Journal of the Korean Data & Information Science Society
2012, **23**(6), 1249–1257

http://dx.doi.org/10.7465/jkdi.2012.23.6.1249
한국데이터정보과학회지

Estimation for generalized half logistic distribution based on records

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Received 12 October 2012, revised 6 November 2012, accepted 12 November 2012

Abstract

In this paper, we derive maximum likelihood estimators (MLEs) and approximate MLEs (AMLEs) of the unknown parameters in a generalized half logistic distribution when the data are upper record values. As an illustration, we examine the validity of our estimation using real data and simulated data. Finally, we compare the proposed estimators in the sense of the mean squared error (MSE) through a Monte Carlo simulation for various record values of size.

Keywords: Approximate maximum likelihood estimator, generalized half logistic distribution, record values.

1. Introduction

Inferences for the half logistic distribution were discussed by several authors. Balakrishnan and Puthenpura (1986) introduced the best linear unbiased estimators of location and scale parameters of the half logistic distribution through linear functions of order statistics. Balakrishnan and Wong (1991) obtained AMLEs for the location parameter and the scale parameter of the half logistic distribution with Type-II right censored samples. Kang and Park (2005) derived AMLE of the scale parameter in a half logistic distribution based on multiply Type-II censored samples. Kang *et al.* (2008) derived AMLEs and MLE of the scale parameter in a half logistic distribution based on progressively Type-II censored samples. Kang *et al.* (2009) proposed AMLEs of the scale parameter in a half logistic distribution based on double hybrid censored samples. Kang *et al.* (2010) considered the modified empirical distribution function type tests using AMLEs and the modified normalized sample Lorenz curve plot to test for the half logistic distribution based on multiply Type-II censored samples. Arora *et al.* (2010) obtained MLE and its asymptotic variance of the generalized half logistic distribution under Type-I progressive censoring with changing failure rates. They also provided some results including total expected waiting time in case of interval censoring schemes. Kim *et al.* (2011) derived approximated profile MLE of the scale parameter in a generalized half logistic distribution based on progressively Type-II censoring. Kim *et al.* (2011) proposed Bayes estimators of the shape parameter and reliability function in

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a generalized half logistic distribution based on progressively Type-II censored data under the various loss functions.

The cumulative distribution function (cdf) and the probability density function (pdf) of a random variable X with a generalized half logistic distribution are, respectively,

$$F(x) = 1 - \left(\frac{2e^{-x/\sigma}}{1+e^{-x/\sigma}}\right)^{\lambda}, \quad x > 0,\ \lambda,\, \sigma > 0 \tag{1.1}$$

and

$$f(x) = \frac{\lambda}{\sigma}\left(\frac{2e^{-x/\sigma}}{1+e^{-x/\sigma}}\right)^{\lambda}\frac{1}{1+e^{-x/\sigma}}, \tag{1.2}$$

where σ is scale parameter and λ is shape parameter. As a special case, if $\lambda = 1$, this distribution is a half logistic distribution.

In many cases, the maximum likelihood estimation method does not provide explicit estimators based on complete and censored samples. Hence it is desirable to develop an approximation to this estimation method which would provide us estimators that are explicit functions of order statistics. The approximate maximum likelihood estimation method was first developed by Balakrishnan (1989) for the purpose of providing explicit estimators of the scale parameter in the Rayleigh distribution. Han *et al.* (2007) discussed the estimation method for the reliability function with AMLEs in the exponentiated logistic distribution based on multiply Type-II censoring. Han and Kang (2008) derived AMLEs of the scale parameter in the half triangle distribution based on progressively Type-II censored samples. Kang and Seo (2011) developed two type AMLEs of the scale parameter in an exponentiated half logistic distribution based on progressively Type-II censored samples.

Chandler (1952) was the first to examine record values and documents a number of basic properties of records. Record values arise in many real-life situations involving weather, sports, economics, and life tests. A record model is closely related to the model of order statistics, and both are widely employed in statistical applications as well as in statistical modeling and inferences because they can be viewed as order statistics from a sample whose size is determined by the value and order of the occurrence of observations. In particular, Balakrishnan *et al.* (1992) established some recurrence relationships for single and double moments of lower record values from the Gumble distribution. Recently, Ahmadi and Balakrishnan (2011) discussed the prediction of future order statistics based on the largest and smallest observations when there is a new record.

The outline of the remaining sections is as follows. In sections 2 and 3, we derive MLEs and AMLEs of unknown parameters in a generalized half logistic distribution based on upper record values. In section 4, we details entropy estimation method of upper record values from the generalized half logistic distribution. Finally, in section 5, the proposed estimators are compared in the sense of the MSE using a Monte Carlo simulation study.

2. Maximum likelihood estimation

In this section, we discuss MLEs of the shape parameter λ and the scale parameter σ when data are upper record values.

Let X_1, X_2, $X_3, \dots$ be a sequence of independent and identically distributed (iid) random variables with a cdf $F(x)$ and a pdf $f(x)$. Setting $Y_n = \max(X_1, X_2, \dots, X_n)$, $n \geq 1$, we say that X_j is a upper record and denoted by $X_{U(j)}$ if $Y_j > Y_{j-1}$, $j > 1$. The indices for which upper record values occur are given by the record times $\{U(n), n \geq 1\}$, where $U(n) = \max\{j|j > U(n-1), X_j > X_{U(n-1)}\}$, $n > 1$, with $U(1) = 1$. From now on, we denote a sequence of upper record values $x_{U(1)}, x_{U(2)}, \cdots, x_{U(n)}$ by $x_1, x_2, \cdots, x_n$ for simplicity. The corresponding likelihood function of the first n upper record values, $x_1, x_2, \dots, x_n$ is

$$L(\lambda, \sigma) = f(x_n) \prod_{i=1}^{n-1} \frac{f(x_i)}{1 - F(x_i)}. \tag{2.1}$$

Suppose we observe n upper record values $x_1, x_2, \dots, x_n$ from the generalized half logistic distribution with pdf (1.2). It follows, from (1.1), (1.2), and (2.1), that

$$L(\lambda, \sigma) = \left(\frac{\lambda}{\sigma}\right)^n \left(\frac{2e^{-x_n/\sigma}}{1 + e^{-x_n/\sigma}}\right)^\lambda \prod_{i=1}^{n} \frac{1}{1 + e^{-x_i/\sigma}}. \tag{2.2}$$

The natural logarithm of the likelihood function (2.2) is given by

$$\log L(\lambda, \sigma) = n \log \lambda - n \log \sigma + \lambda \log \left(\frac{2e^{-x_n/\sigma}}{1 + e^{-x_n/\sigma}}\right) + \sum_{i=1}^{n} \log \left(\frac{1}{1 + e^{-x_i/\sigma}}\right). \tag{2.3}$$

From (2.3), the likelihood equations for λ and σ are, respectively,

$$\begin{aligned} \frac{\partial}{\partial \lambda} \log L(\lambda, \sigma) &= \frac{n}{\lambda} - h_1(x_n; \sigma) \\ &= 0 \end{aligned} \tag{2.4}$$

and

$$\begin{aligned} \frac{\partial}{\partial \sigma} \log L(\lambda, \sigma) &= -\frac{1}{\sigma} \left[n - \lambda \frac{x_n}{\sigma}(1 - h_2(x_n; \sigma)) + \sum_{i=1}^{n} \frac{x_i}{\sigma} h_2(x_i; \sigma) \right] \\ &= 0, \end{aligned} \tag{2.5}$$

where

$$h_1(x_n; \sigma) = \log \left(\frac{1 + e^{-x_n/\sigma}}{2e^{-x_n/\sigma}}\right),$$
$$h_2(x_n; \sigma) = \frac{e^{-x_i/\sigma}}{1 + e^{-x_i/\sigma}}.$$

Assuming that the scale parameter σ is known, the MLE of the shape parameter λ is obtained as

$$\hat{\lambda}(\sigma) = \frac{n}{h_1(x_n; \sigma)}. \tag{2.6}$$

Let $Y = n/h_1(X_n;\sigma)$. In Ahsanullah (1995), because the pdf of X_n is definded as

$$f_{X_n}(x) = \frac{1}{\Gamma(n)}\left[-\log(1-F(x))\right]^{n-1} f(x), \tag{2.7}$$

the pdf of Y is written as

$$f_Y(y) = \frac{(\lambda n)^n}{\Gamma(n)} y^{-n-1} e^{-\lambda n/y}, \quad y > 0, \tag{2.8}$$

which is a inverse gamma distribution with the shape parameter and the scale parameter as n and λn, respectively.

Hence, the MLE $\hat{\lambda}$ has the following expectation and varinace.

$$E(\hat{\lambda}) = \frac{\lambda n}{n-1} \tag{2.9}$$

and

$$Var(\hat{\lambda}) = \frac{(\lambda n)^2}{(n-1)^2(n-2)}. \tag{2.10}$$

From (2.9), we see that since bias of λ is $\lambda/(n-1)$, although the MLE $\hat{\lambda}$ is not an unbiased estimator of λ, it is an asymptotically unbiased estimator of λ.

If the scale parameter σ is unknown, we can find the MLE of σ, denote by $\hat{\sigma}$, by solving the Equation (2.5). Unfortunately, since the Equation (2.5) is cannot be solved explicitly, it may be solved by using the Newton-Raphson method that performs nonlinear optmization. To do this, it is required a initial value for λ, which is obtained by

$$\lambda_0 = \frac{n}{\left[h_1(x_n;\sigma)\right]_{\sigma=1}}. \tag{2.11}$$

Then we can obtain the MLE $\hat{\sigma}$ by updating σ value. From (2.6), the MLE $\hat{\lambda} = \hat{\lambda}(\hat{\sigma})$ can be calculated easily.

3. Approximate maximum likelihood estimation

As discussed earlier, because the Equation (2.5) is very complicated, it does not allow an explicit solution for σ. Therefore, we derive the AMLE of σ by solving the approximate likelihood equation.

Let $Z_i = X_i/\sigma$. Then we can write the likelihood equation (2.5) as

$$\begin{aligned}\frac{\partial}{\partial\sigma}\log L(\lambda,\sigma) &= -\frac{1}{\sigma}\left[n - \lambda(1-h_2(z_n))z_n + \sum_{i=1}^{n} h_2(z_i)z_i\right] \\ &= 0,\end{aligned} \tag{3.1}$$

where

$$h_2(z_i) = \frac{e^{-z_i}}{1+e^{-z_i}}.$$

Let

$$\xi_i = F^{-1}(p_i) = -\log\left[\frac{q_i^{1/\lambda}}{2-q_i^{1/\lambda}}\right],$$

where $q_i = 1 - p_i$ and p_i is a uniformly distributed random variate.

Using Taylor series, we approximate the following function:

$$h_2(z_i) \approx \alpha_i + \beta_i z_i, \tag{3.2}$$

where

$$\alpha_i = \frac{e^{-\xi_i}}{1+e^{-\xi_i}} + \frac{e^{-\xi_i}}{(1+e^{-\xi_i})^2}\xi_i,$$
$$\beta_i = -\frac{e^{-\xi_i}}{(1+e^{-\xi_i})^2}.$$

By using the Equation (3.2), we obtain the following approximate likelihood equation:

$$\frac{\partial}{\partial\sigma}\log L(\lambda,\,\sigma) \simeq -\frac{1}{\sigma}\left[n - \lambda(1-\alpha_n-\beta_n z_n)z_n + \sum_{i=1}^{n}(\alpha_i+\beta_i z_i)z_i\right]$$
$$= 0. \tag{3.3}$$

After solving the quadratic Equation (3.3) for σ, by substituting of the MLE $\hat{\lambda}$, we obtain the AMLE of σ as

$$\tilde{\sigma} = \frac{-B+\sqrt{(B^2-4nC)}}{2n}, \tag{3.4}$$

where

$$B = \hat{\lambda}x_n(\alpha_n - 1) + \sum_{i=1}^{n} x_i\alpha_i,$$
$$C = \hat{\lambda}x_n^2\beta_n + \sum_{i=1}^{n} x_i^2\beta_i.$$

As in the case of the MLE $\hat{\lambda}$, we obtain the AMLE of the shape parameter λ, denoted by $\tilde{\lambda}$, by replacing σ with $\tilde{\sigma}$ in the Equation (2.6). Note that the AMLE $\tilde{\sigma}$ is always positive because $\beta_i < 0$.

4. Entropy of records values

In this section we develope entropy estimation method of upper record values from generalized half logistic distribution.

Let X be a random variable with a cdf $F(x)$ and a pdf $f(x)$. Then the entropy of X is defined as

$$H(X) = -\int_{-\infty}^{\infty} f(x)\log f(x)dx. \tag{4.1}$$

Hence the entropy of X_n can be expressed as

$$H_n = -\int_{-\infty}^{\infty} f_{X_n}(x)\log f_{X_n}(x)dx, \tag{4.2}$$

where $f_{X_n}(x)$ is given by (2.7).

By Baratpour *et al.* (2007), the entropy H_n can be written as

$$H_n = \sum_{j=1}^{n-1}\left(\log j - \frac{n-1}{j}\right) + (n-1)C - I(u), \tag{4.3}$$

where C is the Euler's constant and

$$I(u) = \frac{1}{(n-1)!}\int_0^{\infty} u^{n-1}e^{-u}\log f\left(F^{-1}\left(1-e^{-u}\right)\right)du. \tag{4.4}$$

Then, we have

$$I(u)=\frac{1}{(n-1)!}\left[\log\left(\frac{\lambda}{\sigma}\right)\int_0^{\infty}u^{n-1}e^{-u}du-\int_0^{\infty}u^{n}e^{-u}du+\int_0^{\infty}u^{n-1}e^{-u}\log\left(1-\frac{e^{-u/\lambda}}{2}\right)du\right], \tag{4.5}$$

because of

$$\log f\left(F^{-1}\left(1-e^{-u}\right)\right) = \log\left(\frac{\lambda}{\sigma}\right) - u + \log\left(1-\frac{e^{-u/\lambda}}{2}\right). \tag{4.6}$$

Finally, using $\log(1-x) = -\sum_{j=1}^{\infty}\frac{x^j}{j}$,

$$I(u) = \log\left(\frac{\lambda}{\sigma}\right) - n - \sum_{j=1}^{\infty}\left(1+\frac{j}{\lambda}\right)^{-n}\frac{2^{-j}}{j}. \tag{4.7}$$

Therefore, the entropy H_n from the generalized half logistic distribution is given by

$$H_n(x) = n + (n-1)C - \log\left(\frac{\lambda}{\sigma}\right) + \sum_{j=1}^{n-1}\left(\log j - \frac{n-1}{j}\right) + \sum_{j=1}^{\infty}\left(1+\frac{j}{\lambda}\right)^{-n}\frac{2^{-j}}{j}. \tag{4.8}$$

We can obtain an estimator of entropy function (4.2), denote by $\hat{H}_n$, by replacing λ and σ with $\hat{\lambda}$ and $\hat{\sigma}$ in (4.8). Likewise, we can obtain another estimator of entropy function (4.2), denote by $\tilde{H}_n$, by replacing λ and σ with $\tilde{\lambda}$ and $\tilde{\sigma}$ in (4.8).

5. Illustrative example

In this section, we present an example to validate the estimation method and assess the performance of estimators discussed in the previous sections.

5.1. Real data

Consider the real data given by Hinkley (1977), which represents the thirty successive values of March precipitation (in inches) in Minneapolis/StPau (see Table 5.1) over a period of 30 years. Because the distribution of this data is skewed to the right, it has been used to introduce the concept of transformation. Torabi and Bagheri (2010) showed that this real data follow an extended generalized half logistic through Kolmogorov-Smirnov test.

Table 5.1 The thirty successive values of March precipitation (in inches) in Minneapolis/StPau.

0.77	1.74	0.81	1.20	1.95	1.20	0.47	1.43	3.37	2.20
3.00	3.09	1.51	2.10	0.52	1.62	1.31	0.32	0.59	0.81
2.81	1.87	1.18	1.35	4.75	2.48	0.96	1.89	0.90	2.05

From the above data, five upper records are observed, they are

$$0.77, \quad 1.74, \quad 1.95, \quad 3.37, \quad 4.75.$$

Using the formulas in sections 2 and 3, we obtain MLEs and AMLEs of the shape parameter λ and the scale parameter σ. In addition, we calculate the estimators of the entropy from (4.8). These values are given in Table 5.2. To check the goodness of fit for the generalized half logistic distribution with $\hat{\lambda}$ and $\hat{\sigma}$, we conduct a simple test.

The moment of upper record values is

$$E\left(X_i^k\right) = \frac{1}{\Gamma(i)} \int_{-\infty}^{\infty} x^k \left[-\log(1-F(x))\right]^{i-1} f(x)dx \quad \text{for} \quad i = 1, 2, \dots, \tag{5.1}$$

where $f(\cdot)$ and $F(\cdot)$ are given in (1.2) and (1.1), respectively. From (5.1), we compute for $k = 1$ the expected upper record values from the generalized half logistic distribution with $\hat{\lambda}$ and $\hat{\sigma}$ by using numerical integration. These values are given in Table 5.3. A simple plot of 5 upper records of the precipitation in Minneapolis/StPau against the expected values $E(X_i)$ in Table 5.3 indicates a strong correlation (0.97303). In addition, we have nearly the same results for the AMLEs, which provides support for the assumption that these upper record values are follow the generalized half logistic distribution.

Table 5.2 The MLEs and the AMLEs of λ and σ for the real data.

$\hat{\lambda}$	$\tilde{\lambda}$	$\hat{\sigma}$	$\tilde{\sigma}$	$\hat{H}_n$	$\tilde{H}_n$
0.37299	0.51624	0.33692	0.46193	3.96854	3.94917

Table 5.3 The expected value of the first generalized half logistic upper record values.

i	1	2	3	4	5
$E(X_i)$	1.12042	2.08003	2.99489	3.89616	4.79309

5.2. Simulation results

To assess the performance of the proposed estimators, we simulate the MSEs of all proposed estimators through Monte Carlo simulation method. Samples of upper record values with size $n = 8(1)13$, are generated from the standard generalized half logistic distribution with $\lambda = 0.5$. Using this samples, the MSEs of the estimators are simulated by the Monte Carlo method based on $10,000$ runs. For $\lambda = 4$, the same simulation method is carry out. The results are presented in Table 5.4.

From Table 5.4, we can see that the AMLE $\tilde{\lambda}$ is more efficient than the MLE $\hat{\lambda}$ for the shape parameter λ. For the scale parameter σ, while the MLE $\hat{\sigma}$ is more efficient than the AMLE $\tilde{\sigma}$ when $\lambda = 0.5$, the AMLE $\tilde{\sigma}$ is generally superior to the MLE $\hat{\sigma}$ when $\lambda = 4$. In the case of entropy, $\hat{H}_n$ has lower MSEs than $\tilde{H}_n$ when $\lambda = 0.5$ but $\tilde{H}_n$ has lower MSEs than $\hat{H}_n$ when $\lambda = 4$. That is, the AMLEs show an overall better performance than the MLEs for $\lambda = 4$. Also, as expected, the MSEs of all estimators decrease as sample size n increases.

Table 5.4 The relative MSEs for the MLEs and the AMLEs of λ and σ.

			$\lambda = 0.5$ and $\sigma = 1$			
n	$\hat{\lambda}$	$\tilde{\lambda}$	$\hat{\sigma}$	$\tilde{\sigma}$	$\hat{H}_n$	$\tilde{H}_n$
8	0.11027	0.07560	0.08582	0.09551	0.36612	0.36731
9	0.10108	0.06959	0.07157	0.07942	0.30140	0.30232
10	0.09239	0.06326	0.05984	0.06625	0.24919	0.24992
11	0.08288	0.05674	0.05233	0.05672	0.20723	0.20778
12	0.07466	0.05133	0.04831	0.05111	0.17899	0.17940
13	0.06475	0.04458	0.04734	0.04803	0.15412	0.15442
			$\lambda = 4$ and $\sigma = 1$			
n	$\hat{\lambda}$	$\tilde{\lambda}$	$\hat{\sigma}$	$\tilde{\sigma}$	$\hat{H}_n$	$\tilde{H}_n$
8	0.01565	0.00946	0.18054	0.17278	0.26717	0.26686
9	0.01099	0.00628	0.15460	0.14840	0.22119	0.22061
10	0.00768	0.00418	0.13245	0.12758	0.18414	0.18348
11	0.00500	0.00268	0.11284	0.10975	0.15458	0.15406
12	0.00340	0.00181	0.10070	0.09879	0.13479	0.13446
13	0.00258	0.00139	0.09038	0.08965	0.11710	0.11703

6. Concluding remarks

This paper develop MLEs and AMLEs of unknown parameters in a generalized half logistic distribution based on upper record values. The corresponding estimators of entropy function also are calculated. Because the MLE of the scale parameter cannot solved explicitly, we propose the AMLE as an alternative to that. When comparing these estimators in terms of the MSE, because the AMLEs show an overall better performance than the MLEs when the shape parameter is large, we would recommend the use of the AMLEs provided that the shape parameter has large value. Also, the results from the propsed estimators can be useful guidelines on design of experiments in various statistical fields such that modeling, inference, and life tests.

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