Journal of Electrical Engineering & Technology Vol. 7, No. 1, pp. 17~22, 2012 **17**
http://dx.doi.org/10.5370/JEET.2012.7.1.17

Evaluation of Two Lagrangian Dual Optimization Algorithms for Large-Scale Unit Commitment Problems

Wen Fan*, Yuan Liao*, Jong-Beom Lee† and Yong-Kab Kim**

Abstract – Lagrangian relaxation is the most widely adopted method for solving unit commitment (UC) problems. It consists of two steps: dual optimization and primal feasible solution construction. The dual optimization step is crucial in determining the overall performance of the solution. This paper intends to evaluate two dual optimization methods – one based on subgradient (SG) and the other based on the cutting plane. Large-scale UC problems with hundreds of thousands of variables and constraints have been generated for evaluation purposes. It is found that the evaluated SG method yields very promising results.

Keywords: Dual optimization, Lagrangian relaxation, Resource scheduling, Unit commitment

1. Introduction

Unit commitment (UC) is a special optimization problem for determining the startup and shutdown schedules of generation units in a power system considering unit characteristics, power network operation constraints, and fuel costs, among others [1, 2]. Among the different study objectives related to UC, one common aim is to minimize the total operating cost over a study horizon while satisfying the demands.

UC problems are normally formulated as mixed integer programming problems consisting of integer variables (startup and shutdown of generators) and continuous variables (dispatch level of turned-on generators). The solution resolution is usually one hour, and the study horizon could be one day, one week, or one month. Solving this type of problem could be very difficult due to the huge size of the problem.

While there are a great deal of efforts to tackle this problem utilizing diverse techniques such as genetic algorithm and benders decomposition, the most promising one remains to be the Lagrangian relaxation (LR) method, which mainly comprises two steps [3, 4]. The first step is called dual optimization, while the second step is called primal feasible solution generation [5-7].

The dual optimization step is crucial in determining the overall performance of an LR method in terms of speed and optimality [8, 9]. While various algorithms for dual optimization have been proposed in the past, there are only a few studies utilizing large-scale UC problems for evaluating dual optimization algorithms. Meanwhile, benchmarking studies are helpful for the better understanding of the performance and characteristics of the algorithms. This paper intends to provide certain evaluation results for large-scale UC problems. Two common approaches for updating Lagrangian multipliers, namely the subgradient (SG) based approach and the cutting-plane (CP) based approach, have been evaluated [2, 3]. A library of large-scale UC cases is generated for this purpose. Due to the diverse variants of algorithms published in the literature, this paper does not intend to cover the evaluation of all the multiplier update approaches. Instead, we focus on two selected dual optimization methods.

The LR-based optimization method is first reviewed. The SG-based and CP-based multiplier update approaches are then presented. The development of a library of large-scale UC cases is also described, and the sizes of the various cases are reported. Evaluation studies on the multiplier update methods using the generated cases are then presented, followed by the conclusion.

2. LR and Multiplier Update

The following generic formulation for resource optimization problems, such as UC problems [2, 3], is considered:

$$\underset{x}{Minimize} \qquad f(x)=\sum_i f_i(x_i) \tag{1}$$

Subject to:

$$s_i(x_i)\geq 0$$
$$\sum_i g_i(x_i)\geq G$$
$$\sum_i h_i(x_i)=H$$

† Corresponding Author: Department of Electrical Engineering, Wonkwang University, Korea (ipower@wonkwang.ac.kr)
* Department of Electrical and Computer Engineering, University of Kentucky, Lexington, KY, USA (yliao@engr.uky.edu)
** Department of Information and Communication Engineering (ykim@wonkwang.ac.kr)
Received: January 5, 2011; Accepted: September 19, 2011

Where $x = (x_i; \forall i)$; $i = 1, 2, ..., N$; the objective function f is additively separable into N functions f_i ; x_i is the variable set associated with f_i ; s_i is the local constraint set involving only variable set x_i; and g_i and h_i are the components for coupling constraints and involve only the variable set x_i. In UC problems, N typically represents the number of generation units, which will be referred to as the primal problem (P). In practical problems, local constraints can include the minimum uptime, minimum downtime, ramping constraint, and the available capacity limits of each generating unit. The equality coupling constraints include energy balance requirements, while the inequality coupling constraints include the reserve requirements. The presence of such coupling constraints makes it difficult to find the solution of the problem P.

The LR of P is derived by dualizing the coupling constraints, resulting in the relaxed problem (RP), as shown below [2]:

$$\underset{x}{Minimize} \quad L(x, \lambda_e, \lambda_g) \tag{2}$$

Subject to $\quad s_i(x_i) \geq 0$, $i = 1, 2, ...N$

Where $L(x, \lambda_e, \lambda_g) = L(x, \lambda) = \sum_i f_i(x_i) + \lambda_e^T (H - \sum_i h_i(x_i)) + \lambda_g^T (G - \sum_i g_i(x_i))$

$\lambda = \begin{bmatrix} \lambda_e \\ \lambda_g \end{bmatrix}$ is the Lagrangian multiplier vector; λ_e and λ_g are the multiplier vectors corresponding to the equality and inequality constraints, respectively; $\lambda_g \geq 0$; and λ_e is unrestricted in sign. All vectors in this study are column vectors.

Once the coupling constraints are removed, the resulting problem, RP, can be decomposed into N independent problems, each of which corresponds to a single generation unit. A single problem is usually of much smaller size and thus is much easier to solve.

LR solution method is based on the theorems of duality, which establish certain properties governing the relationship between solutions to the Lagrangian dual problem, DP, and solutions to P. The Lagrangian dual problem, DP, is defined as follows:

$$\underset{\lambda}{Maximize} \quad L(\lambda) \tag{3}$$

Subject to $\quad \lambda_g \geq 0$

Where $\quad L(\lambda) = \underset{x}{Minimize} \; L(x, \lambda)$

The dual optimization aims to find a set of multipliers that maximize the Lagrangian dual function. There are two major variations of LR method depending on how the Lagrangian multipliers are updated in the iterative process: the SG-based method and CP-based method.

The SG for the Lagrangian dual is calculated as follow [2, 3]:

$$\xi = \begin{bmatrix} G - \sum_i g_i(x_i) \\ H - \sum_i h_i(x_i) \end{bmatrix} \tag{4}$$

The solution of the dual problem usually takes an iterative procedure, where the Lagrangian multipliers are updated iteratively depending on the calculated SG. The method of updating the multipliers is pivotal in determining the speed and quality of the solution. The two commonly used multiplier update methods are presented in the following sections; both are evaluated by applying the developed case library.

2.1 SG-based multiplier update method

Assuming that for iteration k the SG vector for the Lagrangian dual is $\xi^{(k)}$ and the multiplier vector is $\lambda^{(k)}$, then the multiplier vector is updated as follows [2]:

$$\lambda^{(k+1)} = \lambda^{(k)} + \alpha \frac{1}{1+k} \xi^{(k)} \tag{5}$$

Appropriate values for α , the scale factors, are selected according to [2].

2.2 CP-based multiplier update method

The updated multiplier is obtained by solving the following linear programming problem [3]:

$$\underset{z,\lambda}{Maximize} \quad z \tag{6}$$

Subject to $\quad z \leq \phi^{(i)} + [\xi^{(i)}]^T (\lambda - \lambda^{(i)})$,
$i = 1, \cdots, k$
$\lambda_g \geq 0$

Where ϕ is the value of the Lagrangian function, and T is the vector transpose operator.

3. Generation of a UC Case Library

This section describes the generated UC case library based on typical, if not comprehensive, models [11-15]. The UC problem is formulated as a mixed-integer linear programming (MILP) problem with a time resolution of one hour, which minimizes the total production cost subject to specific constraints. The inputs of problem formulation include the startup/shutdown cost, the energy cost, the maximum/minimum power output, the minimum uptime/downtime, and the maximum ramp up/down dispatch increment of each unit, as well as the demand and reserve requirement in each hour. There are four sets of variables for each hour and for each unit: dispatch, commitment status, startup action, and shutdown action. The constraints considered include energy balance and reserve, unit capacity, unit ramp up/ramp down, and unit minimum uptime/downtime constraints. The energy balance constraints and the reserve requirement constraints are the two types of global constraints, or complicating constraints. The objective function is to minimize the total production cost, including the energy, startup, and shutdown costs. The mathematical equations for the constraints and objective function are presented as follows based on references [1, 10, 11].

3.1 Constraints

The constraints of the formulated optimization problem are presented as follows, referring to [1, 10], especially [10].

Unit capacity constraints:

$$P_{\min i} u_{ik} - p_{ik} \le 0 , \quad \forall k \in S_K , \quad \forall i \in S_I \tag{7a}$$
$$p_{ik} - P_{\max i} u_{ik} \le 0 , \quad \forall k \in S_K , \quad \forall i \in S_I \tag{7b}$$

Where,

$P_{\max i}$: Maximum capacity of unit i
$P_{\min i}$: Minimum capacity of unit i
p_{ik} : Dispatch of unit i in hour k, which is a continuous variable
u_{ik} : Commitment status of unit i in hour k, which is a binary variable
S_I : Set $\{1, ..., I\}$
S_K : Set $\{1, ..., K\}$
I : The number of units
K : The number of hours

Startup and shutdown relationship:

$$s_{ik} - d_{ik} - u_{ik} + u_{i(k-1)} = 0 , \quad \forall k \in S_K , \quad \forall i \in S_I \tag{8}$$

where

s_{ik} : Startup action of unit i in hour k, which is a binary variable
d_{ik} : Shutdown action of unit i in hour k, which is a binary variable

Initial unit status and output:

$$u_{i0} = U_{i0} , \quad \forall i \in S_I \tag{9a}$$
$$p_{i0} = P_{i0} , \quad \forall i \in S_I \tag{9b}$$

where

U_{i0} : The unit status preceding hour 1, (1 for online and 0 for off line)
P_{i0} : The initial dispatch of unit i preceding hour 1

Energy balance constraints:

$$\sum_{i=1}^{I} (p_{ik}) = D_k , \quad \forall k \in S_K \tag{10}$$

where D_k is the demand requirement in hour k.

Ramp up constraints:

$$p_{i(k+1)} - p_{ik} \le L_{ui} , \quad k = 0, ..., K-1 , \quad \forall i \in S_I \tag{11}$$

where L_{ui} is the maximum ramp up dispatch increment of unit i.

Ramp down constraints:

$$p_{ik} - p_{i(k+1)} \le L_{di} , \quad k = 0, ..., K-1 , \quad \forall i \in S_I \tag{12}$$

where L_{di} is the maximum ramp down dispatch decrement of unit i.

Reserve requirements:

$$\sum_{i=1}^{I} (P_{\max i} u_{ik}) \ge D_k + R_k , \quad \forall k \in S_K \tag{13}$$

where R_k is the reserve requirement in hour k.

Minimum uptime constraints:

$$\sum_{m=k}^{M_{uti}+k-1} u_{im} \ge M_{uti} s_{ik} , \quad k = 1, ..., K+1-M_{uti} , \quad \forall i \in S_I \tag{14a}$$

$$\sum_{m=k}^{K} u_{im} \ge (K-k+1) s_{ik} , \quad k = K+2-M_{uti}, ..., K-1 , \quad \forall i \in S_I \tag{14b}$$

where M_{uti} is the minimum uptime of unit i.

Minimum downtime constraints:

$$M_{dti} - \sum_{m=k}^{M_{dti}+k-1} u_{im} \ge M_{dti} d_{ik} , \quad k = 1, ..., K+1-M_{dti} , \quad \forall i \in S_I \tag{15a}$$

$$(K-k+1) - \sum_{m=k}^{K} u_{im} \ge (K-k+1) d_{ik} ,$$
$$k = K+2-M_{dti}, ..., K-1 , \quad \forall i \in S_I \tag{15b}$$

where M_{dti} is the minimum downtime of unit i.

The abovementioned constraints are not meant to be all-inclusive, and other constraints such as transmission, contingency, emission, and fuel constraints are not considered in the present study [12, 13].

3.2 Objective function

The objective function is to minimize the total production cost of all units during the entire optimization horizon, including the energy, startup, and shutdown costs.

Minimize

$$\sum_{k=1}^{K}\sum_{i=1}^{I}(C_i p_{ik} + s_{ik}W_i + d_{ik}V_i) \quad (16)$$

where

W_i : Startup cost of unit i

V_i : Shutdown cost of unit i

C_i : Energy rate of unit i

Based on the formulation described above, a case library of large-scale UC problem is generated, which is used for testing the dual optimization problem.

4. Evaluation Studies for Dual Optimization

This section presents the evaluation studies for dual optimization. Various cases are generated, with different number of units and of hours in the optimization horizon. The scale and complexity of the cases are comparable to or exceed those of real-world power system UC problems. The sizes of the generated cases are shown in Table 1, where the number of continuous variables, binary variables, constraints, non-zero coefficients in the constraints, coupling equality constraints (CECs), and coupling inequality constraints (CIECs) are shown. Some of the cases are the same as those reported in [11], and the software development work utilized to generate the case library is detailed in [11].

The name of each test system is composed of the number of generation units and the number of hours in the optimization horizon. For example, UK_500_168 means that the system consists of 500 units and the optimization horizon is 168 hours. The number of CECs and CIECs are the same and equal to the number of hours in the optimization horizon.

The developed cases were applied to evaluate the two multiplier update methods. The dual optimization procedure outlined in section 2 is followed in the present study.

The dual solutions obtained by using the SG and CP methods are shown in Table 2. The maximum number of iteration is set at 300 in the simulation studies. In the tables, the column "Iteration" indicates the least number of iterations required to achieve a dual solution gap no more than 1%; if such solution gap cannot be achieved, the iteration number will be equal to 300. The actual dual value and solution gap achieved are shown in the column "Dual" and "Gap", respectively. The dual solution gap is calculated as the difference between the primal feasible optimal value and the dual value, divided by the primal feasible optimal value. Columns 2–4 show the results for the SG method, and columns 5–7 show the results for the CP method.

Table 1. Sizes of the UC cases under study

System	Num. of Con. Var.	Num. of Bin. Var.	Num. of Cons.	Num. of Non-zeros	Num. of CEC	Num. of CIEC
UK10_24	240	720	1728	5730	24	24
UK10_168	1680	5040	12096	40462	168	168
UK10_720	7200	21600	51840	176914	720	720
UK50_24	1200	3600	8448	28520	24	24
UK50_168	8400	25200	59136	211784	168	168
UK50_720	36000	108000	253440	920320	720	720
UK100_24	2400	7200	16848	58612	24	24
UK100_168	16800	50400	117936	423666	168	168
UK100_720	72000	216000	505440	1786216	720	720
UK200_24	4800	14400	33648	114390	24	24
UK200_168	33600	100800	235536	845158	168	168
UK500_24	12000	36000	84048	291256	24	24
UK500_168	84000	252000	588336	2094800	168	168

Table 2. Dual solution based on the SG and CP approaches

System	Dual (SG)	Gap (SG)	Iteration (SG)	Dual (CP)	Gap (CP)	Iteration (CP)
UK10-24	2,809,102.84	0.84%	16	2,804,550.96	1.00%	259
UK10-168	18,779,263.04	0.98%	20	13,496,000.00	28.84%	300
UK10-720	73,386,984.64	0.84%	13	0.00	100%	300
UK50-24	15,707,441.13	0.87%	14	15,707,827.17	0.87%	253
UK50-168	90,317,491.80	0.92%	12	89,522,000.00	1.80%	300
UK50-720	371,292,459.77	0.86%	13	0.00	100%	300
UK100-24	25,970,894.64	0.99%	15	25,961,000.00	1.03%	300
UK100-168	195,444,649.06	0.94%	12	169,600,000.00	14.04%	300
UK100-720	810,132,590.10	0.90%	16	0.00	100%	300
UK200-24	52,975,886.11	0.81%	14	52,957,586.08	0.84%	283
UK200-168	387,170,313.04	0.85%	12	280,990,000.00	28.04%	300
UK500-24	125,381,118.14	0.86%	15	125,217,946.00	1.00%	211
UK500-168	913,278,531.05	0.99%	18	755,850,000.00	18.06%	300

The evaluated SG method significantly outperformed the evaluated CP method. The SG method can usually achieve a high-quality dual solution within 20 iterations, while the CP method either requires hundreds of iterations for reaching the desired solution gap or fails to do so within the specified maximum iteration limit.

To better understand the convergence behavior of the evaluated algorithms, the convergence locus for the SG method is depicted in Fig. 1 for case UK200_168. Its zoom-in view is shown in Fig. 2.

The dual optimization convergence locus based on the CP method for case UK200_168 is plotted in Fig. 3.

The dual optimization convergence loci based on the CP method for case UK100_24 are illustrated in Figs. 4 and 5.

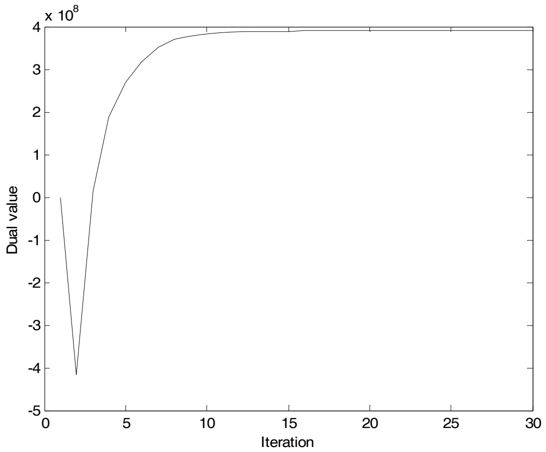

Fig. 1. Dual optimization convergence locus based on SG for UK200_168 – overall view

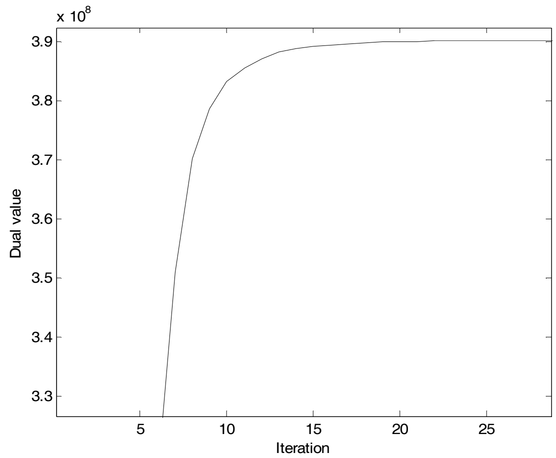

Fig. 2. Dual optimization convergence locus based on SG for UK200_168 – zoom in view

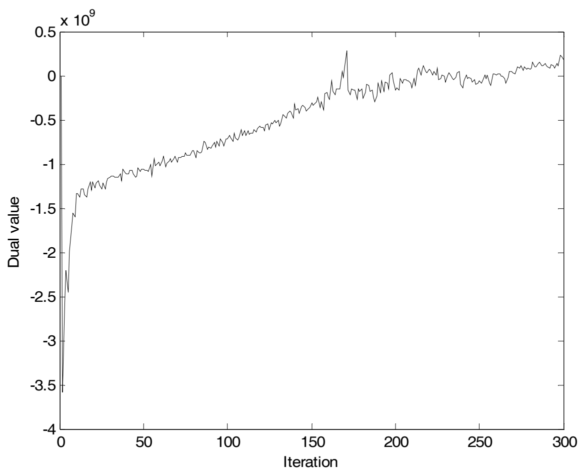

Fig. 3. Dual optimization convergence locus based on CP for UK200_168

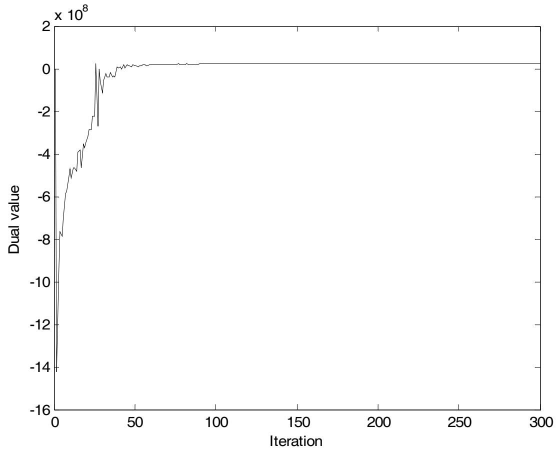

Fig. 4. Dual optimization convergence locus based on CP for case UK100_24 – overall view

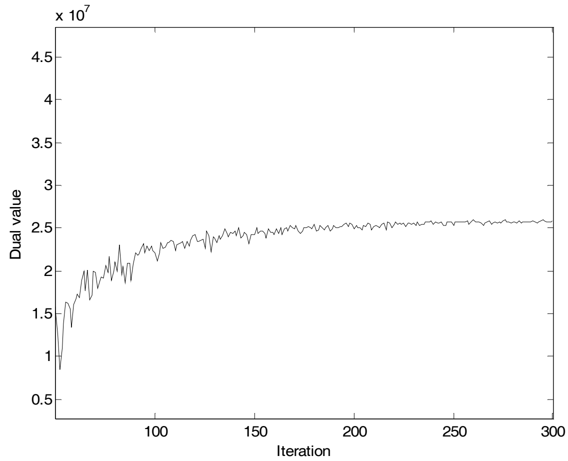

Fig. 5. Dual optimization convergence locus based on CP for case UK100_24 – zoom-in view

The Figs. show that the tested SG method can quickly converge to the desired optimal dual solution with few or no oscillations. Meanwhile, the CP method suffers considerable oscillations and thus takes a significant number of iterations to get close to the dual optimal solution.

5. Conclusions

A library of UC problems, including large-scale problems with hundreds of thousands of variables and constraints, was created and applied for the evaluation of the performance of an SG-based and a CP-based multiplier update methods for Lagrangian dual optimization. The evaluated SG algorithm can achieve high-quality dual solutions, usually within 20 iterations for large-scale UC problems without any significant oscillations. Meanwhile, the evaluated CP method suffers significant oscillations and thus is very slow in reaching the optimal dual solution. The evaluation results can provide researchers with useful benchmarking information for the study of LR-based algorithms to solve large-scale resource scheduling problems such as UC problems.

References

[1] Benjamin F. Hobbs, et al, The Next Generation of Electric Power Unit Commitment Models, Kluwer's International Series, 2001.

[2] X Feng and Yuan Liao, "A new Lagrangian multiplier update approach for Lagrangian relaxation based unit commitment", *Electric Power Components & Systems*, 34 (2006), pp. 857-866.

[3] N. J. Redondo, A. J. Conejo, "Short-term hydro-thermal coordination by Lagrangian relaxation: solution of the dual problem", IEEE Transactions on Power Systems, 14 (1999), pp. 89-95.

[4] C. P. Cheng, C. W. Liu, C. C. Liu, "Unit commitment by Lagrangian relaxation and genetic algorithms", IEEE Transactions on Power Systems, 15(2000), pp. 707-714.

[5] R. T. Rockafellar, "Lagrange multipliers and optimality", SIAM Rev., 35 (1993), pp. 183-238.

[6] Mokhtar S. Bazaraa, Hanif D. Sherali, C. M. Shetty, Nonlinear Programming, Theory and Algorithms, John Wiley & Sons, Inc. 1993.

[7] C. A. Kaskavelis, M. C. Caramanis, "Efficient Lagrangian relaxation algorithm for industry size job-shop scheduling problems", IIE Transactions, 30 (1998), pp 1085-1097.

[8] W. Ongsakul, N. Petcharaks, "Unit commitment by enhanced adaptive Lagrangian relaxation", IEEE Transactions on Power Systems, 19 (2004), pp. 620-628.

[9] A. Borghetti, A. Frangioni, F. Lacalandra, C.A. Nucci, "Lagrangian Heuristics Based on Disaggregated Bundle Methods for Hydrothermal Unit Commitment", IEEE Transactions on Power Systems, 18 (2003), pp. 313-323.

[10] H.P. Williams, Model Building in Mathematical Programming, John Wiley & Sons Ltd.,West Sussex, England, 1999.

[11] Yuan Liao, "Development of a generation resource scheduling case library", 38th Southeastern Symposium on System Theory, Tennessee Technological University, Cookeville, TN, USA, March 5-7, 2006.

[12] Yuan Liao, X Feng and J Pan, "Impact of emission compliance program on competitive power market", invited paper, The 2nd International Conference on Electric Utility Deregulation, Restructuring and Power Technologies, Hong Kong, April 5-8, 2004.

[13] Yuan Liao, X Feng and J Pan, "Analysis of interaction between ancillary service markets and energy market using power market simulator", invited paper, The 2nd International Conference on Electric Utility Deregulation, Restructuring and Power Technologies, Hong Kong, April 5-8, 2004.

[14] K. Chandram, N. Subrahmanyam and M. Sydulu, "Improved pre-prepared power demand table and muller's method to solve the profit based unit commitment problem", Journal of Electrical Engineering & Technology, 4 (2009), pp. 159-167.

[15] Yun-Won Jeong, Woo-Nam Lee, Hyun-Houng Kim, Jong-Bae Park and Joong-Rin Shin, "Thermal unit commitment using binary differential evolution", Journal of Electrical Engineering & Technology, 4 (2009), pp. 323-329.

Wen Fan is currently studying at the Department of Electrical and Computer Engineering at the University of Kentucky. His research interests include power system simulation, analysis and optimization, and smart grid.

Yuan Liao is currently an associate professor at the Department of Electrical and Computer Engineering at the University of Kentucky, Lexington, KY, USA. His research interests include protection, power quality analysis, large-scale resource scheduleing optimization, and network management system/supervisory control and data acquisition system design.

Jong-Beom Lee received his B.S., M.S., and Ph.D degrees in Electrical Engineering from Hanyang University, Korea, in 1981, 1983, and 1986, respectively. He worked at the Korea Electrotechnology Research Institute from 1987 to 1990. He was a visiting scholar at the Technical University of Berlin, Germany, in 1993; the City University, UK, in 1995; Texas A&M University in 1997; and Swiss Federal Institute of Technology (ETH) in 2003 . He is currently a professor in the Department of Electrical Engineering, Wonkwang University, Korea.

Yong-Kab Kim received his B.S. degree in Electronics Engineering from Ajou University and his M.S.E. degree in Electrical Engineering from the University of Alabama in Huntsville. He received his Ph.D. degree in Electrical and Computer Engineering in North Carolina State University. He is currently a professor at School of Electrical Information Communication Engineering, Wonkwang University, Korea. His research interests include remote sensing for visible communication, optical fiber sensing, and power line communication.