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# ON ITERATIVE APPROXIMATION OF COMMON FIXED POINTS OF ASYMPTOTICALLY NONEXPANSIVE MAPPINGS WITH APPLICATIONS

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Abstract. In this paper, the problem of iterative approximation of common fixed points of asymptotically nonexpansive is investigated in the framework of Banach spaces. Weak convergence theorems are established. A necessary and sufficient condition for strong convergence is also discussed. As an application of main results, a variational inequality is investigated.

## 1. Introduction

Recently, iterative algorithms for computing common fixed points of nonlinear mappings has been considered by many authors ([1]–[6]).

From the method of generating iterative sequences, we can divide iterative algorithms into explicit algorithms and implicit algorithms. Recently, both explicit Mann-type iterative algorithms and implicit Mann-type iterative algorithms have been extensively studied for approximating common fixed points of nonlinear mappings ([7]–[16]).

In this paper, we consider the problem of approximating common fixed points of asymptotically nonexpansive mappings based on a general implicit iterative algorithm which includes an explicit iterative process as a special case. As an application of main results, a variational inequality is investigated in a uniformly convex and $q$-uniformly smooth Banach space.

## 2. Preliminaries

Let $E$ be a real Banach space and $E^*$ the dual space of $E$. Let $J_q$, where $q > 1$, denote the generalized duality mapping from $E$ into $2^{E^*}$ give by

$$J_q(x) = \{f^* \in E^* : \langle x, f^* \rangle = \|x\|^q,\ \|f^*\| = \|x\|^{q-1}\}, \quad \forall x \in E,$$

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where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. In particular, $J_2$ is called the normalized duality mapping which is usually denoted by $J$. It is well known (see, for example, [17]) that $J_q(x) = \|x\|^{q-2}J(x)$ if $x \neq 0$.

Let $U_E = \{x \in E : \|x\| = 1\}$. A Banach space $E$ is said to be *strictly convex* if for all $x, y \in E$ which are linearly independent, $\|x + y\| < \|x\| + \|y\|$. This condition is equivalent to the following:

$$\|x\| = \|y\| = 1, \quad \text{and} \quad x \neq y \Longrightarrow \left\| \frac{x+y}{2} \right\| < 1.$$

$E$ is said to be *uniformly convex* if for any two sequences $\{x_n\}$ and $\{y_n\}$ in $E$ such that $\|x_n\| = \|y_n\| = 1$ and $\lim_{n\to\infty} \|x_n + y_n\| = 2$, then $\lim_{n\to\infty} \|x_n - y_n\| = 0$ holds. It is known that a uniformly convex Banach space is reflexive and strictly convex.

A Banach space $E$ is said to be *smooth* if the limit

$$\lim_{t\to 0} \frac{\|x + ty\| - \|x\|}{t}$$

exists for all $x, y \in U_E$. It is said to be *uniformly smooth* if the limit is attained uniformly for all $x, y \in U_E$.

The modulus of smoothness of $E$ is the function $\rho_E : [0, \infty) \to [0, \infty)$ defined by

$$\rho_E(\tau) = \sup \left\{ \frac{1}{2}(\|x + y\| + \|x - y\|) - 1 : \|x\| \leq 1, \ \|y\| \leq \tau \right\}, \quad \forall \tau \geq 0.$$

The Banach space $E$ is uniformly smooth if and only if $\lim_{\tau\to\infty} \frac{\rho_E(\tau)}{\tau} = 0$. A Banach space $E$ is said to be $q$-uniformly smooth if there exists a constant $c > 0$ such that $\rho_E(\tau) \leq c\tau^q$. It is shown in [17] that there is no Banach space which is $q$-uniformly smooth with $q > 2$. Hilbert spaces, $L^p$ (or $l^p$) spaces and Sobolev space $W^p_m$, where $p \geq 2$, are 2-uniformly smooth. Typical examples of both uniformly convex and uniformly smooth Banach spaces are $L^p$, where $p > 1$. More precisely, $L^p$ is $\min\{p, 2\}$-uniformly smooth for every $p > 1$.

$E$ is said to satisfy *Opial's condition* (see [18]) if, for each sequence $\{x_n\}$ in $E$, $x_n \rightharpoonup x$, where $\rightharpoonup$ denotes weak convergence, implies that

$$\liminf_{n\to\infty} \|x_n - x\| < \liminf_{n\to\infty} \|x_n - y\|, \quad \forall y \in E \, (y \neq x).$$

Let $C$ be a nonempty subset of $E$ and $T : C \to C$ be a mapping. In this paper, the symbol $F(T)$ stands for the fixed point set of $T$. $T$ is said to be *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall x, y \in C.$$

$T$ is said to be *asymptotically nonexpansive* if there exists a sequence $\{k_n\} \subset [1, \infty)$ with $k_n \to 1$ as $n \to \infty$ such that

$$\|T^n x - T^n y\| \leq k_n \|x - y\|, \quad \forall x, y \in C, \ \forall n \geq 1.$$

The class of asymptotically nonexpansive mappings was introduced by Goebel and Kirk [19] as a generalization of the class of nonexpansive mappings. They proved that if $C$ is a nonempty, closed, convex, and bounded subset of a real uniformly convex Banach space, then every asymptotically nonexpansive self mapping has a fixed point (see [19]).

In order to prove our main results, we still need the following lemmas.

**Lemma 2.1.** ([20]) *Let $C$ be a nonempty, closed, and convex subset of a uniformly convex Banach space $E$. Let $T : C \to C$ be an asymptotically nonexpansive mapping. Then $I - T$ is demiclosed at zero, that is, $x_n \rightharpoonup x$ and $x_n - Tx_n \to 0$ imply that $x = Tx$.*

**Lemma 2.2.** ([21]) *Let $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ be nonnegative sequences satisfying the following condition:*

$$a_{n+1} \leq (1 + b_n)a_n + c_n, \quad \forall n \geq n_0,$$

*where $n_0$ is some nonnegative integer, $\sum_{n=1}^{\infty} b_n < \infty$ and $\sum_{n=1}^{\infty} c_n < \infty$. Then the limit $\lim_{n\to\infty} a_n$ exists.*

**Lemma 2.3.** ([15]) *Let $E$ be a uniformly convex Banach space, $r > 0$ a positive number and $B_r(0)$ a closed ball of $E$ with the center at zero. Then there exits a continuous, strictly increasing and convex function $g : [0, \infty) \to [0, \infty)$ with $g(0) = 0$ such that*

$$\| \sum_{s=1}^{m} (\alpha_s x_s) \|^2 \leq \sum_{s=1}^{m} (\alpha_s \|x_s\|^2) - \alpha_i \alpha_j g(\|x_i - x_j\|), \quad \forall i, j \in \{1, 2, \ldots, r\},$$

*where $x_1, x_2, \ldots, x_m \in B_r(0)$ and $\alpha_1, \alpha_2, \ldots, \alpha_m \in (0, 1)$ such that $\sum_{i=1}^{m} \alpha_i = 1$.*

## 3. Main results

Let $C$ be a nonempty, closed and convex subset of a Banach space $E$. Let $T : C \to C$ be an asymptotically nonexpansive mapping with the sequence $\{k_n\}$. For every $u \in C$ and $t_n \in (0, 1)$, define a mapping $T_n : C \to C$ by

$$T_n x = t_n u + (1 - t_n)T^n x, \quad \forall x \in C, \ \forall n \geq 1.$$

If $(1 - t_n)k_n < 1$, for every $n \geq 1$, then $T_n$ is a contraction. Hence, by the Banach contraction principal, there exists a unique fixed point of $T_n$, for every $n \geq 1$.

Let $x_0$ be chosen and $r \geq 1$ a positive integer. Let $\{\alpha_n\}$, $\{\beta_{n,1}\}$, $\{\beta_{n,2}\}, \ldots$, $\{\beta_{n,r}\}$, $\{\gamma_{n,1}\}$, $\{\gamma_{n,2}\}, \ldots$, $\{\gamma_{n,r}\}$ be real sequences in $(0, 1)$ such that

$$\alpha_n + \sum_{m=1}^{r} \beta_{n,m} + \sum_{m=1}^{r} \gamma_{n,m} = 1.$$

Let $S_m, T_m : C \to C$ be asymptotically nonexpansive mappings, for every $m \in \{1, 2, \ldots, r\}$.

Find $x_1$ by solving the following equation

$$x_1 = \alpha_1 x_0 + \sum_{m=1}^{r} \beta_{1,m} S_m x_0 + \sum_{m=1}^{r} \gamma_{1,m} T_m x_1.$$

Find $x_2$ by solving the following equation

$$x_2 = \alpha_2 x_1 + \sum_{m=1}^{r} \beta_{2,m} S_m^2 x_1 + \sum_{m=1}^{r} \gamma_{2,m} T_m^2 x_2.$$

$$\cdots.$$

Find $x_n$ by solving the following equation

$$x_n = \alpha_n x_{n-1} + \sum_{m=1}^{r} \beta_{n,m} S_m^n x_{n-1} + \sum_{m=1}^{r} \gamma_{n,m} T_m^n x_n.$$

$$\cdots$$

In view of the above, we have the following implicit iterative algorithm

$$x_0 \in C, \quad x_n = \alpha_n x_{n-1} + \sum_{m=1}^{r} \beta_{n,m} S_m^n x_{n-1} + \sum_{m=1}^{r} \gamma_{n,m} T_m^n x_n, \quad \forall n \geq 1. \tag{3.1}$$

If $S_m = I$, where $I$ is the identity mapping, for every $m \in \{1, 2, \ldots, r\}$, then (3.1) is reduced the following.

$$x_0 \in C, \quad x_n = (\alpha_n + \sum_{m=1}^{r} \beta_{n,m}) x_{n-1} + \sum_{m=1}^{r} \gamma_{n,m} T_m^n x_n, \quad \forall n \geq 1. \tag{3.2}$$

If $T_m = I$, where $I$ stands for the identity mapping, for every $m \in \{1, 2, \ldots, r\}$, then (3.1) is reduced the following.

$$x_0 \in C, \;\; x_n = \frac{\alpha_n}{1 - \sum_{m=1}^{r} \gamma_{n,m}} x_{n-1} + \frac{\sum_{m=1}^{r} \beta_{n,m}}{1 - \sum_{m=1}^{r} \gamma_{n,m}} S_m^n x_{n-1}, \;\forall n \geq 1. \tag{3.3}$$

Now, we neeed the following proposition for our main results.

**Proposition 3.1.** *Let $C$ be a nonempty, closed and convex subset of a uniformly convex Banach space $E$. Let $S_m, T_m : C \to C$ be asymptotically nonexpansive mappings with the sequence $\{s_{n,m}\}$ and $\{t_{n,m}\}$, for every $m \in \{1, 2, \ldots, r\}$, where $r \geq 1$. Assume that $\mathscr{F} = \cap_{m=1}^{r} F(S_m) \bigcap \cap_{m=1}^{r} F(T_m)$ is nonempty. Let $t_n = \max\{t_{n,m} : 1 \leq m \leq r\}$ and $s_n = \max\{s_{n,m} : 1 \leq m \leq r\}$. Assume that $\sum_{n=1}^{\infty} (k_n - 1) < \infty$, where $k_n = \max\{s_n, t_n : 1 \leq m \leq r\}$. Let $\{x_n\}_{n=0}^{\infty}$ be a sequence generated by (3.1), where $\{\alpha_n\}$, $\{\beta_{n,1}\}$, $\{\beta_{n,2}\}, \ldots,$ $\{\beta_{n,r}\}$, $\{\gamma_{n,1}\}$, $\{\gamma_{n,2}\}, \ldots,$ $\{\gamma_{n,r}\}$ are real number sequences in $(0, 1)$ such that $\alpha_n + \sum_{m=1}^{r} \beta_{n,m} + \sum_{m=1}^{r} \gamma_{n,m} = 1$. Assume that the control sequences $\{\alpha_n\}$, $\{\beta_{n,1}\}$, $\{\beta_{n,2}\}, \ldots,$ $\{\beta_{n,r}\}$, $\{\gamma_{n,1}\}$, $\{\gamma_{n,2}\}, \ldots,$ $\{\gamma_{n,r}\}$ are satisfied*

(a) $\liminf_{n\to\infty} \alpha_n \beta_{n,m} > 0$, *and* $\liminf_{n\to\infty} \alpha_n \gamma_{n,m} > 0$, $\forall m \in \{1, 2, \ldots, r\}$;
(b) $\sum_{m=1}^{r} \gamma_{n,m} t_n < 1$.

*Then*

$$\lim_{n\to\infty} \|x_n - S_m x_n\| = \lim_{n\to\infty} \|x_n - T_m x_n\| = 0, \quad \forall m \in \{1, 2, \ldots, r\}.$$

*Proof.* By the condition (b), we see that the sequence $\{x_n\}$ generated by iterative process (3.1) is well defined. For $p \in \mathscr{F}$, we see that

$$\begin{aligned}\|x_n - p\| &\le \alpha_n \|x_{n-1} - p\| + \sum_{m=1}^{r} \beta_{n,m} \|S_m^n x_{n-1} - p\| + \sum_{m=1}^{r} \gamma_{n,m} \|T_m^n x_n - p\| \\ &\le (\alpha_n + \sum_{m=1}^{r} \beta_{n,m} k_n) \|x_{n-1} - p\| + \sum_{m=1}^{r} \gamma_{n,m} k_n \|x_n - p\|.\end{aligned}$$

In view of $\liminf_{n\to\infty} \alpha_n \beta_{n,m} > 0$ and $\alpha_n + \sum_{m=1}^{r} \beta_{n,m} + \sum_{m=1}^{r} \gamma_{n,m} = 1$, we see that there exists some positive integer $n_1$ and a real number $a$, where $a \in (0, 1)$, such that

$$\sum_{m=1}^{r} \gamma_{n,m} \le a, \quad \forall n \ge n_1.$$

Since $\sum_{n=1}^{\infty} (k_n - 1) < \infty$, there exists some positive integer $n_2$ such that $k_n \le 1 + \frac{1-a}{2a}$, for all $n \ge n_2$. It follows that

$$\sum_{m=1}^{r} \gamma_{n,m} k_n \le b < 1, \quad \forall n \ge n_3,$$

where $b = a(1 + \frac{1-a}{2a})$ and $n_3 = \max\{n_1, n_2\}$. It follows that

$$\begin{aligned}\|x_n - p\| &\le \frac{\alpha_n + \sum_{m=1}^{r} \beta_{n,m} k_n}{1 - \sum_{m=1}^{r} \gamma_{n,m} k_n} \|x_{n-1} - p\| \\ &\le \big(1 + \frac{\alpha_n + \sum_{m=1}^{r} \beta_{n,m} k_n + \sum_{m=1}^{r} \gamma_{n,m} k_n - 1}{1 - \sum_{m=1}^{r} \gamma_{n,m} k_n}\big) \|x_{n-1} - p\| \qquad (3.4) \\ &\le \big(1 + \frac{k_n - 1}{1 - b}\big) \|x_{n-1} - p\|.\end{aligned}$$

It follows from Lemma 2.2 that $\lim_{n\to\infty} \|x_n - p\|$ exists. This implies that the sequence $\{x_n\}$ is bounded.

On the other hand, we find from Lemma 2.3 that

$$\begin{aligned}\|x_n - p\|^2 \le& \alpha_n \|x_{n-1} - p\|^2 + \sum_{m=1}^{r} \beta_{n,m} \|S_m^n x_{n-1} - p\|^2 + \sum_{m=1}^{r} \gamma_{n,m} \|T_m^n x_n - p\|^2 \\ &- \alpha_n \beta_{n,m} g(\|x_{n-1} - S_m^n x_{n-1}\|) \\ \le& (\alpha_n + \sum_{m=1}^{r} \beta_{n,m} k_n) \|x_{n-1} - p\|^2 + \sum_{m=1}^{r} \gamma_{n,m} k_n \|x_n - p\|^2 \\ &- \alpha_n \beta_{n,m} g(\|x_{n-1} - S_m^n x_{n-1}\|), \quad \forall m \in \{1, 2, \ldots, N\}.\end{aligned}$$

This implies that

$$\begin{aligned}&\alpha_n\beta_{n,m}g(\|x_{n-1}-S_m^nx_{n-1}\|)\\&\le(\alpha_nk_n+\sum_{m=1}^r\beta_{n,m}k_n)\|x_{n-1}-p\|^2+\sum_{m=1}^r\gamma_{n,m}k_n\|x_n-p\|^2\\&\quad-k_n\|x_n-p\|^2+(k_n-1)\|x_n-p\|^2\\&\le(\alpha_nk_n+\sum_{m=1}^r\beta_{n,m}k_n)(\|x_{n-1}-p\|^2-\|x_n-p\|^2)\\&\quad+(k_n-1)\|x_n-p\|^2,\quad\forall m\in\{1,2,\ldots,r\}.\end{aligned}$$

Since $\lim_{n\to\infty}\|x_n-p\|$ exists, from the condition (a) we have that

$$\lim_{n\to\infty}g(\|x_{n-1}-S_m^nx_{n-1}\|)=0,$$

for every $m\in\{1,2,\ldots,r\}$. It follows that

$$\lim_{n\to\infty}\|x_{n-1}-S_m^nx_{n-1}\|=0,\quad\forall m\in\{1,2,\ldots,r\}.\tag{3.5}$$

From the Lemma 2.3, we obthain that

$$\begin{aligned}\|x_n-p\|^2\le&\alpha_n\|x_{n-1}-p\|^2+\sum_{m=1}^r\beta_{n,m}\|S_m^nx_{n-1}-p\|^2+\sum_{m=1}^r\gamma_{n,m}\|T_m^nx_n-p\|^2\\&-\alpha_n\gamma_{n,m}g(\|x_{n-1}-T_m^nx_n\|)\\\le&(\alpha_n+\sum_{m=1}^r\beta_{n,m}k_n)\|x_{n-1}-p\|^2+\sum_{m=1}^r\gamma_{n,m}k_n\|x_n-p\|^2\\&-\alpha_n\gamma_{n,m}g(\|x_{n-1}-T_m^nx_n\|),\quad\forall m\in\{1,2,\ldots,r\}.\end{aligned}$$

This implies that

$$\begin{aligned}&\alpha_n\gamma_{n,m}g(\|x_{n-1}-T_m^nx_n\|)\\&\le(\alpha_nk_n+\sum_{m=1}^r\beta_{n,m}k_n)\|x_{n-1}-p\|^2+\sum_{m=1}^r\gamma_{n,m}k_n\|x_n-p\|^2\\&\quad-k_n\|x_n-p\|^2+(k_n-1)\|x_n-p\|^2\\&\le(\alpha_nk_n+\sum_{m=1}^r\beta_{n,m}k_n)(\|x_{n-1}-p\|^2-\|x_n-p\|^2)\\&\quad+(k_n-1)\|x_n-p\|^2,\quad\forall m\in\{1,2,\ldots,N\}.\end{aligned}$$

Since $\lim_{n\to\infty}\|x_n-p\|$ exists, from the condition (a) we have that

$$\lim_{n\to\infty}g(\|x_{n-1}-T_m^nx_n\|)=0,$$

for every $m\in\{1,2,\ldots,r\}$. It follows that

$$\lim_{n\to\infty}\|x_{n-1}-T_m^nx_n\|=0,\quad\forall m\in\{1,2,\ldots,r\}.\tag{3.6}$$

Notice that

$$\|x_n - x_{n-1}\| = \sum_{m=1}^{r} \beta_{n,m}\|S_m^n x_{n-1} - x_{n-1}\| + \sum_{m=1}^{r} \gamma_{n,m}\|T_m^n x_n - x_{n-1}\|.$$

From the (3.5) and (3.6), we find that

$$\lim_{n\to\infty} \|x_{n-1} - x_n\| = 0. \tag{3.7}$$

Notice that

$$\|x_n - T_m^n x_n\| \le \|x_n - x_{n-1}\| + \|x_{n-1} - T_m^n x_n\|, \quad \forall m \in \{1, 2, \dots, r\}.$$

This implies from (3.6), and (3.7) that

$$\lim_{n\to\infty} \|x_n - T_m^n x_n\| = 0, \quad \forall m \in \{1, 2, \dots, r\}. \tag{3.8}$$

On the other hand, we have

$$\begin{aligned}\|x_n - S_m^n x_n\| \le &\|x_n - x_{n-1}\| + \|x_{n-1} - S_m^n x_{n-1}\| \\ &+ \|S_m^n x_{n-1} - S_m^n x_n\|, \quad \forall m \in \{1, 2, \dots, r\}.\end{aligned}$$

Since $S_m$ is Lipschitz for every $m \in \{1, 2, \dots, r\}$, from (3.5) and (3.7) we know that

$$\lim_{n\to\infty} \|x_n - S_m^n x_n\| = 0, \quad \forall m \in \{1, 2, \dots, r\}. \tag{3.9}$$

Notice that

$$\begin{aligned}\|x_n - S_m x_n\| &\le \|x_n - x_{n+1}\| + \|x_{n+1} - S_m^{n+1} x_{n+1}\| \\ &\quad + \|S_m^{n+1} x_{n+1} - S_m^{n+1} x_n\| + \|S_m^{n+1} x_n - S_m x_n\| \\ &\le (1 + M)\|x_n - x_{n+1}\| + \|x_{n+1} - S_m^{n+1} x_{n+1}\| \\ &\quad + M\|S_m^n x_n - x_n\|,\end{aligned}$$

where $M = \sup_{n\ge1}\{k_n\}$. It follows from (3.7) and (3.9) that

$$\lim_{n\to\infty} \|x_n - S_m x_n\| = 0, \quad \forall m \in \{1, 2, \dots, r\}. \tag{3.10}$$

On the other hand, we have

$$\begin{aligned}\|x_n - T_m x_n\| &\le \|x_n - x_{n+1}\| + \|x_{n+1} - T_m^{n+1} x_{n+1}\| \\ &\quad + \|T_m^{n+1} x_{n+1} - T_m^{n+1} x_n\| + \|T_m^{n+1} x_n - T_m x_n\| \\ &\le (1 + M)\|x_n - x_{n+1}\| + \|x_{n+1} - T_m^{n+1} x_{n+1}\| \\ &\quad + M\|T_m^n x_n - x_n\|.\end{aligned}$$

It follows from (3.7) and (3.8) that

$$\lim_{n\to\infty} \|x_n - T_m x_n\| = 0, \quad \forall m \in \{1, 2, \dots, r\}. \tag{3.11}$$

This completes the proof. □

Now, we give the following weak convergence theorems with Opial's condition.

**Theorem 3.2.** *Let $C$ be a nonempty, closed, and convex subset of a uniformly convex Banach space $E$ which has Opial's condition. Let $S_m, T_m : C \to C$ be asymptotically nonexpansive mapping with the sequence $\{s_{n,m}\}$ and $\{t_{n,m}\}$, for every $m \in \{1, 2, \ldots, r\}$, where $r \geq 1$. Assume that $\mathscr{F} = \cap_{m=1}^{r} F(S_m) \bigcap \cap_{m=1}^{r} F(T_m)$ is nonempty. Let $t_n = \max\{t_{n,m} : 1 \leq m \leq r\}$ and $s_n = \max\{s_{n,m} : 1 \leq m \leq r\}$. Assume that $\sum_{n=1}^{\infty}(k_n - 1) < \infty$, where $k_n = \max\{s_n, t_n : 1 \leq m \leq r\}$. Let $\{x_n\}$ be a sequence generated by (3.1), where $\{\alpha_n\}$, $\{\beta_{n,1}\}$, $\{\beta_{n,2}\}, \ldots$, $\{\beta_{n,r}\}$, $\{\gamma_{n,1}\}$, $\{\gamma_{n,2}\}, \ldots$ , $\{\gamma_{n,r}\}$ are real number sequences in $(0, 1)$ such that $\alpha_n + \sum_{m=1}^{r} \beta_{n,m} + \sum_{m=1}^{r} \gamma_{n,m} = 1$. Assume that restrictions (a) and (b) as in Proposition 3.1 are satisfied. Then $\{x_n\}$ converges weakly to some point in $\mathscr{F}$.*

*Proof.* Since $\{x_n\}$ is bounded, we find that there exists a subsequence $\{x_{n_i}\} \subset \{x_n\}$ such that $\{x_{n_i}\}$ converges weakly to a point $\bar{x} \in C$. It follows from Lemma 2.1 and Proposition 3.1 that $\bar{x} \in \mathscr{F}$. Assume that there exists another subsequence $\{x_{n_j}\} \subset \{x_n\}$ such that $\{x_{n_j}\}$ converges weakly to a point $\hat{x} \in C$. It follows from Lemma 2.1 that $\hat{x} \in \mathscr{F}$. If $\bar{x} \neq \hat{x}$, then

$$\begin{aligned} \lim_{n\to\infty} \|x_n - \bar{x}\| &= \liminf_{i\to\infty} \|x_{n_i} - \bar{x}\| < \liminf_{i\to\infty} \|x_{n_i} - \hat{x}\| \\ &= \liminf_{j\to\infty} \|x_{n_j} - \hat{x}\| < \liminf_{j\to\infty} \|x_{n_j} - \bar{x}\| \\ &= \lim_{n\to\infty} \|x_n - \bar{x}\|. \end{aligned}$$

This is a contradiction. Hence $\bar{x} = \hat{x}$. Hence every subsequence converges to same point $\bar{x}$. This completes the proof. □

If $r = 1$, then Theorem 3.2 is reduced to the following.

**Corollary 3.3.** *Let $C$ be a nonempty, closed and convex subset of a uniformly convex Banach space $E$ which has Opial's condition. Let $S, T : C \to C$ be an asymptotically nonexpansive mappings with the sequences $\{s_n\}$ and $\{t_n\}$. Assume that $\mathscr{F} = F(S) \bigcap F(T)$ is nonempty. Assume that $\sum_{n=1}^{\infty}(k_n - 1) < \infty$, where $k_n = \max\{s_n, t_n : 1 \leq m \leq r\}$. Let $\{x_n\}$ be a sequence generated by the following*

$$x_0 \in C, \quad x_n = \alpha_n x_{n-1} + \beta_n S^n x_{n-1} + \gamma_n T^n x_n, \quad \forall n \geq 1,$$

*where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are real number sequences in $(0, 1)$ such that $\alpha_n + \beta_n + \gamma_n = 1$. Assume that the following restrictions imposed on the control sequences $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are satisfied*

(a) $\liminf_{n\to\infty} \alpha_n\beta_n > 0$ *and* $\liminf_{n\to\infty} \alpha_n\gamma_n > 0$*;*
(b) $\gamma_n t_n < 1$.

*Then $\{x_n\}$ converges weakly to some point in $\mathscr{F}$.*

If $S_m = I$, then Theorem 3.2 is reduced to the following.

**Corollary 3.4.** *Let $C$ be a nonempty, closed, and convex subset of a uniformly convex Banach space $E$ which has Opial's condition. Let $T_m : C \to C$*

*be an asymptotically nonexpansive mapping with the sequence* $\{t_{n,m}\}$*, for every* $m \in \{1, 2, \ldots, r\}$*, where* $r \geq 1$*. Assume that* $\mathscr{F} = \cap_{m=1}^{r} F(T_m)$ *is nonempty. Assume that* $\sum_{n=1}^{\infty}(t_n - 1) < \infty$*, where* $t_n = \max\{t_{n,m} : 1 \leq m \leq r\}$*. Let* $\{x_n\}$ *be a sequence generated by* (3.2)*, where* $\{\alpha_n\}$, $\{\beta_{n,1}\}$, $\{\beta_{n,2}\}, \ldots,$ $\{\beta_{n,r}\}$, $\{\gamma_{n,1}\}$, $\{\gamma_{n,2}\}, \ldots,$ $\{\gamma_{n,r}\}$ *are real number sequences in* $(0, 1)$ *such that* $\alpha_n + \sum_{m=1}^{r} \beta_{n,m} + \sum_{m=1}^{r} \gamma_{n,m} = 1$*. Assume that restrictions* $(a)$ *and* $(b)$ *in Proposition* 3.1 *are satisfied. Then* $\{x_n\}$ *converges weakly to some point in* $\mathscr{F}$*.*

If $T_m = I$, then Theorem 3.2 is reduced to the following.

**Corollary 3.5.** *Let* $C$ *be a nonempty, closed and convex subset of a uniformly convex Banach space* $E$ *which has Opial's condition. Let* $S_m : C \to C$ *be an asymptotically nonexpansive mapping with the sequence* $\{s_{n,m}\}$*, for every* $m \in \{1, 2, \ldots, r\}$*, where* $r \geq 1$ *with* $\mathscr{F} = \cap_{m=1}^{r} F(S_m)$ *is nonempty. Assume that* $\sum_{n=1}^{\infty}(s_n - 1) < \infty$*, where* $s_n = \max\{s_{n,m} : 1 \leq m \leq r\}$*. Let* $\{x_n\}$ *be a sequence generated by* (3.3)*, where* $\{\alpha_n\}$, $\{\beta_{n,1}\}$, $\{\beta_{n,2}\}, \ldots,$ $\{\beta_{n,r}\}$, $\{\gamma_{n,1}\}$, $\{\gamma_{n,2}\}, \ldots,$ $\{\gamma_{n,r}\}$ *are real number sequences in* $(0, 1)$ *such that* $\alpha_n + \sum_{m=1}^{r} \beta_{n,m} + \sum_{m=1}^{r} \gamma_{n,m} = 1$*. Assume that the condition* $(a)$ *in Proposition* 3.1 *are satisfied. Then* $\{x_n\}$ *converges weakly to some point in* $\mathscr{F}$*.*

Next, we give a necessary and sufficient condition for the strong convergence of (3.1).

**Theorem 3.6.** *Let* $C$ *be a nonempty, closed and convex subset of a uniformly convex Banach space* $E$*. Let* $S_m, T_m : C \to C$ *be asymptotically nonexpansive mappings with the sequences* $\{s_{n,m}\}$ *and* $\{t_{n,m}\}$*, for every* $m \in \{1, 2, \ldots, r\}$*, where* $r \geq 1$*. Assume that* $\mathscr{F} = \cap_{m=1}^{r} F(S_m) \bigcap \cap_{m=1}^{r} F(T_m)$ *is nonempty. Let* $t_n = \max\{t_{n,m} : 1 \leq m \leq r\}$ *and* $s_n = \max\{s_{n,m} : 1 \leq m \leq r\}$*. Assume that* $\sum_{n=1}^{\infty}(k_n - 1) < \infty$*, where* $k_n = \max\{s_n, t_n : 1 \leq m \leq r\}$*. Let* $\{x_n\}$ *be a sequence generated by* (3.1)*, where* $\{\alpha_n\}$, $\{\beta_{n,1}\}$, $\{\beta_{n,2}\}, \ldots,$ $\{\beta_{n,r}\}$, $\{\gamma_{n,1}\}$, $\{\gamma_{n,2}\}, \ldots,$ $\{\gamma_{n,r}\}$ *are real number sequences in* $(0, 1)$ *such that* $\alpha_n + \sum_{m=1}^{r} \beta_{n,m} + \sum_{m=1}^{r} \gamma_{n,m} = 1$*. Assume that the conditions* $(a)$ *and* $(b)$ *in Proposition* 3.1 *are satisfied. Then* $\{x_n\}$ *converges strongly to some point in* $\mathscr{F}$ *if and only if*

$$\liminf_{n\to\infty} dist(x_n, \mathscr{F}) = 0.$$

*Proof.* The necessity of the proof is obvious. We only show the sufficiency of the proof. Assume that $\liminf_{n\to\infty} dist(x_n, \mathscr{F}) = 0$. In view of (3.4), we know from Lemma 2.2 that $\lim_{n\to\infty} dist(x_n, \mathscr{F})$ exists. From the hypothesis, it follows that $\lim_{n\to\infty} dist(x_n, \mathscr{F}) = 0$.

Next, we show that the sequence $\{x_n\}$ is Cauchy. For positive integers $m, n$, where $m > n$, we see from (3.4) that $\|x_n - p\| \leq e^{h_n}\|x_{n-1} - p\|$, where $h_n = \frac{k_n - 1}{1 - a}$. This in turn implies that

$$\|x_m - p\| \leq B\|x_n - p\|,$$

where $B = e^{\sum_{n=1}^{\infty} h_n}$. It follows that

$$\|x_n - x_m\| \leq \|x_n - p\| + \|x_m - p\| \leq (1 + B)\|x_n - p\|.$$

Taking the infimum over all $p \in \mathscr{F}$, we find that $\{x_n\}$ is a Cauchy sequence in $C$. Assume that $\{x_n\}$ converges strongly to some $\bar{q} \in C$. Since $T_m$ and $S_m$ are Lipschitz for each $m \in \{1, 2, \ldots, N\}$, we know that $\mathscr{F}$ is closed. This in turn implies that $\bar{q} \in \mathscr{F}$. This completes the proof. □

If $S_m = I$, then Theorem 3.6 is reduced to the following.

**Corollary 3.7.** *Let $C$ be a nonempty, closed and convex subset of a uniformly convex Banach space $E$. Let $T_m : C \to C$ be an asymptotically nonexpansive mapping with the sequence $\{t_{n,m}\}$, for every $m \in \{1, 2, \ldots, r\}$, where $r \geq 1$. Assume that $\mathscr{F} = \cap_{m=1}^{r} F(T_m)$ is nonempty. Assume that $\sum_{n=1}^{\infty}(t_n - 1) < \infty$, where $t_n = \max\{t_{n,m} : 1 \leq m \leq r\}$. Let $\{x_n\}$ be a sequence generated by* (3.2), *where $\{\alpha_n\}$, $\{\beta_{n,1}\}$, $\{\beta_{n,2}\}, \ldots, \{\beta_{n,r}\}$, $\{\gamma_{n,1}\}$, $\{\gamma_{n,2}\}, \ldots, \{\gamma_{n,r}\}$ are real number sequences in* $(0, 1)$ *such that $\alpha_n + \sum_{m=1}^{r} \beta_{n,m} + \sum_{m=1}^{r} \gamma_{n,m} = 1$. Assume that the conditions* (a) *and* (b) *in Proposition* 3.1 *are satisfied. Then $\{x_n\}$ converges strongly to some point in $\mathscr{F}$ if and only if* $\liminf_{n\to\infty} dist(x_n, \mathscr{F}) = 0$.

If $T_m = I$, then Theorem 3.6 is reduced to the following.

**Corollary 3.8.** *Let $C$ be a nonempty, closed and convex subset of a uniformly convex Banach space $E$. Let $S_m : C \to C$ be an asymptotically nonexpansive mapping with the sequence $\{s_{n,m}\}$, for every $m \in \{1, 2, \ldots, r\}$, where $r \geq 1$ is some positive integer. Assume that $\sum_{n=1}^{\infty}(s_n - 1) < \infty$, where $s_n = \max\{s_{n,m} : 1 \leq m \leq r\}$. Let $\{x_n\}$ be a sequence generated by* (3.2), *where $\{\alpha_n\}$, $\{\beta_{n,1}\}$, $\{\beta_{n,2}\}, \ldots, \{\beta_{n,r}\}$, $\{\gamma_{n,1}\}$, $\{\gamma_{n,2}\}, \ldots, \{\gamma_{n,r}\}$ are real number sequences in* $(0, 1)$ *such that $\alpha_n + \sum_{m=1}^{r} \beta_{n,m} + \sum_{m=1}^{r} \gamma_{n,m} = 1$. Assume that the condition* (a) *in Proposition* 3.1 *are satisfied. Then $\{x_n\}$ converges strongly to some point in $\mathscr{F}$ if and only if* $\liminf_{n\to\infty} dist(x_n, \mathscr{F}) = 0$.

## 4. Applications

Finally, we consider the problem of approximation solutions of variational inequalities as an application of main results.

Let $C$ be a nonempty, closed and convex subset of a smooth Banach space $E$ and $A : C \to E$ an operator. Find an $x \in C$ such that

$$\langle Ax, J(y - x)\rangle \geq 0, \quad \forall y \in C. \tag{4.1}$$

In what follows, the symbol $VI(C, A)$ stands for the solution set of the above inequality (4.1).

$A$ is said to be accretive if

$$\langle Ax - Ay, J(x - y)\rangle \geq 0, \quad \forall x, y \in C.$$

$A$ is said to be $\alpha$-inverse-strongly accretive if there exists a positive constant $\alpha$ such that

$$\langle Ax - Ay, J(x - y)\rangle \geq \alpha\|Ax - Ay\|^2, \quad \forall x, y \in C.$$

Let $K$ be a nonempty subset of $C$ and let $Q : C \to K$ be a mapping. $Q$ is said to be sunny if

$$Qx = Q\big(Qx + t(x - Qx)\big)$$

whenever $Qx + t(x - Qx) \in C$ for $x \in C$ and $t \geq 0$. $Q$ is said to be retraction if $Q^2 = Q$. $Q$ is said to be a sunny nonexpansive retraction if $Q$ is sunny nonexpansive and a retraction onto $K$. A subset $K$ of $C$ is said to be a sunny nonexpansive retract of $C$ if there exists a sunny nonexpansive retraction from $C$ onto $K$.

The following results describe a characterization of sunny nonexpansive retractions on a smooth Banach space; see [22] and [23] for more details.

Let $C$ be a nonempty subset of a smooth Banach space $E$. Let $Q_C$ be a sunny nonexpansive retraction from $E$ onto $C$. Then the following are equivalent:

(a) $Q_C$ is sunny and nonexpansive;
(b) $\langle x - Qx, J(Qx - y)\rangle$, $\forall x \in C, y \in K$.

The following lemma can be found in [17] and [24].

**Lemma 4.1.** *Let $E$ be a $q$-uniformly smooth Banach space with $q$-uniformly smoothness constant $C_q > 0$. Then the following holds*

$$\|x + y\|^q \leq \|x\|^q + q\langle y, J_q x\rangle + C_q\|y\|^q, \quad \forall x, y \in E.$$

Now, we are in a position to give the main results of this section.

**Theorem 4.2.** *Let $E$ be a uniformly convex and $q$-uniformly smooth Banach space with $q$-uniformly smoothness constant $C_q > 0$ and $C$ be a nonempty, closed and convex subset of $E$. Let $Q_C$ be a sunny nonexpansive retraction from $E$ onto $C$. Let $A_m : C \to E$ be a $a_m$-inverse-strongly accretive operator and $B_m : C \to E$ a $b_m$-inverse-strongly accretive operator, for every $m \in \{1, 2, \ldots, r\}$, where $r \geq 1$. Assume that $\mathscr{F} = \cap_{m=1}^{r} VI(C, A_m) \bigcap \cap_{m=1}^{r} VI(C, B_m)$ is nonempty. Let $\{x_n\}$ be a sequence generated by the following: $x_0 \in C$,*

$$\begin{aligned} x_n =& \alpha_n x_{n-1} + \sum_{m=1}^{r} \beta_{n,m} Q_C(x_{n-1} - \mu_m A_m x_{n-1}) \\ &+ \sum_{m=1}^{r} \gamma_{n,m} Q_C(x_n - \nu_m B_m x_n), \quad \forall n \geq 1, \end{aligned} \tag{4.2}$$

*where $\{\alpha_n\}$, $\{\beta_{n,1}\}$, $\{\beta_{n,2}\}$, $\cdots$, $\{\beta_{n,r}\}$, $\{\gamma_{n,1}\}$, $\{\gamma_{n,2}\}$, $\cdots$ , $\{\gamma_{n,r}\}$ are real number sequences in $(0, 1)$ such that $\alpha_n + \sum_{m=1}^{r} \beta_{n,m} + \sum_{m=1}^{r} \gamma_{n,m} = 1$ and $\mu_1, \mu_2, \ldots, \mu_r, \nu_1, \nu_2, \ldots, \nu_r$ are real numbers such that $\mu_m \leq \big(\frac{qa_m}{C_q}\big)^{\frac{1}{q}}$ and $\nu_m \leq \big(\frac{qb_m}{C_q}\big)^{\frac{1}{q}}$, for every $m \in \{1, 2, \ldots, r\}$. Assume that the condition (a) in Proposition 3.1 are satisfied. If $E$ has Opial's condition, then $\{x_n\}$ converges weakly to some point in $\mathscr{F}$.*

*Proof.* From Lemma 2.7 of Aoyama, Iiduka and Takahashi [24], we find, for every $m \in \{1, 2, \cdots, r\}$, that $VI(C, A_m) = F(Q_C(I - \lambda A))$ and $VI(C, A_m) =$

$F(Q_C(I-\lambda B))$ for all $\lambda > 0$. Notice that $Q_C(I-\mu_m A_m)$ and $Q_C(I-\nu_m B_m)$ are nonexpansive. Indeed, we find from Lemma 4.1 that

$$\begin{aligned}
&\|Q_C(I-\mu_m A_m)x - Q_C(I-\mu_m A_m)y\|^q \\
&\le \|(x-y) - \mu_m(A_m x - A_m y)\|^q \\
&\le \|x-y\|^q - q\mu_m\langle A_m x - A_m y, J_q(x-y)\rangle + C_q\mu_m^q\|A_m x - A_m y\|^q \\
&\le \|x-y\|^q - qa_m\mu_m\|A_m x - A_m y\|^q + C_q\mu_m^q\|A_m x - A_m y\|^q \\
&= \|x-y\|^q - (qa_m\mu_m - C_q\mu_m^q)\|A_m x - A_m y\|^q \\
&= \|x-y\|^q, \quad \forall x, y \in C.
\end{aligned}$$

This proves that $Q_C(I-\mu_m A)$ is nonexpansive, so is $Q_C(I-\mu_m B)$. Since nonexpansive mappings are asymptotically nonexpansive mappings with the sequence $\{1\}$, we can easily conclude from Theorem 3.2 the desired conclusion. This completes the proof. □

**Theorem 4.3.** *Let $E$ be a uniformly convex and q-uniformly smooth Banach space with q-uniformly smoothness constant $C_q > 0$ and $C$ a nonempty, closed and convex subset of $E$. Let $Q_C$ be a sunny nonexpansive retraction from $E$ onto $C$. Let $A_m : C \to E$ be an $a_m$-inverse-strongly accretive operator and $B_m : C \to E$ a $b_m$-inverse-strongly accretive operator, for every $m \in \{1, 2, \dots, r\}$, where $r \ge 1$. Assume that $\mathscr{F} = \cap_{m=1}^r VI(C, A_m) \bigcap \cap_{m=1}^r VI(C, B_m)$ is nonempty and $C_q \le \lambda q$, where $\lambda = \min\{a_m\mu_m, b_m\nu_m : 1 \le m \le r\}$. Let $\{x_n\}$ be a sequence generated by the following: $x_0 \in C$,*

$$\begin{aligned}
x_n =& \alpha_n x_{n-1} + \sum_{m=1}^r \beta_{n,m} Q_C(x_{n-1} - \mu_m A_m x_{n-1}) \\
&+ \sum_{m=1}^r \gamma_{n,m} Q_C(x_n - \nu_m B_m x_n), \quad \forall n \ge 1,
\end{aligned}$$

*where $\{\alpha_n\}$, $\{\beta_{n,1}\}$, $\{\beta_{n,2}\}$, $\cdots$, $\{\beta_{n,r}\}$, $\{\gamma_{n,1}\}$, $\{\gamma_{n,2}\}, \cdots$, $\{\gamma_{n,r}\}$ are real number sequences in $(0,1)$ such that $\alpha_n + \sum_{m=1}^r \beta_{n,m} + \sum_{m=1}^r \gamma_{n,m} = 1$ and $\mu_1, \mu_2, \dots, \mu_r, \nu_1, \nu_2, \dots, \nu_r$ are real numbers such that $\mu_m \le \left(\frac{qa_m}{C_q}\right)^{\frac{1}{q}}$ and $\nu_m \le \left(\frac{qb_m}{C_q}\right)^{\frac{1}{q}}$, for every $m \in \{1, 2, \cdots, r\}$. Assume that the condition (a) in Proposition 3.1 are satisfied. Then $\{x_n\}$ converges strongly to some point in $\mathscr{F}$ if and only if $\liminf_{n\to\infty} dist(x_n, \mathscr{F}) = 0$.*

*Proof.* Notice that $Q_C(I-\mu_m A_m)$, and $Q_C(I-\nu_m B_m)$ are nonexpansive. We can immediately conclude from Theorem 3.6 the desired conclusion. This completes the proof. □

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