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Reliability Assessment on Different Designs of a SMES System Based on the Reliability Index Approach

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Abstract – The current paper presents an effective methodology for assessing the reliability of electromagnetic designs when considering uncertainties of design variables. To achieve this goal, the reliability index approach based on the first-order reliability method is adopted to deal with probabilistic constraint functions, which are expressed in terms of random design variables. The proposed method is applied to three different designs of a superconducting magnetic energy storage system that corresponds to initial, deterministic, and roust designs. The validity and efficiency of the method is investigated with reference values obtained from Monte Carlo simulation.

Keywords: Electromagnetic analysis, Optimization, Reliability, Robustness

1. Introduction

Recently, there has been a growing interest in the robust or reliability-based optimization of products, with the aim of minimizing the performance variation or guaranteeing product quality at a specified confidence level under uncertainties, such as manufacturing errors, operating conditions, material properties, etc. Most electromagnetic designs have been carried out based on deterministic optimization methods without considering the randomness of design variables.

Until now, only Monte Carlo simulation (MCS) has been used in electromagnetic design problems to evaluate quantitatively the statistical property of the performance function of interest, considering the random variables [1-4]. However, the method has a major drawback: huge computation time is required for its numerical implementation because there are many function calls, although the probability information of random variables is given. Therefore, in applying robust or reliability-based optimization based on the statistical probability to electromagnetic devices, a more effective reliability assessment method with an acceptable level of accuracy should first be introduced.

In the current paper, the reliability index approach (RIA), one of the first-order reliability methods (FORM) that calculate reliability by exploiting the first-order Taylor series approximation of a performance function [5-8], is adopted to evaluate the reliability of performance functions with respect to probabilistic design variables. The RIA estimates the probability of failure (i.e., a nominal design point dose not satisfying the given performance function) when the probabilistic information of random variables is known [8]. The proposed method is applied to three different designs (i.e., initial, deterministic, and roust design points) of a superconducting magnetic energy storage system (SMES) [9]. The robustness of each SMES design is assessed quantitatively from the probabilistic statistics point of view. The validity and efficiency of the proposed method are investigated thoroughly with the reference values obtained from MCS.

2. Reliability Analysis

In the reliability analysis, the first step is to decide on the random variables and specific performance criteria [6]. The performance function g can be described mathematically as follows:

$$Z = g(\mathbf{X}) = g(X_1, X_2, \cdots, X_n) \tag{1}$$

where Z is the response of the performance function, and X_1, X_2,…, X_n are random variables. The failure surface or limit state of the performance function is defined as Z=0. The surface is a reference interface between the feasible and the infeasible regions when deciding whether a current design point fulfills the required performances or not. Whereas the design point is considered safe when Z>0, the performance failure occurs when $Z \le 0$.

The probability of failure P_f is calculated by integrating the joint probability distribution function, $f_{\mathbf{X}}(\mathbf{x})$, of all random variables over the infeasible region, such as (2).

$$P_f = P\big(g(\mathbf{X}) \le 0\big) = \int \cdots \int_{g(\mathbf{X}) \le 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{2}$$

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As calculating (2) exactly in most design problems is very difficult, various approximation methods have been developed.

2.1 RIA

To deal with the multiple integrations of (1) effectively, the reliability analysis based on FORM requires a transformation from the original random parameter **X** to the independent and standard normal random parameter **U**. That is, the performance function $G(\mathbf{X})$ in X-space is mapped onto $G(\mathrm{T}(\mathbf{X}))\equiv G(\mathbf{U})$ in U-space. Then, in evaluating the probability of (2), the first-order safety reliability index β, as shown in Fig. 1, is obtained by formulating an optimization problem of (3) with an equality constraint, which is the failure surface.

$$\begin{aligned} &\textbf{minimize} && \|\mathbf{U}\| \\ &\textbf{subject to} && G(\mathbf{U})=0 \end{aligned} \tag{3}$$

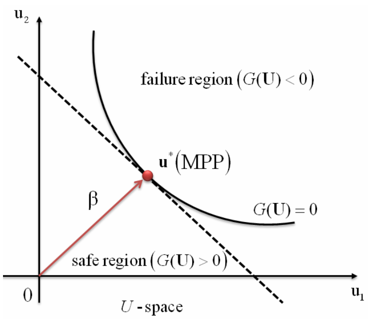

Fig. 1. Reliability index in RIA

The minimum distance point on the failure surface from the origin in U-space is called the most probable point (MPP) $\mathbf{u}^*$. The reliability index is defined by $\beta=\|\mathbf{u}^*\|$. Once the reliability index is obtained, the probability of failure is approximated as

$$P_f \approx \Phi(-\beta) \tag{4}$$

where $\Phi(\bullet)$ is the standard normal cumulative density function.

2.2 MPP search algorithm

The MPP $\mathbf{u}^*$ can be obtained after solving the optimization problem of (3). In doing so, either an MPP search algorithm developed for the first-order reliability analysis or a general optimization algorithm can be used. Owing to simplicity and efficiency, the Hasofer-Lind and Rackwitz-Fiessler (HL-RF) method [8] is employed here. In the method, MPP is determined from the iterative updating process similar to (5).

$$\mathbf{u}_{k+1}^* = \frac{\nabla g(\mathbf{u}_k^*)^T \mathbf{u}_k^* - g(\mathbf{u}_k^*)}{\|\nabla g(\mathbf{u}_k^*)\|^2}\nabla g(\mathbf{u}_k^*) \tag{5}$$

where $\nabla g(\mathbf{u}_k^*)$ is the gradient vector of the performance function at the kth iteration point $\mathbf{u}_k^*$. The iterative procedure usually starts from the origin in the U-space (i.e., $\mathbf{u}_0 = \mathbf{0}$ for the initial value of MPP). This scheme stops when the two convergence criteria below are satisfied.

$$\|\mathbf{u}_k^* - \mathbf{u}_{k-1}^*\| \le \varepsilon_1, \quad |g(\mathbf{u}_k^*)| \le \varepsilon_2 \tag{6}$$

where ε_1 and ε_2 are certain tolerance values.

The flowchart of the HL-RF method illustrated in Fig. 2 consists of the following steps:

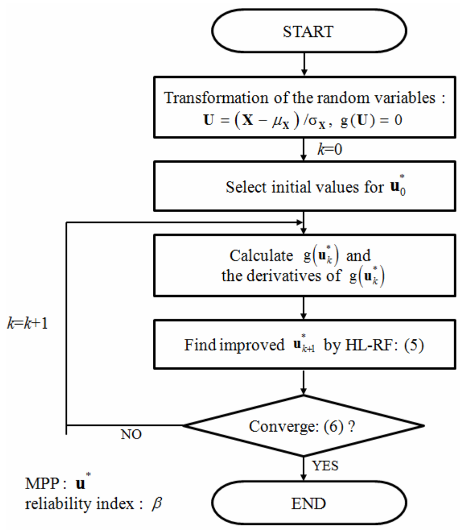

Fig. 2. Flowchart of the HL-RF method

(1) Define random variables **X** and constraint function G(**X**), and transform them into the standard normal U-space, with the transformation $U=(X-\mu_X)/\sigma_X$ (μ_X, σ_X; mean, and standard deviation (SD) of X).
(2) Set the initial value of MPP for $\mathbf{u}_0^*=0$.
(3) Compute the constraint function and its derivative corresponding to the current MPP value.
(4) Update MPP with the HL-RF method of (5).
(5) If the MPP is acceptable for the prescribed convergence criteria of (6), then the optimization process stops.
(6) Otherwise, go to step 3.

3. Results

3.1 Deterministic optimum designs of a SMES

The TEAM benchmark problem 22 in [9] deals with the design optimization of a SMES system shown in Fig. 3. To simplify the design problem, a constraint of the current quench condition on the superconductivity magnet is not considered here. A typical optimization problem for minimizing an objective function subject to a set of constraints is given as follows:

$$\begin{aligned} \textbf{minimize} \quad & f(\mathbf{X}) = \sum_{i=1}^{21} |B_{stray,i}(\mathbf{X})|^2 \\ \textbf{subject to} \quad & G_1(\mathbf{X}) = \left(\frac{E(\mathbf{X}) - E_o}{0.05 \times E_o}\right)^2 - 1 \le 0 \\ & G_2(\mathbf{X}) = (R_1 - R_2) + \frac{1}{2}(D_1 + D_2) \le 0 \end{aligned} \tag{7}$$

where $B_{stray,i}$ is the stray field calculated at the ith measurement point along line a and line b, E is the stored magnetic energy, and E_o is the target energy of 180 MJ. The design variable vector **X** consists of six parameters describing the magnet dimensions and the two current densities.

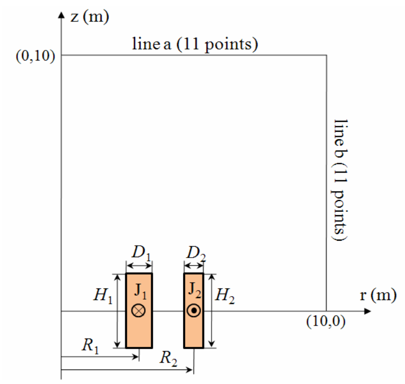

Fig. 3. Configuration of the SMES with eight design variables

The problem is solved with two different optimization methods belonging to the deterministic design method, without considering the probability distributions of design variables. The first method is the deterministic approach based on the continuum design sensitivity analysis (CDSA) [10]; the second method is the robust optimization method utilizing gradient index (GI) [9]. The detailed explanation of the two optimization methods is omitted because the focus of the current paper is on the reliability assessment on different design points of the same SMES magnet.

Starting with an initial design, the deterministic and robust optima are presented in Table 1. The stored energy values obtained from the two methods almost reach the target value of 180 MJ, but the robust optimum produces a better mean value of the stray fields than the deterministic algorithm. We assume that the deterministic optimal solution is trapped in one of the local minima near the constraint boundaries, and a better optimal solution is found by the robust optimization. In Fig. 4, the dimensions of the two optimized magnets are compared with each other. However, note that either of the optimization methods cannot provide designers quantitative information on how robust a current design point is when the uncertainties are included.

Table 1. Design variables and performance indications at the deterministic and robust optima [9]

Design variables	Unit	Lower bound ($\mathbf{x}_L$)	Initial design	CDSA optimum	Robust optimum	Upper bound ($\mathbf{x}_U$)
R_1	mm	1000	2000	2108	1977	4000
D_1	mm	100	400	412	404	800
H_1	mm	200	1500	1504	1507	3600
R_2	mm	2300	2360	2462	2348	5000
D_2	mm	200	300	294	233	350
H_2	mm	1700	1730	1756	1871	1900
J1	A/mm^2	10	17	16.39	16.30	30
J2	A/mm^2	10	17	14.49	16.19	30
$B_{stray,mean}$	μT		5468	153	34	
Energy	MJ		186	183	181	

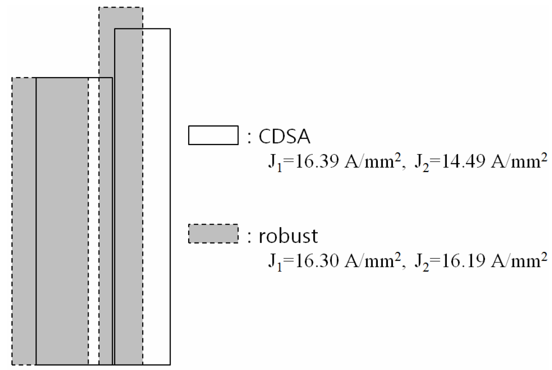

Fig. 4. Comparison of two SMES magnet designs after optimization

3.2 Reliability assessment

The robustness of the three nominal designs (i.e., initial, CDSA, and robust designs) is investigated in terms of the probability of failure based on RIA. For simplicity, only three design parameters, i.e., R_2, D_2 and H_2, of the eight parameters shown in Table 1 are considered random variables. The statistical information of the random variables is presented in Table 2.

Table 2. Statistical information of random variables

	Distribution type	Mean (mm)	Standard deviation (mm)
R_2	Normal	μ_{R_2}	10
D_2	Normal	μ_{D_2}	5
H_2	Normal	μ_{H_2}	10

A failure probability of the constraints associated with the deterministic optimization problem of (7) is written as follows:

$$P_f(G_i) = P(G_i(\mathbf{X}) > 0), \quad i = 1, 2, \cdots. \tag{8}$$

The RIA is applied to assess the robustness of feasibility of the two constraint functions defined in (7). The accuracy and efficiency of RIA are examined with the MCS values, which are assumed exact solutions.

The reliability analysis results on the three design points are compared in Table 3, in which the probability of the second constraint function is analytically obtained because G_2 is a linear function with respect to random variables. In MCS, the 10,000 function calls for G_1 are set to satisfy a significance level of 5%.

Table 3. Reliability analysis results of the three nominal design points by employing RIA and MCS

	Initial design		CDSA optimum		Robust optimum	
Method	RIA	MCS	RIA	MCS	RIA	MCS
$P_f(G_1)$	0.327	0.332	0.152	0.160	0.040	0.062
Function calls (G_1)	21	10,000	28	10,000	35	10,000
$P_f(G_2)$	0.166	0.166	0.461	0.461	1.76×10^{-7}	1.76×10^{-7}
Function calls (G_2)	1	1	1	1	1	1

The first constraint G_1 in (7) is related to the energy requirement that the stored magnetic energy must not exceed the maximum deviation of 5% from the target value of 180 MJ. The initial design gives the lowest feasibility of G_1 among them, which is about 67% probability with the reference value of MCS (i.e., the failure probability is about 33%). This finding is expected as the magnetic energy at the initial design is farthest from the target value. Based on the results, the RIA is observed to produce acceptable probability values of G_1 for the three magnet designs compared with the MCS values. Under the statistical assumption on the random variables, the robust optimum is assumed the best design because it yields the lowest failure probability values among the three designs. Based on Table 3, the proposed method provides adequate accuracy in evaluating the probability, although it requires very few function calls compared with MCS.

The second constraint G_2 is used to prevent two magnets from overlapping with each other. As previously mentioned, G_2 is a linear function with respect to all random variables, of which the probability distributions are assumed normal. Hence, the failure probability of G_2 is calculated analytically by

$$P_f(G_2) = P(G_2(\mathbf{X}) > 0) = \Phi\left(-\frac{\mu_{G_2}}{\sigma_{G_2}}\right) \quad (9)$$

where $\mu_{G_2}=(\mu_{R_2}-R_1)-1/2(\mu_{D_2}+D_1)$, $\sigma_{G_2}=(\sigma_{R_2}{}^2+1/4\ \sigma_{D_2}{}^2)^{1/2}$, and the mean and SD values of R_2, D_2 are denoted as μ_{R_2}, μ_{D_2}, σ_{R_2}, and σ_{D_2}, respectively. In the case of CDSA, the gap between the windings is very small, as shown in Fig. 4. Thus, the failure probability of G_2 increases up to 46%, as shown in Table 3. This finding implies that the deterministic approach, without considering the uncertainties of the design variables, may lead to an infeasible design from the reliability point of view. At the robust optimum, the constraint feasibility of G_2 is improved significantly, and the failure probability is given by a very small value of 1.7599×10^{-7}. In any case, the robust optimum is revealed as the best design, which has high reliability in the two constraints considered.

5. Conclusion

Based on the reliability index approach, three different SMES designs have been assessed quantitatively in terms of the robustness of constraint feasibility. The reliability of the two constraints is much improved at the robust optimum compared with the others. Results show that the proposed reliability analysis method, RIA, provides acceptable accuracy in evaluating the failure probability of constraint functions, although it requires very few function calls compared with MCS.

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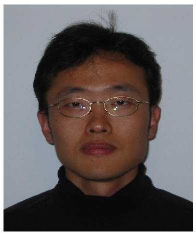

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