

A Stratified Unknown Repeated Trials in Randomized Response Sampling

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Abstract

This paper proposes an alternative stratified randomized response model based on the model of Singh and Joarder (1997). It is shown numerically that the proposed stratified randomized response model is more efficient than Hong *et al.* (1994) (under proportional allocation) and Kim and Warde (2004) (under optimum allocation).

Keywords: Randomized response technique, stratified random sampling, proportional allocation, optimum allocation.

1. Introduction

A randomized response (RR) data gathering device to procure thrust worthy data on sensitive issues was developed by Warner (1965). The Warner model required the interviews to give a 'Yes' or 'No' answer either to the sensitive question or to its negative depending on the outcome of a randomizing device not reported to the interviewer. Several authors including Mangat and Singh (1990), Mangat (1994), Singh and Mangat (1996) and Singh *et al.* (1994) have modified and suggested alternative randomized response procedures applicable to different situations.

Stratified random sampling is generally obtained by dividing the population into non-overlapping groups (called strata) and selecting a simple random sample from each stratum. An RR technique using a stratified random sampling provide group characteristics related to each stratum estimator; in addition, the stratified sample protects a researcher from the possibility of obtaining a poor sample, see Kim and Warde (2004).

Hong *et al.* (1994) suggested a stratified RR technique that applied the same randomization device to every stratum. Mahajan *et al.* (1994) have considered the problem of construction of optimum strata boundaries for scrambled responses. Under Hong *et al.*'s (1994) proportional sampling assumption, it may be easy to derive the variance of the proposed estimator; however, it may cause a high cost because of the difficulty to obtain a proportional sample from some stratum. To rectify this problem, Kim and Warde (2004) presented a stratified randomized response technique using an optimal allocation that is more efficient than a stratified randomized response technique using proportional allocation.

2. Review of Some Related Models

2.1. Unknown repeated trials in randomized sampling due to Singh and Joarder (1997)

Warner (1965) considered a case in which the respondents in a population can be divided into mutually exclusive groups (one group with sensitive character A and the other group without it). To

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estimate π_S , the proportion of respondents in the population belonging to the sensitive group A, a simple random and with replacement sample (SRSWR) of n respondents is selected from the population. To collect information on a sensitive characteristic, Warner (1965) made use of a randomization device. The procedure consists of using a randomization device with outcomes A and not A with known probabilities P and $(1 - P)$ respectively. The respondent observes the outcome of the device which remains unknown to the experimenter in order to protect the privacy. The respondent's answers 'Yes' if he has the characteristic shown by the device's outcome and 'No' otherwise; therefore, the probability θ of a 'Yes' response is

$$\theta = P\pi_S + (1 - P)(1 - \pi_S). \quad (2.1)$$

An unbiased estimator of π_S , the proportion of population belonging to the sensitive group A, considered by Warner is given by

$$\hat{\pi}_S^{(1)} = \frac{\hat{\theta} - 1 + P}{2P - 1}, \quad P \neq 0.5 \quad (2.2)$$

with variance

$$V(\hat{\pi}_S^{(1)}) = \frac{\theta(1 - \theta)}{n(2P - 1)^2}, \quad (2.3)$$

where $\hat{\theta} = n'/n$ and n' is the number of 'Yes' responses from the sample of n individuals. Singh and Joarder (1997) suggested an interesting modification of Warner's (1965) randomized response technique. According to the model of Singh and Joarder (1997), if a respondent belongs to group A, then he/she is requested to repeat the trial in the Warner's (1965) randomization device if in the first trial he/she does not get the statement according to his/her status. The rest of procedure remains the same. The repetition of the trials is known to the interviewee but remains unknown to the interviewer. Singh and Joarder (1997) called this the "unknown repeated trials model".

Assuming completely truthful reporting by the respondents, the probability of a 'Yes' answer is given by

$$\theta_1 = \pi_S[P + (1 - P)P] + (1 - \pi_S)(1 - P). \quad (2.4)$$

Singh and Joarder (1997) suggested an unbiased estimator of π_S as

$$\hat{\pi}_S^{(2)} = \frac{\hat{\theta}_1 - (1 - P)}{2P - 1 + P(1 - P)}, \quad (2.5)$$

where $\hat{\theta}_1$ is the sample proportion of 'Yes' responses in the proposed procedure. The variance of $\hat{\pi}_S^{(2)}$ is given by

$$V(\hat{\pi}_S^{(2)}) = \frac{\theta_1 - (1 - \theta_1)}{n\{2P - 1 + P(1 - P)\}^2}. \quad (2.6)$$

It follows from (2.3) and (2.6) that the estimator $\hat{\pi}_S^{(2)}$ is more efficient than Warner's (1965) estimator $\hat{\pi}_S^{(1)}$ as long as $P > 1/2$, see Singh and Joarder (1997, p.105).

2.2. Hong *et al.* (1994) and Kim and Warde (2004) models

Let the population be partitioned into strata, and a sample is selected by simple random sampling with replacement in each stratum. To get the full advantage from stratification, it is assumed that the number of units in each stratum is known. An individual respondent in the sample of stratum is instructed to use the randomization device R_i that consists of a sensitive question (S) card with probability P_i and its negative question (S) card with probability $(1 - P_i)$, The respondent should answer the question ‘Yes’ or ‘No’ without reporting which question and she or he has. A respondent belonging to the sample in different strata will perform different randomization devices, each having different pre-assigned probabilities. Let n_i denote the number of units in the sample from stratum and n denote the total number of units in the sample from all strata so that $n = \sum_{i=1}^k n_i$. Under the assumption that there ‘Yes’ and ‘No’ reports are made truthfully and $P_i (\neq 0.5)$, is set by the researcher, the probability of a ‘Yes’ answer in a stratum i for this procedure is.

$$Z_i = (P_i\pi_{S_i} + (1 - P_i)(1 - \pi_{S_i})), \quad \text{for } (i = 1, 2, \dots, k), \tag{2.7}$$

where Z_i is the proportion of ‘Yes’ answer in a stratum i , π_{S_i} is the proportion of respondent with the sensitive trait in stratum i and P_i is the probability than a respondent in the sample stratum i has a sensitive question (S) card. An unbiased estimator of π_{S_i} is

$$\hat{\pi}_{S_i} = \frac{\hat{Z}_i - (1 - P_i)}{(2P_i - 1)}, \quad P_i \neq \frac{1}{2}, \quad \text{for } (i = 1, 2, \dots, k), \tag{2.8}$$

where \hat{Z}_i is the proportion of ‘Yes’ answer in a sample is the stratum i . Since each \hat{Z}_i is a binomial distribution $B(n_i, Z_i)$ and the selection in different strata are made independently, an unbiased estimator of $\pi_S = \sum_{i=1}^k w_i\pi_{S_i}$ is given by

$$\hat{\pi}_S = \sum_{i=1}^k w_i\hat{\pi}_{S_i} = \sum_{i=1}^k w_i \left[\frac{\hat{Z}_i - (1 - P_i)}{2P_i - 1} \right], \tag{2.9}$$

where N is the number of units in the entire population, N_i is the number of units in the stratum i and $w_i = (N_i/N)$ for $i = 1, 2, \dots, k$ so that $w = \sum_{i=1}^k w_i = 1$.

The variance of $\hat{\pi}_S$ in (2.9) is given by

$$\begin{aligned} \text{Var}(\hat{\pi}_S) &= \sum_{i=1}^k w_i^2 \text{Var}(\hat{\pi}_{S_i}) \\ &= \sum_{i=1}^k \frac{w_i^2}{n_i} \left[\pi_{S_i}(1 - \pi_{S_i}) + \frac{P_i(1 - P_i)}{(2P_i - 1)^2} \right]. \end{aligned} \tag{2.10}$$

Hong et al. (1994) result

Under proportional allocation (*i.e.* $n_i = n(N_i/N)$), the variance of $\hat{\pi}_S$ in (2.10) reduces to

$$\text{Var}(\hat{\pi}_S)_P = \frac{1}{n} \sum_{i=1}^k w_i \left[\pi_{S_i}(1 - \pi_{S_i}) + \frac{P_i(1 - P_i)}{(2P_i - 1)^2} \right]. \tag{2.11}$$

For $P_i = P$ for all i , (2.11) becomes the following

$$\text{Var}(\hat{\pi}_S)_P^* = \frac{1}{n} \sum_{i=1}^k w_i L_i^2, \quad (2.12)$$

where

$$L_i = \sqrt{\left[\pi_{Si}(1 - \pi_{Si}) + \frac{P(1 - P)}{(2P - 1)^2} \right]}, \quad \text{for } i = 1, 2, \dots, k.$$

Kim and Warde (2004) result

If prior information on π_{Si} is available from past experience, then under the optimum allocation:

$$\frac{n_i}{n} = \frac{w_i V_i}{\sum_{i=1}^k w_i V_i}. \quad (2.13)$$

The minimal variance of the estimator $\hat{\pi}_S$ due to Kim and Warde (2004) is given by

$$\text{Var}(\hat{\pi}_S)_O = \frac{1}{n} \left(\sum_{i=1}^k w_i V_i \right)^2, \quad (2.14)$$

where

$$V_i = \sqrt{\left[\pi_{Si}(1 - \pi_{Si}) + \frac{P_i(1 - P_i)}{(2P_i - 1)^2} \right]}.$$

For $P_i = P$ for all i , (2.14) becomes

$$\text{Var}(\hat{\pi}_S)_O^* = \frac{1}{n} \left(\sum_{i=1}^k w_i L_i \right)^2. \quad (2.15)$$

From (2.12) and (2.14) we have

$$\text{Var}(\hat{\pi}_S)_P^* - \text{Var}(\hat{\pi}_S)_O^* = \frac{1}{n} \sum_{i=1}^k w_i \left\{ L_i - \left(\sum_{i=1}^k w_i L_i \right) \right\}^2 \quad (2.16)$$

which is always positive. It follows that the stratified RR technique due to Kim and Warde (2004) is more efficient than Hong *et al.* (1994) stratified RR technique when $P_i = P$ for all i . In the case that $P_i \neq P$ for all i , it is difficult to derive the mathematical condition. In this paper we present a stratified randomized response model based on Singh and Joarder (1997) model using (i) proportional allocation, and (ii) optimal allocation. A comparative study is also presented; in addition, numerical illustrations are given in support of the present study.

3. Proposed Model

In this model, if an individual respondent in the sample of stratum i belongs to the sensitive group, then he/she is requested to repeat the trial in the Hong *et al.* (1994) and Kim and Warde (2004) (originally Warner's (1965)) randomization device if in the first trial he/she does not get the statement according to his/her status. The rest of the procedure remains the same. The repetition of the trial is known to the interviewee but remains unknown to the interviewer (see Singh and Joarder, 1997). Thus, the model may be known as "unknown repeated trial model in stratified sampling". Assuming completely truthful reporting by the respondent, the probability of a 'Yes' answer in a stratum i for this procedure is

$$\theta_{1i} = \pi_{Si}[P_i + (1 - P_i)P_i] + (1 - \pi_{Si})(1 - P_i), \tag{3.1}$$

where θ_{1i} is the proportion of 'Yes' answer in a stratum i , π_{Si} is the proportion of respondents with the sensitive trait in a stratum i and P_i is the probability that a respondent in the sample stratum i has a sensitive question (S) card.

An unbiased estimator of π_{Si} is shown to be

$$\hat{\pi}_{Si}^* = \frac{\hat{\theta}_{1i} - (1 - P_i)}{2P_i - 1 + P_i(1 - P_i)}. \tag{3.2}$$

Since each $\hat{\theta}_{1i}$ is a binomial distribution $B(n_i, \theta_{1i})$ and the selections in different strata are made independently and unbiased estimator of π_S is shown to be

$$\hat{\pi}_S^* = \sum_{i=1}^k w_i \hat{\pi}_{Si}^* = \sum_{i=1}^k w_i \left[\frac{\hat{\theta}_{1i} - (1 - P_i)}{2P_i - 1 + P_i(1 - P_i)} \right]. \tag{3.3}$$

As each estimator $\hat{\pi}_{Si}^*$ is unbiased for π_{Si} , the expected value of $\hat{\pi}_S^*$ is

$$E(\hat{\pi}_S^*) = E \left(\sum_{i=1}^k w_i \hat{\pi}_{Si}^* \right) = \sum_{i=1}^k w_i E(\hat{\pi}_{Si}^*) = \sum_{i=1}^k w_i \pi_{Si} = \pi_S. \tag{3.4}$$

Since each unbiased estimator $\hat{\pi}_{Si}^*$ has its own variance, the variance of $\hat{\pi}_S^*$ is

$$\begin{aligned} \text{Var}(\hat{\pi}_S^*) &= \text{Var} \left(\sum_{i=1}^k w_i \hat{\pi}_{Si}^* \right) = \sum_{i=1}^k w_i^2 \text{Var}(\hat{\pi}_{Si}^*) \\ &= \sum_{i=1}^k \frac{w_i^2}{n_i} \frac{\theta_{1i}(1 - \theta_{1i})}{\{2P_i - 1 + P_i(1 - P_i)\}^2} \\ &= \sum_{i=1}^k \frac{w_i^2}{n_i} \left[\pi_{Si}(1 - \pi_{Si}) + \frac{P_i(1 - P_i)}{\{2P_i - 1 + P_i(1 - P_i)\}^2} - \frac{\pi_{Si}P_i(1 - P_i)}{\{2P_i - 1 + P_i(1 - P_i)\}} \right]. \end{aligned} \tag{3.5}$$

(i) Proportional allocation

Under the assumption $n_i = nw_i$, the variance in (3.5) reduces to

$$\text{Var}(\hat{\pi}_S^*)_P = \frac{1}{n} \sum_{i=1}^k w_i V_i^*, \tag{3.6}$$

where

$$V_i^* = \left[\pi_{S_i}(1 - \pi_{S_i}) + \frac{P_i(1 - P_i)}{\{2P_i - 1 + P_i(1 - P_i)\}^2} - \frac{\pi_{S_i}P_i(1 - P_i)}{\{2P_i - 1 + P_i(1 - P_i)\}} \right].$$

For $P_i = P$ for all i , (3.5) becomes the following.

$$\begin{aligned} \text{Var}(\hat{\pi}_S^*)_P &= \frac{1}{n} \sum_{i=1}^k w_i \left[\pi_{S_i}(1 - \pi_{S_i}) + \frac{P(1 - P)}{\{2P - 1 + P(1 - P)\}^2} - \frac{\pi_{S_i}P(1 - P)}{\{2P - 1 + P(1 - P)\}} \right] \\ &= \frac{1}{n} \sum_{i=1}^k w_i L_i^{*2} \end{aligned} \quad (3.7)$$

where

$$L_i^* = \left[\pi_{S_i}(1 - \pi_{S_i}) + \frac{P(1 - P)}{\{2P - 1 + P(1 - P)\}^2} - \frac{\pi_{S_i}P(1 - P)}{\{2P - 1 + P(1 - P)\}} \right]^{\frac{1}{2}}. \quad (3.8)$$

(ii) Optimum allocation

Information on π_{S_i} is usually, unavailable; however, any prior information on π_{S_i} available from the past experience helps to devise the following optimal allocation formula.

Theorem 1. *The optimal allocation of n to n_1, n_2, \dots, n_{k-1} and n_k to obtain the minimum variance of the $\hat{\pi}_S^*$ subject to $n = \sum_{i=1}^k n_i$ is approximately given by*

$$\frac{n_i}{n} = \frac{w_i \sqrt{V_i^*}}{\sum_{i=1}^k w_i \sqrt{V_i^*}}. \quad (3.9)$$

Proof is simple so omitted.

The minimal variance of the estimator $\hat{\pi}_S^*$ is given by

$$\text{Var}(\hat{\pi}_S^*) = \frac{1}{n} \left(\sum_{i=1}^k w_i \sqrt{V_i^*} \right)^2. \quad (3.10)$$

The unbiased minimal variance of an estimator $\hat{\pi}_S^*$ follows on replacing n by $n - 1$. For $P_i = P$ for all i , (3.10) becomes

$$\text{Var}(\hat{\pi}_S^*)_O = \frac{1}{n} \left(\sum_{i=1}^k w_i \sqrt{L_i^*} \right)^2. \quad (3.11)$$

From (3.7) and (3.11) we have

$$\begin{aligned} \text{Var}(\hat{\pi}_S^*)_P - \text{Var}(\hat{\pi}_S^*)_O &= \frac{1}{n} \left[\sum_{i=1}^k w_i L_i^* - \left(\sum_{i=1}^k w_i \sqrt{L_i^*} \right)^2 \right] \\ &= \frac{1}{n} \sum_{i=1}^k w_i \left\{ L_i^* - \left(\sum_{i=1}^k w_i L_i^* \right) \right\}^2 \end{aligned} \quad (3.12)$$

Table 1: The relative efficiency of $(\hat{\pi}_S^*)$ under optimum allocation with respect to $(\hat{\pi}_S^*)^*$ under proportional allocation.

				$P = P_1$							
				0.6		0.7		0.8		0.9	
π_{S1}	π_{S2}	w_1	w_2	P_2 0.7	P_2 0.8	P_2 0.8	P_2 0.9	P_2 0.9	P_2 0.95	P_2 0.93	P_2 0.95
0.08	0.13	.7	.3	1.206	1.354	1.354	1.480	1.480	1.542	1.517	1.542
0.08	0.13	.3	.7	1.596	2.215	2.215	2.920	2.920	3.351	3.169	3.351
0.28	0.33	.7	.3	1.188	1.309	1.309	1.397	1.397	1.434	1.420	1.434
0.28	0.33	.3	.7	1.543	2.010	2.010	2.442	2.442	2.650	2.567	2.650
0.48	0.53	.7	.3	1.186	1.299	1.299	1.372	1.372	1.400	1.390	1.400
0.48	0.53	.3	.7	1.530	1.970	1.970	2.320	2.320	2.467	2.410	2.467
0.68	0.73	.7	.3	1.203	1.325	1.325	1.398	1.398	1.422	1.413	1.422
0.68	0.73	.3	.7	1.594	2.095	2.095	2.466	2.466	2.600	2.550	2.600
0.88	0.93	.7	.3	1.268	1.440	1.440	1.549	1.549	1.579	1.569	1.579
0.88	0.93	.3	.7	1.830	2.735	2.735	3.509	3.509	3.746	3.675	3.746

Table 2: The relative efficiency of $(\hat{\pi}_S^*)_P$ under proportional allocation with respect to Hong *et al.* (1994) $(\hat{\pi}_S^*)_P$ under proportional allocation.

				$P = P_1$							
				0.6		0.7		0.8		0.9	
π_{S1}	π_{S2}	w_1	w_2	P_2 0.7	P_2 0.8	P_2 0.8	P_2 0.9	P_2 0.9	P_2 0.95	P_2 0.93	P_2 0.95
0.08	0.13	.7	.3	4.779	4.779	4.779	4.779	4.779	4.779	4.779	4.779
0.08	0.13	.3	.7	4.773	4.773	4.773	4.773	4.773	4.773	4.773	4.773
0.28	0.33	.7	.3	4.826	4.826	4.826	4.826	4.826	4.826	4.826	4.826
0.28	0.33	.3	.7	4.843	4.843	4.843	4.843	4.843	4.843	4.843	4.843
0.48	0.53	.7	.3	5.126	5.126	5.126	5.126	5.126	5.126	5.126	5.126
0.48	0.53	.3	.7	5.173	5.173	5.173	5.173	5.173	5.173	5.173	5.173
0.68	0.73	.7	.3	5.794	5.794	5.794	5.794	5.794	5.794	5.794	5.794
0.68	0.73	.3	.7	5.891	5.891	5.891	5.891	5.891	5.891	5.891	5.891
0.88	0.93	.7	.3	7.211	7.211	7.211	7.211	7.211	7.211	7.211	7.211
0.88	0.93	.3	.7	7.430	7.430	7.430	7.430	7.430	7.430	7.430	7.430

which is always positive. The relative efficiency(RE) of the two variances is

$$RE = \frac{\text{Var}(\hat{\pi}_S^*)_P}{\text{Var}(\hat{\pi}_S^*)_O} > 1. \tag{3.13}$$

The value of relative efficiency is more than one and the proposed stratified RR technique under optimum allocation is more efficient than that of under proportional allocation, when $P_i = P$ for all i . It then becomes difficult to obtain the mathematical condition of the relative efficiency comparison from (3.10) and $\text{Var}(\hat{\pi}_S^*)_P$. Therefore, we conduct an empirical study on the relative efficiency. Suppose that there are two strata in a population and $P = P_1$ and $P_2 > P_1$. Findings of the relative efficiencies are given in Table 1. Table 1 shows that the value of the relative efficiency are greater than one for all parametric values considered in Table 1 and justifies the theoretical results.

Comparisons of the proposed stratified RR technique under proportional allocation with that of Hong et al (1994) stratified RR technique

From (2.12) and (3.7) the relative efficiency of the proposed stratified RR technique under propo-

Table 3: The relative efficiency of $(\hat{\pi}_S^*)_O$ under optimum allocation with respect to Kim and Warde (2004) $(\hat{\pi}_S)_O$ under optimum allocation.

				$P = P_1$							
				0.6		0.7		0.8		0.9	
π_{S1}	π_{S2}	w_1	w_2	P_2	P_2	P_2	P_2	P_2	P_2	P_2	P_2
				0.7	0.8	0.8	0.9	0.9	0.95	0.93	0.95
0.08	0.13	.7	.3	4.112	4.042	4.045	4.093	4.093	4.143	4.121	4.143
0.08	0.13	.3	.7	3.098	2.760	2.760	2.724	2.724	2.779	2.750	2.779
0.28	0.33	.7	.3	4.130	4.023	4.023	4.021	4.021	4.034	4.028	4.034
0.28	0.33	.3	.7	3.116	2.722	2.722	2.597	2.597	2.579	2.583	2.579
0.48	0.53	.7	.3	4.392	4.261	4.261	4.226	4.226	4.219	4.221	4.219
0.48	0.53	.3	.7	3.344	2.897	2.897	2.706	2.706	2.646	2.668	2.646
0.68	0.73	.7	.3	5.001	4.874	4.874	4.807	4.807	4.775	4.788	4.775
0.68	0.73	.3	.7	3.915	3.407	3.407	3.123	3.123	3.001	3.123	3.001
0.88	0.93	.7	.3	6.477	6.441	6.441	6.383	6.383	6.297	6.330	6.297
0.88	0.93	.3	.7	5.467	5.150	5.150	4.810	4.810	4.489	4.632	4.489

rtional allocation with respect to the Hong *et al.* (1994) stratified RR technique is:

$$RE((\hat{\pi}_S^*)^*, (\hat{\pi}_S)^*) = \frac{\text{Var}(\hat{\pi}_S)_P^*}{\text{Var}(\hat{\pi}_S^*)_P^*}$$

Findings of relative efficiencies are shown in Table 2.

Comparison of the proposed stratified RR technique under optimum allocation with that of Kim and Warde (2004) stratified RR technique.

From (2.14) and (3.11) the relative efficiency of the proposed stratified RR technique under optimum allocation with respect to Kim and Warde (2004) stratified RR technique is

$$RE[(\hat{\pi}_S^*)_O^*, (\hat{\pi}_S)_O] = \frac{\text{Var}(\hat{\pi}_S)_O}{\text{Var}(\hat{\pi}_S^*)_O^*}$$

Findings of relative efficiencies are shown in Table 3.

Table 2 and Table 3 clearly indicate that the proposed stratified RR technique under (i) proportional allocation and (ii) optimum allocation, is better than Hong *et al.* (1994) and Kim and Warde (2004) estimators respectively for the given values of $\pi_{S1}, \pi_{S2}, w_1, w_2, P = P_1 = P_2$.

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