on X with $(f^{-1}(\nu))^c < \delta < 1$. In fact, if there exists a fuzzy semiopen set δ^c such that $\delta^c \le f^{-1}(\nu)$, then it is a contradiction. So $(f^{-1}(\nu))^c$ is a fuzzy semidense set on X. Hence $f((f^{-1}(\nu))^c)$ is a fuzzy semidense set on Y. But $f((f^{-1}(\nu))^c) = f(f^{-1}(\nu^c)) \ne \nu^c < 1$. This is a contradiction to the fact that $f((f^{-1}(\nu))^c)$ is fuzzy semidense on Y. Hence there exists a semiopen set $\mu \ne 0_X$ on X such that $\mu \le f^{-1}(\nu)$. Consequently, f is somewhat fuzzy irresolute continuous. □

Theorem 3.6. Let X_1 be product related to X_2 and let Y_1 be product related to Y_2. Then the product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of somewhat fuzzy irresolute continuous mappings $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ is also somewhat fuzzy irresolute continuous.

Proof. Let $\lambda = \bigvee_{i,j}(\mu_i \times \nu_j)$ be a fuzzy semiopen set on $Y_1 \times Y_2$ where $\mu_i \ne 0_{Y_1}$ and $\nu_j \ne 0_{Y_2}$ are fuzzy semiopen sets on Y_1 and Y_2 respectively. Then $(f_1 \times f_2)^{-1}(\lambda) = \bigvee_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j))$. Since f_1 is somewhat fuzzy irresolute continuous, there exists a fuzzy semiopen set $\delta_i \ne 0_{X_1}$ such that $\delta_i \le f_1^{-1}(\mu_i) \ne 0_{X_1}$. And, since f_2 is somewhat fuzzy irresolute continuous, there exists a fuzzy semiopen set $\eta_j \ne 0_{X_2}$ such that $\eta_j \le f_2^{-1}(\nu_j) \ne 0_{X_2}$. Now $\delta_i \times \eta_j \le f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j) = (f_1 \times f_2)^{-1}(\mu_i \times \nu_j)$ and $\delta_i \times \eta_j \ne 0_{X_1 \times X_2}$ is a fuzzy semiopen set on $X_1 \times X_2$. Hence $\bigvee_{i,j}(\delta_i \times \eta_j) \ne 0_{X_1 \times X_2}$ is a fuzzy semiopen set on $X_1 \times X_2$ such that $\bigvee_{i,j}(\delta_i \times \eta_j) \le \bigvee_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j)) = (f_1 \times f_2)^{-1}(\bigvee_{i,j}(\mu_i \times \nu_j)) = (f_1 \times f_2)^{-1}(\lambda) \ne 0_{X_1 \times X_2}$. Therefore, $f_1 \times f_2$ is somewhat fuzzy irresolute continuous. □

Theorem 3.7. Let $f : X \to Y$ be a mapping. If the graph $g : X \to X \times Y$ of f is a somewhat fuzzy irresolute continuous mapping, then f is also somewhat fuzzy irresolute continuous.

Proof. Let ν be a fuzzy semiopen set on Y. Then $f^{-1}(\nu) = 1 \wedge f^{-1}(\nu) = g^{-1}(1 \times \nu)$. Since g is somewhat fuzzy irresolute continuous and $1 \times \nu$ is a fuzzy semiopen set on $X \times Y$, there exists a fuzzy semiopen set $\mu \ne 0_X$ on X such that $\mu \le g^{-1}(1 \times \nu) = f^{-1}(\nu) \ne 0_X$. Therefore, f is somewhat fuzzy irresolute continuous. □

Definition 3.8. A mapping $f : X \to Y$ is called somewhat fuzzy irresolute semiopen if there exists a fuzzy semiopen set $\nu \ne 0_Y$ on Y such that $\nu \le f(\mu) \ne 0_Y$ for any fuzzy semiopen set $\mu \ne 0_X$ on X.

It is clear that every fuzzy irresolute semiopen mapping is a somewhat fuzzy irresolute semiopen mapping. And every somewhat fuzzy irresolute semiopen mapping is a fuzzy semiopen mapping. Also, every fuzzy semiopen mapping is a somewhat fuzzy semiopen mapping from the above definitions. But the converses are not true in general as the following examples show.

Example 3.9. Let μ_1, μ_2 and μ_3 be fuzzy sets on $X = \{a, b, c\}$ and let ν_1, ν_2 and ν_3 be fuzzy sets on $Y = \{x, y, z\}$ with

$$\mu_1(a) = 0.1, \mu_1(b) = 0.1, \mu_1(c) = 0.1,$$
$$\mu_2(a) = 0.2, \mu_2(b) = 0.2, \mu_2(c) = 0.2,$$
$$\mu_3(a) = 0.5, \mu_3(b) = 0.5, \mu_3(c) = 0.5 \text{ and}$$
$$\nu_1(x) = 0.0, \nu_1(y) = 0.1, \nu_1(z) = 0.0,$$
$$\nu_2(x) = 0.0, \nu_2(y) = 0.2, \nu_2(z) = 0.0,$$
$$\nu_3(x) = 0.0, \nu_3(y) = 0.5, \nu_3(z) = 0.0.$$

Let $\tau = \{0_X, \mu_1, \mu_2, 1_X\}$ be fuzzy topologies on X and let $\tau^* = \{0_Y, \nu_1, \nu_1^c, 1_Y\}$ be fuzzy topologies on Y. Consider the mapping $f : (X, \tau) \to (Y, \tau^*)$ defined by $f(a) = y$, $f(b) = y$ and $f(c) = y$. Since $f(\mu_1) = \nu_1$, $\nu_1 \leq f(\mu_2) = \nu_2$ and $\nu_1 \leq f(\mu_3) = \nu_3$, f is somewhat fuzzy irresolute semiopen. But $f(\mu_2) = \nu_2$ and $f(\mu_3) = \nu_3$ are not fuzzy semiopen sets. Hence f is not a fuzzy irresolute semiopen mapping. □

Example 3.10. Let μ_1, μ_2 and μ_3 be fuzzy sets on $I = [0, 1]$ with

$$\mu_1(x) = 0.4, 0 \leq x \leq 1,$$
$$\mu_2(x) = 0.5, 0 \leq x \leq 1 \text{ and}$$
$$\mu_3(x) = 0.6, 0 \leq x \leq 1$$

Let $\tau = \{0_I, \mu_1, 1_I\}$ and $\tau^* = \{0_I, \mu_3, 1_I\}$ be fuzzy topologies on I. Consider the mapping $f : (I, \tau) \to (I, \tau^*)$ defined by $f(x) = 1 - x$, $0 \leq x \leq 1$. Since $f(\mu_1) = \mu_3$ is a fuzzy semiopen set on (I, τ^*), $f : (I, \tau) \to (I, \tau^*)$ is fuzzy semiopen. But μ_2 is a fuzzy semiopen set on (I, τ) and $f(\mu_2) = \mu_2$ is not a fuzzy semiopen set on $(I, \tau*)$. Also, there is no non-zero fuzzy semiopen set smaller than $f(\mu_2) = \mu_2 \neq 0$. Hence f is not a somewhat fuzzy irresolute semiopen mapping. □

Example 3.11. Let μ_1, μ_2 and μ_3 be fuzzy sets on $X = \{a, b, c\}$ with

$$\mu_1(a) = 0.1, \mu_1(b) = 0.1, \mu_1(c) = 0.1,$$
$$\mu_2(a) = 0.3, \mu_2(b) = 0.3, \mu_2(c) = 0.3 \text{ and}$$
$$\mu_3(a) = 0.8, \mu_3(b) = 0.8, \mu_3(c) = 0.8.$$

Let $\tau = \{0_X, \mu_3, 1_X\}$ and $\tau^* = \{0_X, \mu_1, \mu_2, 1_X\}$be fuzzy topologies on X. Consider the identity mapping $i_X : (X, \tau) \to (X, \tau^*)$. Then we have $\mu_2^c \leq i_X(\mu_3) = \mu_3$. Since μ_2^c is a fuzzy semiopen set on (X, τ), i_X is somewhat fuzzy semiopen. But $i_X(\mu_3) = \mu_3$ is not a fuzzy semiopen set on (X, τ^*). Hence i_X is not a fuzzy semiopen mapping. □

Theorem 3.12. Let $f : X \to Y$ be a bijection. Then the following are equivalent:

(1) f is somewhat fuzzy irresolute semiopen.

(2) If μ is a fuzzy semiclosed set on X such that $f(\mu) \neq 1_Y$, then there exists a fuzzy semiclosed set $\nu \neq 1_Y$ on Y such that $f(\mu) < \nu$.

Proof. (1) implies (2): Let μ be a fuzzy semiclosed set on X such that $f(\mu) \neq 1_Y$. Since f is bijective and μ^c is a fuzzy semiopen set on X, $f(\mu^c) = (f(\mu))^c \neq 0_Y$. And, since f is somewhat fuzzy irresolute semiopen, there exists a semiopen set $\delta \neq 0_Y$ on Y such that $\delta < f(\mu^c) = (f(\mu))^c$. Consequently, $f(\mu) < \delta^c = \nu \neq 1_Y$ and ν is a fuzzy semiclosed set on Y.

(2) implies (1): Let μ be a fuzzy semiopen set on X such that $f(\mu) \neq 0_Y$. Then μ^c is a fuzzy semiclosed set on X and $f(\mu^c) \neq 1_Y$. Hence there exists a fuzzy semiclosed set $\nu \neq 1_Y$ on Y such that $f(\mu^c) < \nu$. Since f is bijective, $f(\mu^c) = (f(\mu))^c < \nu$. Thus $\nu^c < f(\mu)$ and $\nu^c \neq 0_X$ is a fuzzy semiopen set on Y. Therefore, f is somewhat fuzzy irresolute semiopen. □

Theorem 3.13. Let $f : X \to Y$ be a surjection. Then the following are equivalent:

(1) f is somewhat fuzzy irresolute semiopen.

(2) If ν is a fuzzy semidense set on Y, then $f^{-1}(\nu)$ is a fuzzy semidense set on X.

Proof. (1) implies (2): Let ν be a fuzzy semidense set on Y. Suppose $f^{-1}(\nu)$ is not fuzzy semidense on X. Then there exists a fuzzy semiclosed set μ on X such that $f^{-1}(\nu) < \mu < 1$. Since f is somewhat fuzzy irresolute semiopen and μ^c is a fuzzy semiopen set on X, there exists a fuzzy semiopen set $\delta \neq 0_Y$ on Y such that $\delta \leq f(\text{ Int}\mu^c) \leq f(\mu^c)$. Since f is surjective, $\delta \leq f(\mu^c) < f(f^{-1}(\nu^c)) = \nu^c$. Thus there exists a semiclosed set δ^c on Y such that $\nu < \delta^c < 1$. This is a contradiction. Hence $f^{-1}(\nu)$ is fuzzy semidense on X.

(2) implies (1): Let μ be a fuzzy open set on X and $f(\mu) \neq 0_Y$. Suppose there exists no fuzzy semiopen $\nu \neq 0_Y$ on Y such that $\nu \leq f(\mu)$. Then $(f(\mu))^c$ is a fuzzy set on Y such that there exists no fuzzy semiclosed set δ on Y with $(f(\mu))^c < \delta < 1$. This means that $(f(\mu))^c$ is fuzzy semidense on Y. Thus $f^{-1}((f(\mu))^c)$ is fuzzy semidense on X. But $f^{-1}((f(\mu))^c) = (f^{-1}(f(\mu)))^c \leq \mu^c < 1$. This is a contradiction to the fact that $f^{-1}((f(\nu))^c$ is fuzzy

semidense on X. Hence there exists a semiopen set $\nu \neq 0_Y$ on Y such that $\nu \leq f(\mu)$. Therefore, f is somewhat fuzzy irresolute semiopen. □

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Young Bin Im
Faculty of General Education, Dongguk University, Seoul 100-715, Korea
Research Area: Fuzzy topology, Fuzzy matrix
E-mail : philpen@dongguk.edu

Joo Sung Lee
Department of Mathematics, Dongguk University, Seoul 100-715, Korea
Research Area: Fuzzy topology, Topological dynamics
E-mail : jsl@dongguk.edu

Yung Duk Cho
Faculty of General Education, Dongguk University, Seoul 100-715, Korea
Research Area: Fuzzy topology, Fuzzy algebra
E-mail : joyd@dongguk.edu