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COMMON FIXED POINT THEOREM FOR OCCASIONALLY WEAKLY BAISED MAPPINGS AND ITS APPLICATION TO BEST APPROXIMATION

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Abstract. The aim of this paper is to prove a common fixed point theorem in normed linear spaces for discontinuous, occasionally weakly biased mappings without assuming completeness of the space. We give an example to illustrare our theorem. We also give an application of our theorem to best approximation theory. Our theorem improve the results of Gregus [9], Jungck [12], Pathak, Cho and Kang [22], Sharma and Deshpande [26]-[28].

1. Introduction and preliminaries

Sessa [24] defined a generalization of commutativity, which is called weak commutativity. Further Jungck [11] generalized the concept of weakly compatible maps. The concept of compatible maps further widened by Jungck and Rhoades [13], with the notion of weakly compatible maps.

It may be observed in this context that it is known since the paper of Kannan [17] in 1968 that there exist maps that have discontinuity in their domain but which have fixed points. However, the maps involved were continuous at the fixed points.

The study of common fixed points of noncompatible mappings is also very interesting. Work along these lines has recently been initiated by Pant [20], [21].

Jungck and Pathak [15], introduced weakly biased maps. Bouhadjera and Djoudi [4], generalized weakly biased maps and introduced the concept of occasionally weakly biased maps in metric spaces.

Definition 1. ([11]) Let X be a normed linear space and let $A, B : X \to X$ be two mappings. A and B are said to be compatible if whenever $\{x_n\}$ is a

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sequence in X such that Ax_n, $Bx_n \to t \in X, n \to \infty$ then

$$||ABx_n - BAx_n|| \to 0 \quad (\text{as } n \to \infty).$$

Definition 2. ([13]) Two self mappings A and B of a normed linear space X are said to be weakly compatible if they commute at their coincidence points.

It is easy to see that two compatible maps are weakly compatible but converse need not true.

Definition 3. ([1]) Let A and B be two self-mappings of a normed linear space X. A and B satisfy the property (E.A) if there exists a sequence $\{x_n\}$ such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = t, \qquad \text{for some } t \in X.$$

Remark 1. ([1]) It is clear from Jungck's definition [10] that two self-mappings A and B of a normed linear space X will be noncompatible if there exists at least one sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = t$, for some $t \in X$ but $\lim_{n\to\infty} ||ABx_n - BAx_n||$ is either non zero or non-existent. Therefore two noncompatible self-mappings of a normed linear space X satisfy the property (E.A).

Definition 4. ([14]) Two self maps A and B on a normed linear space (X, d) are said to be occasionally weakly compatible (owc) iff there exists some point x in X such that Ax = Bx and ABx = BAx.

Definition 5. ([4]) The pair $\{A, B\}$ is weakly B-biased and A-biased, respectively iff $Au = Bu$ implies

$$||ABu - Au|| \leq ||BAu - Bu||,$$
$$||BAu - Bu|| \leq ||ABu - Au||,$$

respectively. Cleary, every biased maps are weakly biased maps (see Propostion 1.1 in [14]) but the converse is false in general.

Definition 6. ([4]) Let A and B be self-maps of a normed linear space X. The pair $\{A, B\}$ is said to be occasionally weakly A-biased and B-biased, respectively, if and only if, there exists a point u in X such that $Au = Bu$ implies

$$||ABu - Au|| \leq ||BAu - Bu||,$$
$$||BAu - Bu|| \leq ||ABu - Au||,$$

respectively.

Of course, weakly A-biased maps and B-biased, respectively, are occasionally weakly A-biased maps and B-biased, respectively. However, the converses are not true in general.

The following example shows comparision between compatible, weakly compatible, weakly biased, occasionally weakly biased mappings.

Example 1. Let $X = [0, \infty)$ with the usual metric $d(x, y) = |x - y|$. Define $A, B : X \to X$ by

$$A(x) = \begin{cases} 2x, & \text{if } x \in (0, 1] \\ \frac{6}{x}, & \text{if } x \in (1, \infty), \end{cases}$$

$$B(x) = \begin{cases} 1, & \text{if } x \in (0, 1] \\ 2x, & \text{if } x \in (1, \infty). \end{cases}$$

Consider a sequence $\{x_n\} = \{\frac{1}{2} - \frac{1}{n}\}$ in X then

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = 1,$$
$$\lim_{n\to\infty} ||ABx_n - BAx_n|| = 1 \neq 0.$$

Thus the pair $\{A, B\}$ is non compatible.
We have $Ax = Bx$ if and only if $x = \frac{1}{2}$ or $x = \sqrt{3}$, $AB(\frac{1}{2}) = 2 \neq BA(\frac{1}{2}) = 1$.

Also

$$AB(\sqrt{3}) = \sqrt{3} \neq BA(\sqrt{3}) = 4\sqrt{3}.$$

Thus A and B are neither weakly compatible maps nor occasionally weakly compatible maps.

We further observe that

$$\sqrt{3} = ||AB(\sqrt{3}) - A(\sqrt{3})|| \leq ||BA(\sqrt{3}) - B(\sqrt{3})|| = 2\sqrt{3}.$$

Also

$$1 = ||AB(\frac{1}{2}) - A(\frac{1}{2})|| \nleq ||BA(\frac{1}{2}) - B(\frac{1}{2})|| = 0,$$
$$1 = d(AB(\frac{1}{2}), A(\frac{1}{2})) \nleq d(BA(\frac{1}{2}), B(\frac{1}{2})) = 0.$$

Thus A and B are not weakly A-biased, but A and B are occasionally weakly A-biased.

Let C be a subset of a normed linear space X and $A : X \to X$. The set of fixed points of A on X is denoted by $F(A)$. If $\bar{x}$ is a point of X, then for $0 < a \leq 1$, we define the set Da of best (C, a) -approximants to $\bar{x}$ consists of the point y in C such that

$$a||y - \bar{x}|| = \inf\{||z - \bar{x}|| : z \in C\}.$$

For $a = 1$our definition reduces to the set D of best C-approximants to $\bar{x}$. A subset C of X is said to be starshaped with respect to a point $p \in C$ if, for all x in C and for all $\lambda \in [0, 1]$, $\lambda x + (1 - \lambda)p \in C$. The point p is called the star-centre of C. A convex set is star shaped with respect to each of its points, but not conversely. For an example the set $C = \{0\} \times [0, 1] \cup [1, 0] \times \{0\}$ is star shaped with respect to $(0, 0) \in C$ as the star-centre of C, but it is not convex.

Many authors have studied the applications of fixed point theorems to best approximation theory including [2], [3], [5], [6]-[8], [10], [16], [18], [19], [22], [23], [25]-[34].

In this paper, we prove a common fixed point theorem in normed linear spaces for discontinuous, occasionally weakly biased mappings without assuming completeness of the space. We given an example to illustrate our theorem. Our theorem improve the results of Gregus [9], Jungck [12], Pathak, Cho and Kang [22], Sharma and Deshpande [28]. We also give an application of our main theorem to best approximation theory. Our application improves the results Pathak, Cho and Kang [22], Sharma and Deshpande [26]-[28].

2. Common fixed point theorem

Theorem 1. *Let S and T be two mappings of a normed space X into itself satisfying*

$$\begin{aligned} ||Tx-Ty||^p \leq a||Sx-Sy||^P &+ b\max\{||Tx-Sx||^p, ||Ty-Sy||^p\} \\ &+ c||Ty-Sy||^p, \end{aligned} \tag{1.1}$$

for all x, y in C, where a, b, $c > 0$, $a+b+c=1$, $a+2b<c$. If the pair $\{S,\ T\}$ is occasionally weakly $S-$biased. Then S and T have a unique common fixed point.

Proof. Since the pair {S, T} is occasionally weakly $S-$biased maps, Therefore there exists $u \in X$ such that

$$Su = Tu \Rightarrow ||STu - Su|| \leq ||TSu - Tu||.$$

Assume that $TTu \neq Tu$. Then using (1.1), we get

$$\begin{aligned} ||TTu-Tu||^p \leq a||STu-Su||^P &+ b\max\{||TTu-Su||^p, ||Tu-Su||^p\} \\ &+ c||Tu-Su||^p \end{aligned}$$

Since the pair {S, T} is occasionally weakly $S-$biased. Therefore

$$\begin{aligned} ||TTu-Tu||^p &\leq a||TTu-Tu||^P + b\max\{2||TTu-Tu||^p, ||Tu-Tu||^p\} \\ &\quad + c||Tu-Tu||^p \\ &\leq (a+2b)||TTu-Tu||^p \\ &< c||TTu-Tu||^p, \end{aligned}$$

which is a contradiction, so we have $TTu = Tu$ and so $STu = Su$, i. e. Tu is common fixed point of S and T.

If $Tu = z \in X$, then z is a common fixed point S and T .

The uniqueness of the common fixed point fallows from (1.1). For if $z_1 \in X$ is another common fixed point of S and T. Then by using (1.1), we get

$$\begin{aligned} ||z-z_1||^p &= ||Tz-Tz_1||^p \\ &\leq a||Sz-Sz_1||^P + b\max\{||Tz-Tz_1||^p, \|Tz_1-Sz_1\|^p\} \\ &\quad + c||Tz_1-Sz_1||^p \end{aligned}$$

$$= a||Tz - Tz_1||^P$$
$$< a||z - z_1||^p,$$

which is a contradiction. Therefore $z = z_1$. This completes the proof. □

If we put $p = 1$ in Theorem 1 we get the following:

Corollary 1. *Let S and T be two mappings of a normed space X into itself satisfying*

$$\begin{aligned} ||Tx - Ty|| \leq a||Sx - Sy|| + b \max\{||Tx - Sx||, ||Ty - Sy||\} \\ + c||Ty - Sy||, \end{aligned} \tag{1.2}$$

where $a, b, c > 0$, $a + b + c = 1$, $a + 2b < c$. If the pair $\{S, T\}$ is occasionally weakly $S-$ biased. Then S and T have a unique common fixed point.

Example 2. Consider $X = [0, 10)$, with the usual norm $||x - y|| = |x - y|$. Define $S, T : X \to X$ by

$$Sx = \begin{cases} x^2, & \text{if } x \in [0, 1) \\ x, & \text{if } x \in [1, 10), \end{cases}$$

$$Tx = \begin{cases} \frac{1}{4}, & \text{if } x \in [0, 1) \\ \frac{2}{x}, & \text{if } x \in [1, 10). \end{cases}$$

Consider a sequence $\{x_n\} = \{\frac{1}{2} + \frac{1}{n}\}$ in X then

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = \frac{1}{4},$$
$$\lim_{n\to\infty} ||STx_n - TSx_n|| = \frac{3}{64} \neq 0.$$

Thus the pair $\{S, T\}$ is non compatible. We have $Sx = Tx$ if and only if $x = \frac{1}{2}$ or $x = \sqrt{2}$, $\frac{1}{16} = ST(\frac{1}{2}) \neq TS(\frac{1}{2}) = \frac{1}{4}$.
Also

$$\sqrt{2} = ST(\sqrt{2}) = TS(\sqrt{2}) = \sqrt{2}.$$

Thus the pair $\{S, T\}$ is not weakly compatible.

We further observe that

$$\frac{3}{16} = ||ST(\frac{1}{2}) - S(\frac{1}{2})|| \nleq ||TS(\frac{1}{2}) - T(\frac{1}{2})|| = 0.$$

Also

$$0 = ||ST(\sqrt{2}) - S(\sqrt{2})|| \leq ||TS(\sqrt{2}) - T(\sqrt{2})|| = 0.$$

Thus S and T are not weakly S-biased but S and T are occasionally weakly S-biased.

If we take $a = \frac{1}{8}$, $b = \frac{3}{16}$, $c = \frac{11}{16}$, we can see that S and T satisfy all the conditions of Theorem 1 and have a unique common fixed point $\sqrt{2} \in X$.

Since two non compatible self-mappings of a normed linear space X satisfy the property (E.A), we get the following result:

3. Application of common fixed point theorem in best approximation

Theorem 2. *Let S and T be two mappings of a normed linear space X into itself and C be a nonempty, closed subset of X such that $A : \partial C \to C$ and $\bar{x} \in F(S) \cap F(T)$. Further, suppose that S and T satisfy (1.1) for all x, y in $D\prime a = Da \cup \{\bar{x}\} \cup E$, where*

$$E = \{q \in X : Sx_n, Tx_n \to q, \quad \{x_n\} \subset Da\},$$

$a, b, c > 0$, $a + b + c = 1$, $a + 2b < c$. If S and T are continuous on Da and the pair $\{S, T\}$ is occasionally weakly $S-$biased in Da. If Da is nonempty, compact convex and $T(Da) = Da$ then $Da \cap F(S) \cap F(T) \neq \phi$.

Proof. Let $y \in Da$ and hence Ty is in Da since $T(Da) = Da$. Further, if $y \in \partial C$ then Sy in C. Since $S(\partial C) \subset C$, from (1.1), it follows that

$$\begin{aligned} ||Ty - \bar{x}||^p &= ||Ty - T\bar{x}||^p \\ &\leq a||S\bar{x} - Sy||^p + b\max\{||Ty - Sy||^p, ||T\bar{x} - S\bar{x}||^p + c||Ty - Sy||^p \\ &\leq a||S\bar{x} - Sy||^p + b\max\{||Ty - \bar{x}||^p + ||\bar{x} - Sy||^p\} + c\{||Ty - \bar{x}||^p \\ &\quad + ||\bar{x} - Sy||^p\}, \end{aligned}$$

which implies a $||Sy - \bar{x}||^p \leq ||Ty - \bar{x}||^p$ and so Sy is in Da.

Since $\{S, T\}$ is occasionally weakly $S-$biased in Da there exist and $u \in Da$ such that

$$Su = Tu \Rightarrow ||STu - Su|| \leq ||TSu - Tu||.$$

Let $Su = Tu = z \in Da$ then we have

$$||Sz - z||^p \leq ||Tz - z||^p.$$

Next we claim that $Sz = Tz$ if not then by (1.1), we have

$$||Sz - Tz||^p \leq (1 - a)||Sz - Tz||^p$$

a contradiction. So we have $Sz = Tz$ by (1.1), we have

$$\begin{aligned} ||Tz - \bar{x}||^p &= ||Tz - T\bar{x}||^p \\ &\leq a||Sz - S\bar{x}||^p + b\max\{||Tz - Sz||^p, ||T\bar{x} - S\bar{x}||^p + c||T\bar{x} - S\bar{x}||^p \\ &\leq a||Tz - \bar{x}||^p, \end{aligned}$$

which is contraction. So $Sz = Tz = \bar{x}$.

Since S and T are noncompatible on Da so S and T satisfy the property $(E.A)$, therefore

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z, \quad \text{for some } z \in D(a). \tag{2.1}$$

Next we consider

$$\begin{aligned} ||Tz - Tx_n||^p &\leq a||Sz - Sx_n||^p + b\max\{||Tz - Sz||^p, ||Tx_n - Sx_n||^p\} \\ &\quad + c||Tx_n - Sx_n||^p. \end{aligned}$$

Taking the limit $n \to \infty$ yields

$$||\bar{x} - z||^p \leq a||\bar{x} - z||^p,$$

which is a contradiction, so $\bar{x} = z$ i. e., $z = Sz = Tz$. By Theorem 1, z must be unique. Hence $E = \{z\}$, then $D\prime a = Da \cup \{z\}$.

Let $\{e_n\}$ be a monotonically nondecreasing sequence of real numbers such that $0 \leq e_n < 1$ and $\overline{\lim}_{n\to\infty} e_n = 1$. Let $\{x_j\}$be a sequence in $D\prime a$ satisfying (2.1). For each $n \in N$, define a mapping $A_n : D\prime a \to D\prime a$ by

$$S_n x_j = e_n S x_j + (1 - e_n)p.$$

It is possible to define such a mapping S_n for each $n \in N$ since $D\prime a$ is starshaped with respect to $k \in F(T)$. We have

$$\begin{aligned}\lim_{j\to\infty} S_n x_j &= e_n \lim_{j\to\infty} S x_j + (1 - e_n)z \\ &= e_n z + (1 - e_n)z \\ &= z.\end{aligned}$$

Now, $S_n z = Tz = z$ and clearly S_n and T are occasionally weakly S_n-biased maps on $D\prime a$ for each n. On the other hand by (1.1), for all $x, y \in D\prime a$, we have for all $j \geq n$ and n fixed,

$$\begin{aligned}||S_n x - S_n y||^p &= e_n||Sx - Sy||^p \\ &\leq e_j||Sx - Sy||^p \\ &< ||Tx - Ty||^p \\ &\leq a||Sx - Sy||^P + b\max\{||Tx - Sx||^p, ||Ty - Sy||^p\} \\ &\quad + c||Ty - Sy||^p \\ &\leq a||Sx - Sy||^p + b\max\{||Sx - S_n x||^p + ||S_n x - Tx||^p, \\ &\quad ||Sy - S_n y||^p + ||S_n y - Ty||^p\} \\ &\quad + c\{||Sy - S_n y||^p + ||S_n y - Ty||^p\} \\ &\leq a||Sx - Sy||^p + b\max\{(1 - e_n)||Sx - k||^p + ||S_n x - Tx||^p, \\ &\quad (1 - e_n)||Sy - k||^p + ||S_n y - Ty||^p\} \\ &\quad + c\{(1 - e_n)||Sy - k||^p + ||S_n y - Ty||^p\}.\end{aligned}$$

Hence for all $j \geq n$, we have

$$\begin{aligned}||S_n x - S_n y||^p \leq\, & a||Sx - Sy||^p + b\max\{(1 - e_j)||Sx - k||^p + ||S_n x - Tx||^P, \\ & (1 - e_j)||Sy - k||^p \\ & + ||S_n y - Ty||^p\} \\ & + c\{(1 - e_j)||Sy - k||^p + ||S_n y - Ty||^p.\end{aligned} \tag{2.2}$$

Thus, since $\overline{\lim}_{j\to\infty} e_j = 1$, from (2.2) for every $n \in N$, we have

$$\begin{aligned}||S_n x - S_n y||^p \le \overline{\lim_{j\to\infty}}[a||Sx - Sy||^p &+ b\max\{(1-e_j)||Sx - k||^p \\ &+ ||S_n x - Tx||^p, (1-e_j)||Sy - k||^p + ||S_n y - Ty||^p\} \\ &+ c\{(1-e_j)||Sy - k||^p + ||S_n y - Ty||^p\},\end{aligned}$$

which implies

$$\begin{aligned}||S_n x - S_n y||^p = a||Sx - Sy||^p &+ b\max\{||S_n x - Tx||^p, ||S_n y - Ty||^p \\ &+ c||S_n y - Ty||^p\end{aligned}$$

for all $x, y \in D\prime a$. Therefore by Theorem 1, for every $n \in N$, S_n and T have a unique common fixed point x_n in $D\prime a$, i.e., for every $n \in N$, we have

$$F(S_n) \cap F(T) = \{x_n\}.$$

Now the compactness of Da ensures that $\{x_n\}$ has a convergent subsequence $\{x_{n_i}\}$ which converges to a point w in Da. Since

$$\begin{aligned}x_{n_i} &= S_{n_i} x_{n_i} \\ &= e_{n_i} S_{n_i} + (1 - e_{n_i})k\end{aligned} \tag{2.3}$$

and S is continuous, we have as $i \to \infty$ in (2.3) $w = Sw$, i.e., $w \in Da \cap F(S)$. Further, the continuity of T implies that

$$\begin{aligned}Tw &= T(\lim_{i\to\infty} x_{n_i}) \\ &= \lim_{i\to\infty} Tx_{n_i} \\ &= \lim_{i\to\infty} x_{n_i} \\ &= w,\end{aligned}$$

i.e., $w \in F(T)$. Therefore $w \in Da \cap F(S) \cap F(T)$ and so $Da \cap F(S) \cap F(T) \neq \phi$. This completes the proof.

□

References

[1] M. Aamri and D. El. Moutawakil, *Some new common fixed point theorems under strict contractive condition*, J. Math. Anal. Appl. **270** (2002), 181–188.

[2] A. Bano, A. R. Khan and A. Latif, *Coincidence points and best approximations in p-normed spaces*, Radovi, Mathematicki **12** (2003), 27–36.

[3] I. Beg, N. Shahzad and M. Iqbal, *Fixed point theorems and best approximation in convex metric spaces*, Approx. Theory and its Appl. **9** (1992), 97–105.

[4] H. Bouhadjera and A. Djoudi, *Fixed points for occasionally weakly biased maps*, SEA. Bull. of Maths. accepted for publication.

[5] B. Brosowski, *Fixpunktsatze in der approximation-theorie Mathematica (Cluj)* **11** (1969), 195–220.

[6] A. Carbone, *Applications of Fixed Points to Approximation Theory*, Jnanabha **19** (1989), 63–67.

[7] A. Carbone, *Applications of fixed point theorems*, Jnanabha **22** (1992), 85–91.

[8] E. W. Cheney, *Applications of fixed point Theory to Approximation Theory*, Proc. Approximation Theory and Application & Academic Press (1976), 1–8.

[9] M. Gregus, *A fixed point theorem in Banach space*, Boll. Um. Math. Ital. **17-A(5)** (1980), 221–225.

[10] T. L. Hicks and M. D. Humphries, *A note on fixed point theorems*, J. Approx. Theory **34** (1982), 221–225.

[11] G. Jungck, *Compatible mappings and common fixed points*, Internat J. Math. and Math. Sci. **9** (1986), 771–779.

[12] G. Jungck, *On a fixed point theorem of Fisher and Sessa*, Internat. J. Math. and Math. Sci. **13** (1990), 497–500.

[13] G. Jungck and B. E. Rhoades, *Fixed point for set valued functions without continuity*, Ind. J. Pure Appl. Math. **29(3)** (1998), 227–238.

[14] G. Jungck and Rhoads B. E., *Fixed point theorems for occasionally weakly compatible mappings*, Fixed Point Theory **7(2)** (2006), 287–296.

[15] G. Jungck and H. K. Pathak, *Fixed points via biased maps*, Proc. Amer. Math. Soc. **123 (7)** (1995), 2049–2060.

[16] G. Jungck and S. Sessa, *Fixed point theorems in best approximation theory*, Math. Japon **42(2)** (1995), 249–252.

[17] R. Kannan, *Some results on fixed points*, Bull. Cal. Math. Soc. **60** (1968), 71–76.

[18] T. D. Narang, *Applications of fixed point theorems to approximation theory*, Math vesnik **36** (1994), 69–75.

[19] A. Naz, *Best approximation in strongly M-starshped metric spaces*, Radovi Mathematicki **10** (2001), 203–207.

[20] R. P. Pant, *R-weak commutativity and common fixed points*, Soochow J. Math. **25** (1999), 37–42.

[21] R. P. Pant, *Common fixed points of contractive maps*, J. Math. Anal. Appl. **226** (1998), 251–258.

[22] H. K Pathak, Y. J. Cho and S. M. Kang, *An application of fixed point theorems in best approximation theory*, Internat. J. Math. and Math. Sci. **21(3)** (1998), 467–470.

[23] S. A. Sahab, M. S. Khan and S. Sessa, *A result in best approximation theory*, J. Approx. Theory **55** (1988), 349–351.

[24] S. Sessa, *On a weak commutativity condition of mappings in fixed point considerations*, Publ. Inst. Math. **32(46)** (1982), 149–153.

[25] N. Shahzad, *Noncommting maps and best approximation*, Radovi Mathematicki **10** (2001), 77–83.

[26] S. Sharma and B. Deshapnde, *Fixed point theorems and its application to best approximation theory*, Bull. Cal. Math. Soc. **93(2)** (2001), 155–166.

[27] S. Sharma and B. Deshpande, *Fixed point theorems for weakly compatible mappings and its application to best approximation theory*, J. Ind. Math. Soc. **69** (2002), 161–171.

[28] S. Sharma and B. Deshpande, *Fixed point theorems for noncompatible discontinuous mappings and best approximation*, East Asian Math. J. **24 (2)** (2008), 169–176.

[29] K. L. Sing, *Applications of fixed points to approximation theory*, Proc. Approximation Theory and Applications, Pitman, London (1985), 198-213. Ed S. P. Singh.

[30] S. P. Singh, *An application of a fixed point theorem to approximation theory*, J. Approx. Theory **25** (1979), 89–90.

[31] S. P. Singh, *Application of fixed point theorems in approximation theory*, Applied Nonlinear Analysis (Edited by V. Lakshikantham), Academic Press, New York (1979).

[32] P. V. Subrahmanyam, *An application of a fixed point theorem to best approximations*, J. Approx Theory **20** (1977), 165–172.

[33] M. A. Thagafi, *Best approximation and fixed points in strong M-starshped metric spaces*, Internat. J. Math. and Math. Sci. **18** (1995), 613–616.

[34] M. A. Thagafi, *Common fixed points and best approximation*, J. approx. theory **85** (1996), 318–323.

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