

# Calculation of the Reactor Impedance of a Planar-type Inductively Coupled Plasma Source

Deuk-Chul Kwon\*, Bong-Sam Jung\*\* and Nam-Sik Yoon†

**Abstract** – A two-dimensional nonlocal heating theory of planar-type inductively coupled plasma source has been previously reported with a filamentary antenna current model. However, such model yields an infinite value of electric field at the antenna position, resulting in the infinite self-inductance of the antenna. To overcome this problem, a surface current model of antenna should be adopted in the calculation of the electromagnetic fields. In the present study, the reactor impedance is calculated based on the surface current model and the dependence on various discharge parameters is studied. In addition, a simpler method is suggested and compared with the surface current calculation.

**Keywords:** ICP, Impedance, Plasma

## 1. Introduction

Inductively coupled plasma (ICP) sources are widely used in semiconductor fabrication processing because they allow high density plasma with good uniformity to be easily obtained under low pressure without the need for an external magnetic field [1-8].

In some previous theoretical studies of electron heating mechanism, Yoon *et al.* [4] calculated the plasma impedance by using the filamentary type antenna model; however, the electric field in such model was infinite at the antenna position. Consequently, it was not possible to calculate the impedance for a whole reactor because of the corresponding singularity, except for the plasma impedance, which could be calculated through an indirect method. Therefore, a surface current model was adopted to calculate the electric field more accurately [9].

Meanwhile, a simple method of antenna reactance calculation of rectangular ICP with a filamentary current model which can exclude the infinity of electric field was reported in Ref. [10].

In this study, we first calculate the impedance for a whole reactor (including both the antenna and plasma regions) using the surface current model for a cylindrical planar-type ICP reactor. A simpler method using the filamentary current model is also applied. The results of both are then compared.

This paper is organized as follows. In section 2, the wave equation, antenna model, and the analytic solution are given. Some of the results of specific applications under various conditions are presented in section 3. Finally,

the conclusion is given in section 4.

## 2. Description of the Calculation

A schematic diagram of the cylindrical planar-type ICP source is shown in Fig. 1(a), where  $R$  is the chamber radius,  $L_s$  is the shielding cap height, and  $L_p$  is the plasma length. The antenna, which has a cross-sectional area of  $\Delta s = \Delta r_c \Delta z_c$ , is placed at  $(r_c, z_c)$ , where  $\Delta r_c$  and  $\Delta z_c$  are the width and height of the antenna, respectively. For convenience, the entire chamber space is divided into two parts: the antenna region, which is surrounded by the shielding cap and the plasma surface, and the plasma region.

Assuming  $\theta$ -symmetry, the wave equation describing the inductive electric field can be shown as the following equation:

$$\frac{\partial^2 E_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial E_\theta}{\partial r} - \frac{E_\theta}{r^2} + \frac{\partial^2 E_\theta}{\partial z^2} + \kappa^2 E_\theta = -\frac{4\pi\omega}{c^2} i(J_c + J_p) \quad (1)$$

Here  $\omega$  is the excitation frequency,  $c$  is the speed of light, and  $\kappa = \omega/c$ .  $J_c$  and  $J_p$  represent the antenna and plasma current densities, respectively. The components of the magnetic field can be calculated from the electric field by the following:

$$B_r(r, z) = \frac{i}{\kappa} \frac{\partial E_\theta(r, z)}{\partial z} \quad (2)$$

$$B_z(r, z) = -\frac{i}{\kappa} \frac{1}{r} \frac{\partial}{\partial r} [r E_\theta(r, z)] \quad (3)$$

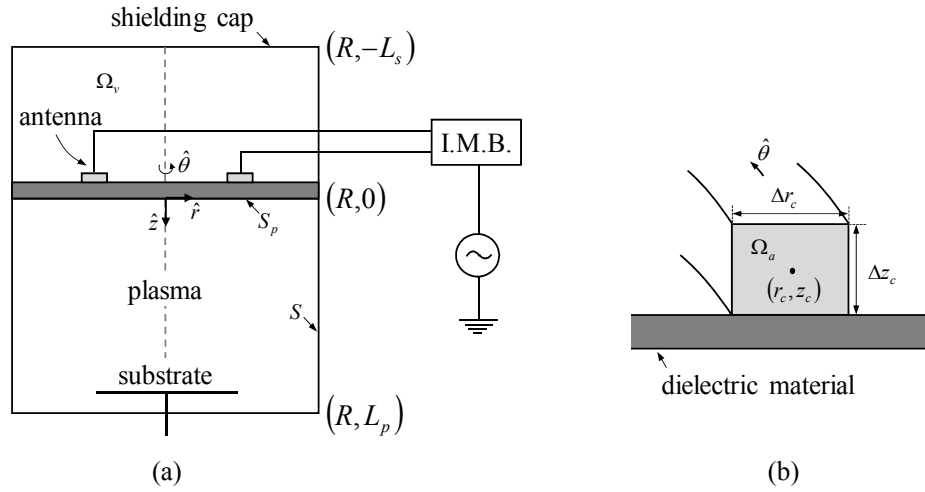
Yoon *et al.* [4] obtained the electromagnetic fields by expanding the Fourier-Bessel series, including the

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**Fig. 1.** A schematic diagram of a planar-type cylindrical ICP reactor and the adopted coordinate system in this calculation

anomalous skin effect, and the plasma impedance is calculated by the field definition of the impedance [4] using a filamentary current model:

$$J_c(r, z) = I_c \delta(r - r_c) \delta(z - z_c) \quad (4)$$

where  $I_c$  is the antenna current and  $\delta(z)$  is the Dirac delta function. However, this calculation yields an infinite value of the electric field at the antenna position, thus it cannot be used in the calculation of antenna reactance.

Instead, we can use the following surface current model:

$$J_c(r, z) = \frac{I_c}{4} \frac{S(r_c, \Delta r_c)}{\Delta r_c} \left\{ \delta \left( z - \left( z_c + \frac{\Delta z_c}{2} \right) \right) + \delta \left( z - \left( z_c - \frac{\Delta z_c}{2} \right) \right) \right\} + \frac{I_c}{4} \frac{S(z_c, \Delta z_c)}{\Delta z_c} \left\{ \delta \left( r - \left( r_c + \frac{\Delta r_c}{2} \right) \right) + \delta \left( r - \left( r_c - \frac{\Delta r_c}{2} \right) \right) \right\} \quad (5)$$

where

$$S(a, b) \equiv \begin{cases} 1. & (a - \frac{b}{2} \leq z \leq a + \frac{b}{2}), \\ 0. & (\text{otherwise}). \end{cases}$$

In the antenna region, the Fourier-Bessel series of the electric field becomes [9]:

$$E_\theta(r, z) = \frac{4\pi k i}{cR^2} \sum_{m=1}^{\infty} \frac{J_1(p_m r)}{J_2^2(\alpha_{1,m}) \beta_m^2} \sum_j I_j e_{mj}, \quad (6)$$

where  $J_1$  is the first-order Bessel function,  $p_m \equiv \alpha_{1,m}/R$ , and  $\alpha_{1,m}$  is the  $m$ th zero of  $J_1$ .

$$e_{mj} = \psi_{mj} (\zeta_{mj} - \zeta_{mj}) + B_{c,m} \zeta_{mj}, \quad (7)$$

$$\psi_{mj} = A_{c,m} \cosh \left( \beta_m \frac{\Delta z_c}{2} \right) + B_{c,m} \cosh \left( \beta_m \frac{\Delta z_c}{2} \right), \quad (8)$$

$$\zeta_{mj} = \frac{\sinh[\beta_m(z + L_s)] \sinh[\beta_m(z_c + L_s)]}{\sinh[2\beta_m L_s] \left[ 1 + 2 \frac{\beta_m}{L_p} \cot(\beta_m L_s) S_m \right]}, \quad (9)$$

$$\xi_{mj} = \begin{cases} \frac{\sinh[\beta_m(z + L_s)] \cosh(\beta_m z_c)}{\cosh(L_s)} & \left( z \leq z_c - \frac{\Delta z_c}{2} \right) \\ \frac{\sinh[\beta_m(z_c + L_s)] \cosh(\beta_m z)}{\cosh(L_s)} & \left( z \geq z_c + \frac{\Delta z_c}{2} \right) \\ \cosh(\beta_m z_c) \cosh(\beta_m z) \tanh(\beta_m L_s) \\ + \frac{\sinh[\beta_m(z + z_c)]}{2} \\ - \frac{\cosh[\beta_m(z - z_c)]}{2} \tanh\left(\beta_m \frac{\Delta z_c}{2}\right) & (\text{otherwise}), \end{cases} \quad (10)$$

$$\varsigma_{mj} = \begin{cases} \frac{1}{2} - \frac{\cosh[\beta_m(z - z_c)]}{2 \cosh\left[\beta_m \frac{\Delta z_c}{2}\right]} & \left( \text{if } z_c - \frac{\Delta z_c}{2} \leq z \leq z_c + \frac{\Delta z_c}{2} \right) \\ 0 & (\text{otherwise}). \end{cases} \quad (11)$$

According to the Poynting theorem for harmonic fields, impedance can be determined by using field quantities [4, 7]. The complex energy equation for harmonic fields is as follows:

$$\frac{1}{2} \int_{\Omega} \mathbf{J}^* \cdot \mathbf{E} d\tau + 2i\omega \int_{\Omega} (w_e - w_m) d\tau + \oint_S \mathbf{S} \cdot \mathbf{n} da = 0 \quad (12)$$

where  $\Omega$  and  $S$  are the volume and surface of the reactor, respectively;  $\mathbf{J}$  is the current density;  $\mathbf{E}$  is the electric field;

$w_e$  and  $w_m$  are the electric and the magnetic field energy density, respectively; and  $\mathbf{S}$  is the Poynting vector. This complex energy conservation equation can be applied to each region of the antenna and plasma, and the result is obtained as follows:

$$-\frac{1}{2} \int_{\Omega_a} \mathbf{J}^* \cdot \mathbf{E} d\tau = \frac{1}{2} \int_{\Omega_v} \mathbf{J}^* \cdot \mathbf{E} d\tau + 2i\omega \int_{\Omega_v} (w_e - w_m) d\tau - \oint_{S_p} \mathbf{S} \cdot \mathbf{n} da \quad (13)$$

where  $\Omega_a$  and  $\Omega_v$  are the volume of the antenna and vacuum, respectively, and  $S_p$  is the interface between the plasma and the antenna regions. The reactor impedance can then be expressed as follows:

$$Z_{rec} = Z_p - iX_a \equiv -\frac{1}{|I_c|^2} \int_{\Omega_a} \mathbf{J}_c^* \cdot \mathbf{E} d\tau \quad (14)$$

where  $Z_p$  is the impedance of the plasma region and  $X_a$  is the reactance of the antenna region. The resistance of the antenna coil is neglected in this calculation by assuming a perfect conductor coil.

If Eqs. (5) and (6) are substituted into Eq. (14), we obtain the following:

$$Z_{rec} = -\frac{2\pi^2 \kappa i}{cR^2} \sum_{m=1}^{\infty} \frac{1}{J_2^2(\alpha_m) \beta_m^3} \sum_j (A_{c,m} U_{mj} + B_{c,m} V_{mj}) \quad (15)$$

with

$$U_{mj} = e_{mj} \left[ \beta_m \left( z_c + \frac{\Delta z_c}{2} \right) \right] + e_{mj} \left[ \beta_m \left( z_c - \frac{\Delta z_c}{2} \right) \right] \quad (16)$$

$$V_{mj} = \beta_m \int_{z_c - \frac{\Delta z_c}{2}}^{z_c + \frac{\Delta z_c}{2}} e_{mj}(\beta_m z) dz. \quad (17)$$

Substituting Eq. (7) into Eqs. (16) and (17) will result in the following:

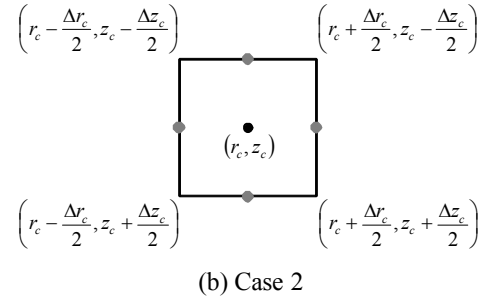
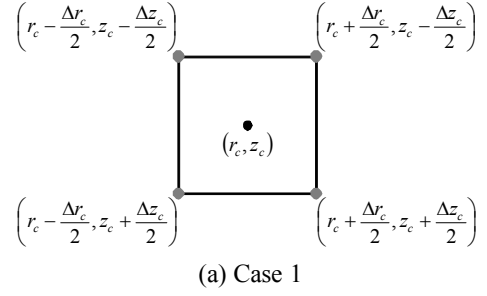
$$U_{mj} = 2 \frac{\sinh[\beta_m(z_c + L_s)]}{\cosh(\beta_m L_s)} \cosh^2 \left( \beta_m \frac{\Delta z_c}{2} \right) \times a_{mj} \left\{ \cosh(\beta_m z_c) - \frac{\sinh[\beta_m(z_c + L_s)]}{\sinh(\beta_m L_s)} \hat{b}_{0,m} \right\}, \quad (18)$$

$$V_{mj} = \frac{1}{2} \sinh(\beta_m \Delta z_c) a_{mj} \times \left[ 2 \cosh^2(\beta_m z_c) \tanh(\beta_m L_s) + \sinh(2\beta_m z_c) - \tanh \left( \beta_m \frac{\Delta z_c}{2} \right) \right] - \frac{1}{2} \sinh(\beta_m \Delta z_c) a_{mj} \frac{4 \sinh^2[\beta_m(z_c + L_s)]}{\sinh(2\beta_m L_s)} \hat{b}_{0,m} + \left[ \beta_m \frac{\Delta z_c}{2} - \tanh \left( \beta_m \frac{\Delta z_c}{2} \right) \right] B_{c,m}, \quad (19)$$

where

$$a_{mj} = A_{c,m} + B_{c,m} \tanh \left( \beta_m \frac{\Delta z_c}{2} \right), \quad (20)$$

$$\hat{b}_{0,m} = \left[ 1 + 2 \frac{\beta_m}{L_p} \cosh(\beta_m L_s) S_m \right]^{-1}. \quad (21)$$



**Fig. 2.** Two types of filamentary current model: (a) corner type (case 1); (b) side type (case 2)

The final form of Eq. (15), which is based on the surface current model, is very accurate; however, it is also very complicated and impractical to apply in a complex antenna configuration.

A simpler approximation, which was adopted in the rectangular ICP reactor [10], can be used. In this filamentary approximation, the antenna current is assumed to be divided into several parts of the filamentary current. Although the electric field at these filamentary coil positions is still infinite, we can calculate the impedance at a point shifted to a distance, which is about the size of a real antenna:

$$Z_{rec} \cong \frac{-I_c \cdot 2\pi(r_c \pm \Delta r_c/2) \cdot E(r_c \pm \Delta r_c/2, z_c \pm \Delta z_c/2)}{|I_c|^2} \quad (22)$$

The approximated form of Eq. (22) is then applied to the relatively simple expression of the electric field with filament antenna, which is given by the following:

$$E_\theta = \frac{4\pi}{c} \kappa i \sum_{m=1}^{\infty} J_1(p_m r) \epsilon_m \quad (23)$$

where

$$\varepsilon_m = \left[ \frac{j_{m0}^c}{2\beta_m^2} + \sum_{n=1}^{\infty} \frac{j_{mn}^c \cosh(k_n z)}{\beta_m^2 + k_n^2} \right] - \frac{L_s \cosh(\beta_m z)}{\beta_m \sinh(2\beta_m L_s)} j_{c,m}^{ch} - \frac{c}{4\pi} \frac{b_{0,m} \sinh[\beta_m(z + L_s)]}{\beta_m \cosh(\beta_m L_s)}, \quad (24)$$

$$j_{mn}^c = \sum_j \frac{4r_{c,j}}{L_s R^2 J_2^2(\alpha_{1,m})} I_{c,j} J_1(p_m r_{c,j}) \cos(q_n z_{c,j}), \quad (25)$$

$$j_{c,m}^{ch} = 2 \frac{\beta_m}{L_s} \sinh(\beta_m L_s) \left[ \sum_{n=1}^{\infty} (-1)^n \frac{j_{mn}^c}{\beta_m^2 + k_n^2} + \frac{j_{m0}^c}{2\beta_m^2} \right], \quad (26)$$

$$b_{0,m} = \frac{2\pi L_s}{c} j_{c,m}^{(sh,L_s)} \left[ \sinh(\beta_m L_s) + 2 \frac{\beta_m}{L_p} \cosh(\beta_m L_s) S_m \right]^{-1}, \quad (27)$$

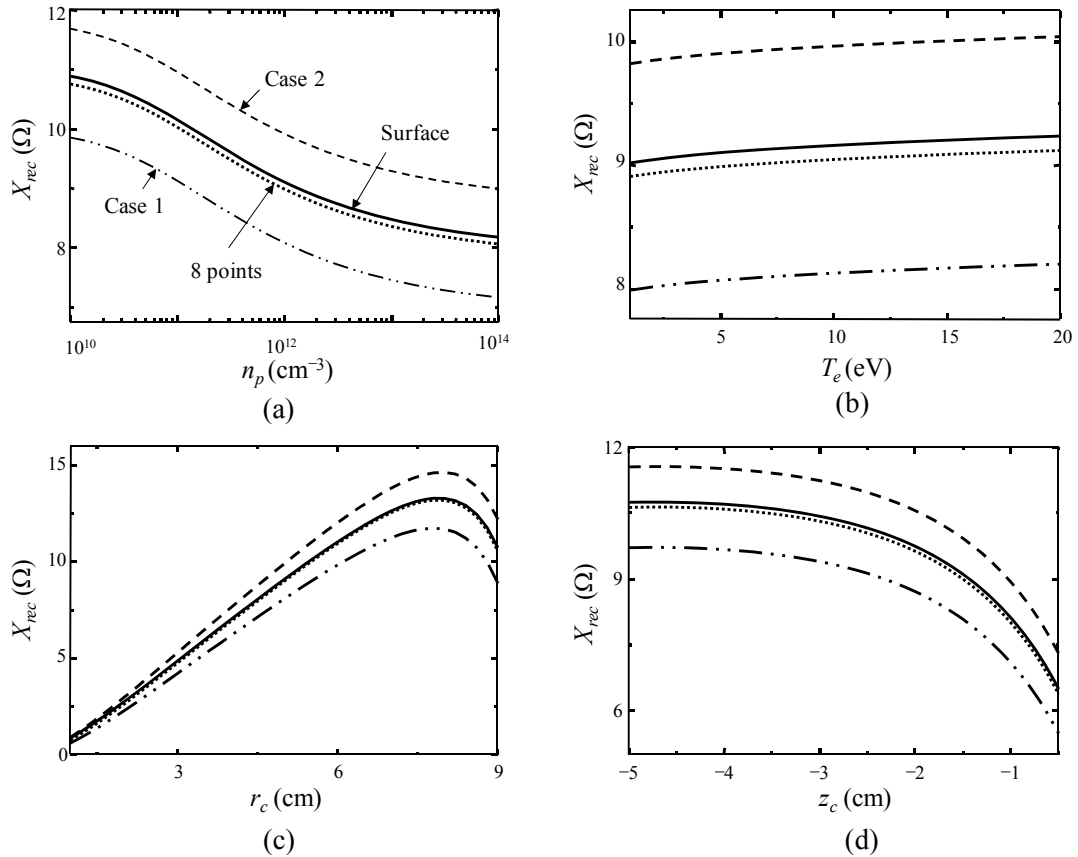
$$j_{c,m}^{(sh,L_s)} \equiv \sum_j \frac{4}{L_s R^2 J_2^2(\alpha_m)} I_{c,j} J_1(p_m r_{c,j}) \sinh[\beta_m(L_s + z_{c,j})]. \quad (28)$$

### 3. Numerical Results and Discussion

In this section, the two forms of Eqs. (14) and (22) are compared numerically. The default parameters used in these calculations are  $R = 10$  cm;  $L_p = 10$  cm;  $L_s = 10$  cm;  $r_c = 5$  cm;  $z_c = -1.5$  cm;  $n_p = 10^{11}$  cm<sup>-3</sup>;  $T_e = 5$  eV;  $P_{RF} = 500$  W;  $\omega/2\pi = 13.56$  MHz; and  $\nu = 0$  s<sup>-1</sup>. The antenna coil is made of a rectangular wire loop with a cross-sectional area of  $1 \times 1$  cm<sup>2</sup>.

Two sets of the filamentary current are used in Eq. (22), as shown in Fig. 2. The first set is a corner current type (case 1) and the other is a side current type (case 2).

Fig. 3 shows the numerical results of the reactor reactance. Solid lines denote the surface current results, while the dashed lines are the filamentary current results. The dependence of the reactance on the various discharge parameters of electron density, temperature, and antenna position are very similar; meanwhile, their absolute values show some differences. The value in case 1 is slightly lower than the surface current results, while case 2 yields higher values. The higher value in case 2 than case 1 is due to the position of each filamentary current being closer to the center, thus the effective area becomes large. Consequently, the effective total current density is higher in case 2 than in case 1, hence the reactance in each case



**Fig. 3.** Dependence of reactance, calculated by two methods, on various discharge parameters: (a) plasma density; (b) electron temperature; (c) antenna radius; (d) axial position of the antenna.

shows the same tendency.

Interestingly, the average of the eight points (the sum of cases 1 and 2) is close to the result of the surface current density model. We, therefore, conclude that this eight-point approximation is a good way to calculate the reactance in this antenna configuration.

Since the filamentary current model is a much easier way to the current configuration, this method will be more powerful in more complicated antenna shapes, especially in some three-dimensional calculation.

#### 4. Conclusion

Two methods were used to calculate the reactor impedance of a planar-type cylindrical ICP source. The first method, which models the antenna current as surface current, is relatively more exact but complicated. The second method, which uses some filamentary current models, is a simpler process. Furthermore, the adequate filamentary current results are in a fine degree of agreement with the more exact results of the surface current model.

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