64 *Journal of Electrical Engineering & Technology Vol. 7, No. 1, pp. 64~68, 2012*
http://dx.doi.org/10.5370/JEET.2012.7.1.64

Shape Optimization for Interior Permanent Magnet Motor based on Hybrid Algorithm

Woo-Gyong Yim†, Kwang-Ok An, Jang-Ho Seo*, Min-Jae Kim* and Hyun-Kyo Jung***

Abstract – In this paper, a design method for minimizing the cogging torque of an Interior Permanent Magnet Motor (IPM) is proposed based on a hybrid algorithm. The suggested optimization algorithm is based on a combination of the Response Surface Method (RSM) and Simplex Method. The results show that the proposed method provides improved characteristics compared to the conventional methods, such as a shorter calculation time and the acquisition of a more correct solution.

Keywords: Simplex method, Response Surface Method (RSM), Hybrid optimization, Notch

1. Introduction

An IPM generates a cogging torque, because it uses a permanent magnet. This induces torque ripple, which results in mechanical noise and vibration. Therefore, the development of methods to reduce the cogging torque is important, and a number of studies on this issue have been carried out. Among the many methods that have been developed recently, the use of a notch has attracted attention as one of the most effective methods [1, 2].

To design the shape of the notch, an optimization algorithm is needed. In this paper, the RSM and Simplex Method are used. The RSM, which is a global approximation method used in statistics, is widely used to find the relationship between the design variables and response result. By means of the RSM, the relationship between the design variables and response result can be quickly approximated to obtain the response surface function. However, despite the fast process of obtaining the optimum solution, it is relatively incorrect, compared to the actual optimum value. Meanwhile, the Simplex Method is one of the deterministic methods [3]. This method has the advantage of optimizing the complex function without its derivatives. However, it is likely to induce an error in determining the optimum peak point, because its dependence on the initial value used for the operation is considerable. Therefore, in this paper, a combination of the Simplex Method and the Shaking technique is used to find the accurate optimum solution. However, this method has the disadvantage of taking an excessive amount of time for the calculation [4].

Therefore, in this paper, a hybrid algorithm using both the RSM and Simplex Method is proposed to solve the problems of each method. By using this method, the time taken for the calculation was reduced compared to that when only the Simplex Method was used, and a more correct solution was obtained than the one calculated by using only the RSM. To confirm the performance of this method, various test functions were tried.

Finally, the optimized shape design for a 400[W] class IPM was proposed by using the hybrid algorithm. The method's usefulness was confirmed by comparison with conventional methods.

† Corresponding Author: Dept. of Electrical Engineering and Computer Science, Seoul Nat'l University, Korea. (yimwg@naver.com)
* Dept. of Electrical Engineering and Computer Science, Seoul Nat'l University, Korea. (jangho78@elecmech.snu.ac.kr, haemish@hanmail.net, hkjung@snu.ac.kr)
** National Rehabilitation Center, Korea. (anko04@korea.kr)
Received: August 14, 2010; Accepted: December 5, 2011

2. Proposed Optimization Algorithm

2.1 Conventional methods

The RSM is composed of two processes. First, the Design of Experiment (DOE) is carried out to find the sampling points. Usually, the Central Composite Design (CCD) is used for this purpose. Next, the least squares method is applied to make the response surface [5].

The Simplex Method starts by making the initial simplex, which consists of (n+1) points in n-dimensions. Next, after calculating the vertices of the simplex, the worst value point is replaced with a better point. The replacement process is made of five operations, viz. the reflection, expansion, contraction, multi-contraction, and shaking techniques. These five operations are shown in Fig. 1 [3, 4].

2.2 Proposed hybrid algorithm

The main advantage afforded by the RSM is its fast calculation time in obtaining the optimum point, but there is a trade-off in that the solution is relatively inaccurate, because it is only an approximation. By contrast, the Simplex Method can be applied to obtain an accurate solution, but this method takes much more time than the RSM.

To combine the advantages of these two methods, we

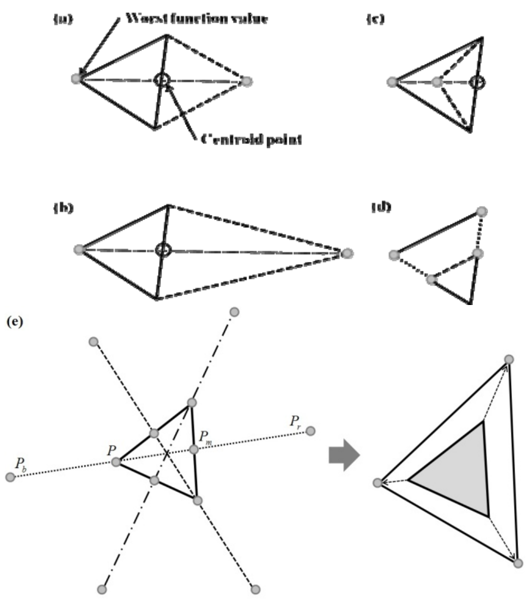

Fig. 1. Basic operations of simplex method: (a) Reflection, (b) Expansion; (c) Contraction; (d) Multi-contraction, (e) Shaking techniques.

propose a hybrid algorithm, which uses both the RMS and the Simplex Method to produce an improved algorithm procedure. The proposed procedure is as follows:

- **Step 1** : Set the objective function, design variables and constraint condition.
- **Step 2** : Select several experiment points (sampling points) using the Central Composite Design in the analysis region (constraint condition), and analyze them using finite element analysis.
- **Step 3** : Create the Response surface (Regression function) using the Least Squares Method. (R^2(Coefficient of Determination) $\geq$ 70 %)
- **Step 4** : Find the Response surface's peak using the Conjugate Gradient Method.
- **Step 5** : Reduce the Analysis region by 50% around the peak point. This region is a new analysis region.
- **Step 6** : Find the optimum solution using the Simplex method in the new analysis region.
- **Step 7** : If the optimum point is found to be in the boundary line of the region, go back to Step 5 and conduct the procedure again.

The overall flow chart of the hybrid algorithm is shown in Fig. 2.

The proposed algorithm is expected to find the correct optimum solution, while reducing the time needed for the calculation. For this purpose, the hybrid algorithm first creates the response surface in order to roughly identify the location of the solution and then reduce the area to 50% of the original analysis region, using the peak point as the center. This reduction in the size of the region is aimed at removing the unnecessary time taken for calculation, by taking into consideration the fact that the solution is likely to be found within the reduced region in most cases.

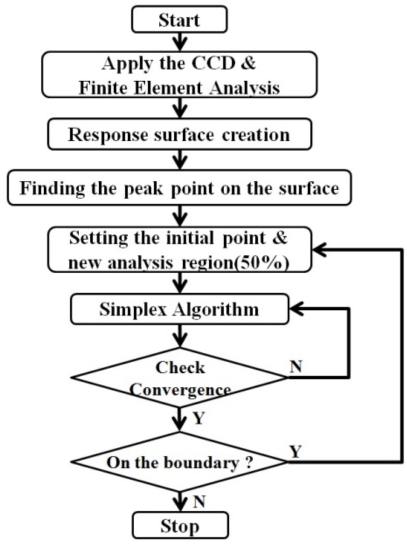

Fig. 2. Flow chart of hybrid algorithm.

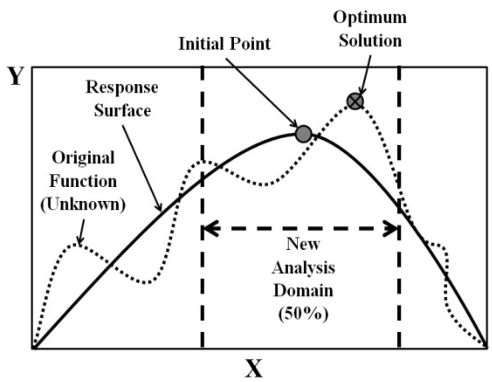

Fig. 3. A schematic illustration of the hybrid algorithm.

This performance of the hybrid algorithm is depicted in Fig. 3.

However, it is still possible that the solution does not lie within the reduced region, in which case the response surface fails to approximate the objective function. In this case, the optimum solution would be expected to be found in the boundary line (error optimum solution) and the hybrid algorithm sets the newly 50% reduced region using the error optimum solution as the center (therefore, being reduced to 25% of the original analysis region) and applies the Simplex Method. This process is shown in Fig. 4.

3. Numerical Test and Results

To compare the performance of the proposed algorithm with conventional methods such as RSM, Simplex Method and Genetic Algorithm(GA), test function shown in Fig. 5 was used. The formula of the test function is as follows.

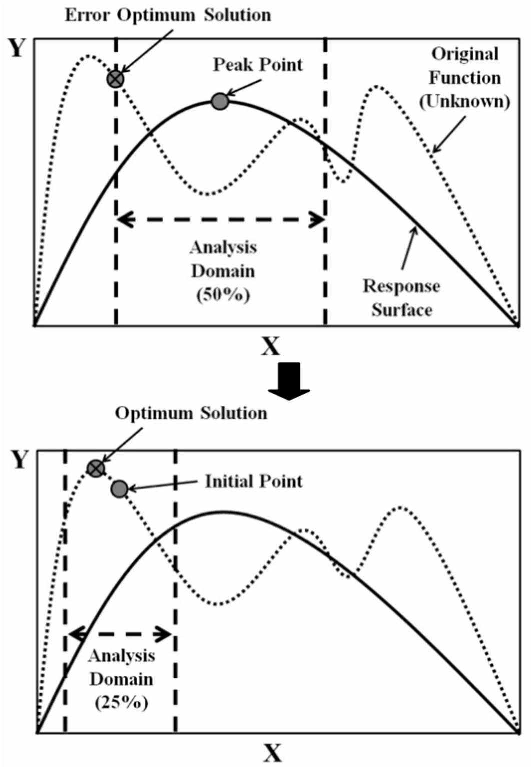

Fig. 4. Exceptional case of hybrid algorithm.

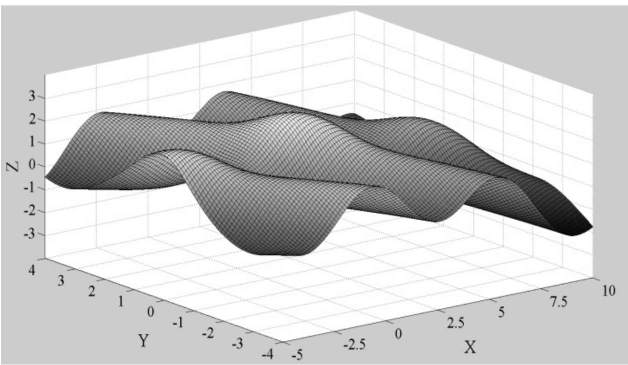

Fig. 5. A test function.

$$Z = \sin(x+y) + 1 - \frac{x^2}{40} + e^{-y^2} \quad . \qquad (1)$$
$$-5 \le x \le 10, -4 \le y \le 4$$

The evaluation of the hybrid algorithm was conducted by comparing the accuracy of the solutions it provides to those given by the RSM method and the number of calls to that required by the Simplex method. For each category, GA and GA + RSM were included as a reference.

From Table 1, it can be seen that the z value is increased for the hybrid algorithm compared with that of the RSM method. The increased z value indicates that the accuracy of the solution is improved. Meanwhile, in Table 2, the number of calls for the hybrid algorithm is decreased by 73 % compared to that for the Simplex Method.

Table 1. Optimum solution comparison of test function

	(X, Y)	Z
RSM	(2.5, 0)	2.44
GA	(1.43, 0.06)	2.94
GA + RSM	(1.43, 0.06)	2.94
Hybrid algorithm	(1.44, 0.01)	2.94

Table 2. The number of function calls of the test function

	Number of function call
GA	974
GA + RSM	496
Simplex method	387
Hybrid algorithm	103

4. Application to the Optimal Design of IPM

We selected a 400[W] class IPM with the notches on the rotor surface as a practical optimization example. The initial structure of this model is shown in Fig. 6 and its basic specifications are given in Table 3.

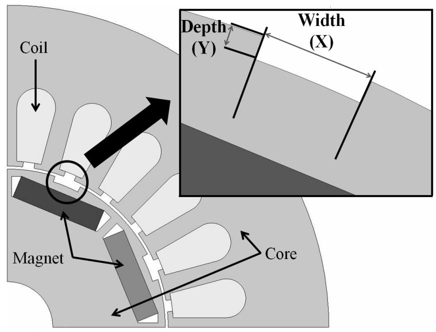

Fig. 6. Initial structure of IPM. (8 pole, 24 slot, Width(X) = 10[deg], Depth = 1.0[mm])

Table 3. Specification of IPM

Rated power	424 [w]
Rated RPM	1800 [r/m]
Stacking length	30 [mm]
Air gap length	0.5 [mm]
Stator outer radius	112 [mm]
Rotor inner radius	16 [mm]
Permanent magnet	NdFeB

4.1 Design variables and constraint condition

We chose the depth and width of the notches as the design variables, as shown in Fig 6. The range of each variable was set as follows;

$$0 \le Width(X)[\text{deg}] \le 10$$
$$0 \le Depth(Y)[mm] \le 1.0 \qquad (2)$$

4.2 Objective function

The optimum objective function is the multi-objective function which takes into consideration the peak to peak value of the cogging torque and the average value of the output torque from the IPM. The objective function is expressed in the following equation;

$$Z = w \times Output\,Torque + (1-w) \times \frac{1}{Cogging\,Toruqe} \quad (3)$$

To find the optimized result, two factors were properly weighted (in Eq. (3), where the ratio of the weights was expressed as w for the output torque and 1-w for the cogging torque). In this paper, we fixed the value of w at 0.1 to focus on the cogging torque and carried out the calculation to find the maximum z value.

4.3 Optimization results

In this paper, we created the response surface, as shown in Fig. 7, to test the proposed algorithm by using the Central Composite Design. The response surface was created by the regression function in Eq. (4).

$$\begin{aligned} f(X,Y) = (&139 + 1640X + 8665Y - 163X^2 \\ &- 7053Y^2 - 29XY) * 10^{-3} \end{aligned} \quad (4)$$

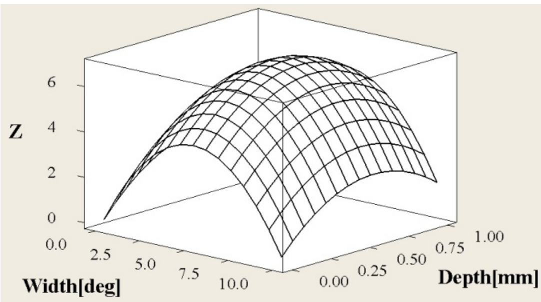

Fig. 7. Response surface of optimized model.

The peak point calculated by the Conjugate Gradient Method was (5, 0.6). Therefore, the reduction of the surface region (50 % of the original surface area) is conducted to include the surface area in (5)

$$\begin{aligned} 2.5 \le Width(X)[\deg] \le 7.5 \\ 0.35 \le Depth(Y)[mm] \le 0.85 \end{aligned} \quad (5)$$

If we apply the improved simplex method in the range of width and depth shown in (5), we obtain the results in Tables 4 and 5. We observe that the calculation time required to find the optimum solution is reduced by around 65% in terms of the number of calls based on the Finite Element Analysis (F. E. A). The width for the optimized performance was determined to be 4.4 [deg], while the depth was 0.7 [mm].

Table 4. Optimum solution comparison

	(X, Y)	Z
RSM	(5, 0.6)	6.69
Hybrid algorithm	(4.4, 0.7)	8.26

Table 5. The number of F.E.A calls

	Number of F.E.A call
Simplex method	24621
Hybrid algorithm	8735

4.4 Simulation results of optimized model

A comparison of the cogging and output torques before and after optimization by F.E.A. as a function of time is displayed in Figs. 8 and 9. The analysis of the graphs is summarized in Table 6.

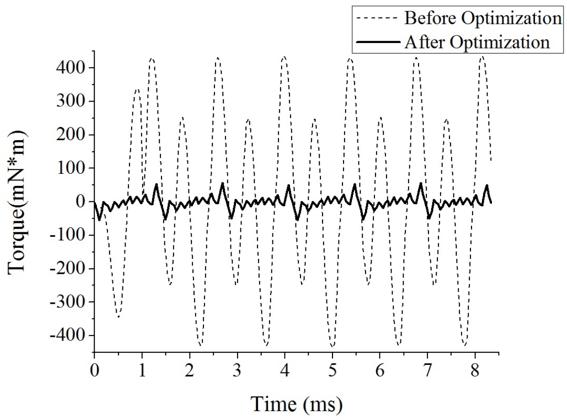

Fig. 8. Cogging torque before/after optimization.

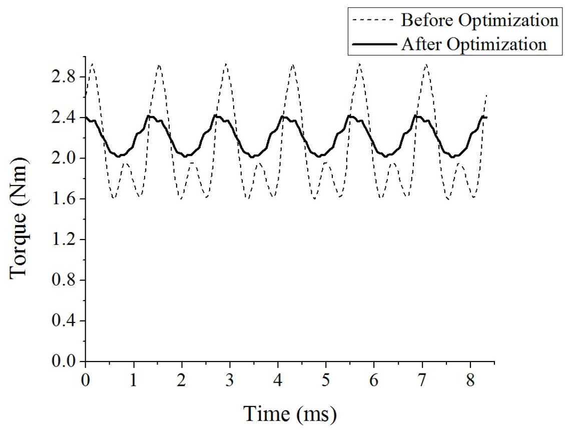

Fig. 9. Output torque before/after optimization.

Table 6. Characteristic changes before/after optimization

	Initial model	Optimized model	
Output torque	2.08[N*m]	2.21[N*m]	+ 6.3%
Torque ripple	1340[mN*m]	413[mN*m]	- 69%
Cogging torque	874[mN*m]	112[mN*m]	- 87%

5. Conclusion

In this paper, we proposed an optimization technique which combines the RSM and Simplex method. The usefulness of the hybrid algorithm was verified by applying it to a test function and the optimal design of an IPM. As a result, we observed that the reduction of the calculation time and improved calculation accuracy were obtained simultaneously.

Therefore, we concluded that the proposed hybrid optimization method can combine the advantages of the conventional methods and obtain the solutions faster and more correctly. Also, this method is thought to be useful not only for determining the optimum shape of notches, but also in designing various parts of motors.

References

[1] C. S. Koh, and J. S. Seol, "New Cogging-Torque Reduction Method for Brushless Permanent- Magnet Motors," *IEEE Trans. on Magnetics*, vol. 39, no. 6, pp. 3503-3506, Nov. 2003.

[2] G. H. Kang, Y. D. Son, G. T. Kim, and J. Hur, "A Novel Cogging Torque Reduction Method for Interior-Type Permanent-Magnet Motor," *IEEE Trans. on Industry Applications*, vol. 45, no. 1, pp. 161-167, Jan/Feb. 2009.

[3] J. A. Nelder and R. Mead, "A Simplex Method for Function Minimization," *Computer Journal*, no. 7, pp. 308-313, 1965

[4] K. O. An, C. H. Im, and H. K. Jung, "Magnetoencephalography source localization using improved simplex method," *Inverse Problems in Science and Engineering,* vol. 16, no. 4, pp. 499-510, Jun. 2008.

[5] C. I. Kim, "Efficient Response Surface Modeling and Design Optimization using Sensitivity," *M.S Thesis*, Department of Mechatronics, Kwangju Institute of Science and Technology, 2001.

Woo-Gyong Yim He received his M.S. degree in electrical engineering from Seoul National University. His research interests are numerical analysis and the design of electrical machines.

Kwang-Ok An She received her Ph.D. degree in electrical engineering from Seoul National University. Her research interests are numerical analysis and bioelectromagnetism.

Jang-Ho Seo He received his Ph.D. degree in electrical engineering from Seoul National University. His research interests are numerical analysis and the design of electrical machines.

Min-Jae Kim He received his M.S. degree in electrical engineering from Seoul National University. His research interests are numerical analysis and the design of electrical machines.

Hyun-Kyo Jung He received his Ph.D. degree in electrical engineering from Seoul National University. His research interests include electric machinery, EM waves, and circuit breakers.