Opportunistic Decode-and-Forward Cooperation in Mixed Rayleigh and Rician Fading Channels

Changqing Yang, Wenbo Wang, Shuang Zhao, and Mugen Peng

We consider a two-hop multiple-relay network implemented with opportunistic decode-and-forward cooperative strategy, where the first hop and second hop links experience different fading (Rayleigh and Rician). We derive the exact expressions of end-to-end outage probability and analyze the approximate results in high signal-to-noise ratio region. The analysis shows that the same diversity order can be achieved even in different mixed fading environments. Simulation results are provided to verify our analysis.

Keywords: Opportunistic cooperation, decode-and-forward, Rayleigh fading, Rician fading.

I. Introduction

Opportunistic cooperation, which is also known as selection cooperation [1], has been shown to be an attractive alternative to cooperative diversity schemes based on maximum ratio combining [2]. To date, the performance analysis of opportunistic cooperation has been studied extensively under various fading channels, including Rayleigh [3], Nakagami-*m* [4], and Rician (Nakagami-*n*) [5]. However, most of the evaluations are obtained assuming that each hop experiences the same kind of channel fading (for example, Rayleigh fading), which does not conform to reality all the time. In practical systems, different links in relay networks may experience asymmetric fading conditions. For example, in relay enhanced cellular networks, base stations and relays may be located outdoors above surrounding building heights where no rich scattered environment exists. Therefore, the propagation between the base station and relay comprises a line of sight (LoS) component and a scattered component, which can be modeled as a Rician fading channel. Meanwhile, user equipment is usually surrounded by multiple scatters, and the relay-mobile link may only experience Rayleigh fading. In [6], the authors have studied the performance of a fixed two-hop amplify-and-forward relay system in mixed Ralyleigh and Rician fading channels, where only the single relay scenario is considered. Little attention has been paid on the performance evaluation of opportunistic cooperation under mixed fading channels.

In this letter, we study the outage probability of a two-hop decode-and-forward (DF) opportunistic cooperative network with multiple relays in an asymmetric fading environment. Specifically, we derive the exact expressions of the outage probability, which is verified through Monte Carlo simulation. We also analyze the approximate outage performance and obtain the diversity order. The main contributions of this letter include: i) the exact closed-form expression of outage probability has been derived and verified through Monte Carlo simulation, which is beneficial to network planning and optimization under practical environments and ii) the diversity order of opportunistic cooperation in mixed fading channels has been obtained through asymptotic analysis in high signal-to-noise ratio (SNR) region.

II. System and Channel Model

We consider a two-hop system where the source S communicates to the destination D with help of N relays

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$\{R_n\}_{n=1}^{N}$. Assume the direct link between the source and the destination is blocked by an intermediate wall, then the destination can only receive the signals from the relays. Due to the half-duplex transceiver equipped at the relays, a whole transmission takes place in two consecutive time slots, and the transmission strategy is the same as [2]. In the first slot, the source broadcasts, and all the relays listen. Those relays that can decode the signals successfully form a decoding set denoted as $D(S)$. In the second slot, a single relay that has the highest channel power gain (the squared amplitude of the wireless channel) to the destination is selected from $D(S)$ to decode, re-encode, and transmit the signals to the destination.

The channel power gains of S–R_n and R_n–D are denoted as α_n and β_n, respectively. Assume S and R_n have the same transmit power P, and the power of the additive white Gaussian noise is N_0. Then, we can get the instantaneous received SNR of the first and second hop as $\xi_n = P\alpha_n / N_0$ and $\zeta_n = P\beta_n / N_0$, respectively. As the mixed channel fading scenarios are considered in this letter, the fading features are different for the first and second hop links, and there are two cases. For case 1, the S–R_n link is subject to Rayleigh fading, and the R_n–D link is subject to Rician fading. For case 2, the S–R_n link is subject to Rician fading, and the R_n–D link is subject to Rayleigh fading. For either case, the links experience independent but not necessarily identical fading. To simplify the notation, we use γ_{Ra} and γ_{Ri} to denote the instantaneously received SNR of a certain hop that experiences Rayleigh fading and Rician fading, respectively.

If a link experiences Rayleigh fading, γ_{Ra} is exponentially distributed whose probability density function can be

$$p_{\gamma_{\mathrm{Ra}}}(\gamma) = \frac{1}{\bar{\gamma}_{\mathrm{Ra}}} e^{-\frac{\gamma}{\bar{\gamma}_{\mathrm{Ra}}}}, \tag{1}$$

where $\bar{\gamma}_{\mathrm{Ra}}$ is the average SNR of the link.

If a link experiences Rician fading, then γ_{Ri} is distributed according to a noncentral-χ^2 distribution given by [6]:

$$p_{\gamma_{\mathrm{Ri}}}(\gamma) = \frac{(K+1)e^{-K}}{\bar{\gamma}_{\mathrm{Ri}}} e^{-\frac{(K+1)\gamma}{\bar{\gamma}_{\mathrm{Ri}}}} I_0\left(2\sqrt{\frac{K(K+1)\gamma}{\bar{\gamma}_{\mathrm{Ri}}}}\right), \tag{2}$$

where K is the Rician K-factor defined as the power ratio of the LoS component to the scattered components. The average SNR of the link is $\bar{\gamma}_{\mathrm{Ri}}$, and $I_0(\cdot)$ is the zeroth order modified Bessel function of the first kind (see equation (8.447.1) of [7]).

III. Outage Performance Analysis

In a cooperative network, an outage only happens i) when the end-to-end transmission rate is below a predefined target R_{th} or ii) when the instantaneous end-to-end SNR falls below an equivalent threshold $\eta = 2^{2R_{\mathrm{th}}} - 1$, where the coefficient 2 before R_{th} models the required spectral efficiency per hop due to the half-duplex constraint.

From the law of total probability, we can get the outage probability of DF opportunistic cooperation as

$$P_{\mathrm{out}} = \sum_{D(S)} \Pr\{D(S)\} \Pr\{\mathrm{outage} \mid D(S)\}, \tag{3}$$

where $\Pr\{D(S)\}$ is the probability that a specific decoding set exists, and $\Pr\{\mathrm{outage} \mid D(S)\}$ is the conditional probability that an outage happens for a known decoding set.

For case 1, the probability that a certain decoding set exists can be obtained from [3] as

$$\begin{aligned}\Pr\{D(S)\} &= \prod_{j\in D(S)} \Pr\{\xi_j \ge \eta\} \prod_{j\notin D(S)} \Pr\{\xi_j < \eta\} \\ &= \prod_{j\in D(S)} e^{-\frac{\eta}{\bar{\xi}_j}} \prod_{j\notin D(S)} \left(1 - e^{-\frac{\eta}{\bar{\xi}_j}}\right),\end{aligned} \tag{4}$$

where $\bar{\xi}_j$ is the average SNR of the first hop for the j-th relay. According to equation (8.447.1) of [7], (2) can be transformed into

$$p_{\gamma_{\mathrm{Ri}}}(\gamma) = \frac{(K+1)e^{-K-\frac{(K+1)\gamma}{\bar{\gamma}_{\mathrm{Ri}}}}}{\bar{\gamma}_{\mathrm{Ri}}} \sum_{k=0}^{\infty} \frac{\left[K(K+1)\gamma/\bar{\gamma}_{\mathrm{Ri}}\right]^k}{(k!)^2}. \tag{5}$$

From equation (2.321.2) of [7], the cumulative distribution function (CDF) of γ_{Ri} can be obtained as

$$F_{\gamma_{\mathrm{Ri}}}(x) = \frac{K+1}{\bar{\gamma}_{\mathrm{Ri}}} e^{-K} \sum_{k=0}^{\infty} \left\{ \begin{array}{l} \dfrac{\left[K(K+1)x/\bar{\gamma}_{\mathrm{Ri}}\right]^k}{(k!)^2} \cdot \\ \left[e^{-\frac{K+1}{\bar{\gamma}_{\mathrm{Ri}}}x} \cdot G(k, K, \bar{\gamma}_{\mathrm{Ri}}, x) + \dfrac{k!}{\left(\dfrac{K+1}{\bar{\gamma}_{\mathrm{Ri}}}\right)^{k+1}} \right] \end{array} \right\}, \tag{6}$$

where $G(k, K, \bar{\gamma}_{\mathrm{Ri}}, x)$ is utilized to denote the expression of

$$G(k, K, \bar{\gamma}_{\mathrm{Ri}}, x) = \sum_{m=0}^{k} \left[(-1)^m m! \binom{k}{m} x^{k-m} \right] \Big/ \left[-\frac{K+1}{\bar{\gamma}_{\mathrm{Ri}}} \right]^{m+1}$$

to simplify the notation.

For simplicity, we use $F(K, \bar{\gamma}, x)$ to denote the CDF of SNR that experiences Rician fading with K-factor be K and the average SNR be $\bar{\gamma}$. Due to the relay selection protocol and the independence of the second hop links, we can get the conditional outage probability as

$$\begin{aligned}\Pr\left\{\text{outage}\middle|D(S)\right\} &= \prod_{j\in D(S)} \Pr\left\{\zeta_j < \eta\right\}\\ &= \prod_{j\in D(S)} F\left(K,\bar{\zeta}_j,\eta\right),\end{aligned} \tag{7}$$

where $\bar{\zeta}_j$ is the average SNR of the second hop for the j-th relay. If we bring (4) and (7) into (3), we can get the outage probability of opportunistic DF cooperation in case 1 as

$$P_{\text{out}1} = \sum_{D(S)} \prod_{j\in D(S)} e^{-\frac{\eta}{\bar{\xi}_j}} F\left(K,\bar{\zeta}_j,\eta\right) \prod_{j\notin D(S)} \left(1-e^{-\frac{\eta}{\bar{\xi}_j}}\right). \tag{8}$$

Similarly for case 2, the probability that a certain decoding set exists is [3]

$$\begin{aligned}\Pr\left\{D(S)\right\} &= \prod_{j\in D(S)} \Pr\left\{\xi_j \ge \eta\right\} \prod_{j\notin D(S)} \Pr\left\{\xi_j < \eta\right\}\\ &= \prod_{j\in D(S)} \left[1-F\left(K,\bar{\xi}_j,\eta\right)\right] \prod_{j\notin D(S)} F\left(K,\bar{\xi}_j,\eta\right).\end{aligned} \tag{9}$$

Since the best relay in $D(S)$ is selected to forward the signals for the source, the conditional outage probability is

$$\begin{aligned}\Pr\left\{\text{outage}\middle|D(S)\right\} &= \prod_{j\in D(S)} \Pr\left\{\zeta_j < \eta\right\}\\ &= \prod_{j\in D(S)} \left(1-e^{-\frac{\eta}{\bar{\zeta}_j}}\right).\end{aligned} \tag{10}$$

If we bring (9) and (10) into (3), we can get the outage probability of opportunistic DF cooperation in case 2 as

$$P_{\text{out}2} = \sum_{D(S)} \prod_{j\in D(S)} \left[1-F\left(K,\bar{\xi}_j,\eta\right)\right]\left(1-e^{-\frac{\eta}{\bar{\zeta}_j}}\right) \prod_{j\notin D(S)} F\left(K,\bar{\xi}_j,\eta\right). \tag{11}$$

Diversity order is another useful metric to evaluate the asymptotic performance of a cooperative diversity protocol. According to [2], we can see that diversity order can be viewed as the power to which $\left(P/N_0\right)^{-1}$ is raised in the approximate expressions of outage probability in a high SNR region. Specifically speaking, diversity order is defined as $d = \lim_{P/N_0\to\infty} -\log\left(P_{\text{out}}\right)/\log\left(P/N_0\right)$, which is equivalent to the minimum power of P^{-1} in the approximate expression. Through asymptotic analysis, we can get the diversity order of the system as below.

Theorem 1. For a two-hop network consisting of N relays and implemented with an opportunistic DF cooperative strategy in mixed Rayleigh and Rician fading channels, the achievable diversity order is N.

Proof. When $\bar{\xi}_j$ and $\bar{\zeta}_j$ are very large, we can get the approximate outage probability of case 1 as

$$P_{\text{out}1} \approx \sum_{D(S)} \prod_{j\in D(S)} \frac{(K+1)e^{-K}}{\bar{\zeta}_j}\left(\eta - \frac{(K+1)\eta^2}{2\bar{\zeta}_j}\right) \prod_{j\notin D(S)} \frac{\eta}{\bar{\xi}_j}. \tag{12}$$

As the different values of the average SNR are not related to the diversity order, the analysis on diversity order can be simplified under a symmetric network. Assume each hop has an identical average SNR respectively, that is, $\bar{\xi}_j = \bar{\xi}$, $\bar{\zeta}_j = \bar{\zeta}$, $\forall j \in \{1,\ldots,N\}$. Then, (12) can be further simplified as

$$P_{\text{out}1} \approx \left(\frac{\eta}{\bar{\xi}}\right)^N \left[1+\frac{\bar{\xi}}{\bar{\zeta}}(K+1)e^{-K}\right]^N. \tag{13}$$

Thus, the diversity order for case 1 is N.

In a high SNR region, the outage probability of case 2 can be simplified as

$$P_{\text{out}2} \approx \sum_{D(S)} \begin{matrix} \prod_{j\in D(S)} \left[1-\frac{(K+1)e^{-K}}{\bar{\xi}_j}\left(\eta-\frac{(K+1)\eta^2}{2\bar{\xi}_j}\right)\right]\frac{\eta}{\bar{\zeta}_j} \\ \cdot \prod_{j\notin D(S)} \frac{(K+1)e^{-K}}{\bar{\xi}_j}\left(\eta-\frac{(K+1)\eta^2}{2\bar{\xi}_j}\right) \end{matrix}. \tag{14}$$

When each hop has an identical average SNR respectively, (14) can be rewritten as

$$P_{\text{out}2} \approx \left(\frac{\eta}{\bar{\zeta}}\right)^N \left[1+\frac{\bar{\zeta}}{\bar{\xi}}(K+1)e^{-K}\right]^N. \tag{15}$$

Thus, the diversity order for case 2 is N. Since for either case 1 or case 2, the diversity order is N, we complete the proof. □

IV. Simulation and Numerical Results

In this section, we present Monte Carlo simulation results to verify our analysis. The exact analytical results (solid curves) are from (8) and (11) using only 20 summations (k=0,…, 19), and the approximate results (dashed curves) are from (13) and (15). The markers are obtained from simulation with 10 million independent snapshots.

From Fig. 1, we can see that the asymmetry of the two hops affects case 1 more significantly than case 2 because the gap between the two curves of case 1 is larger than that of case 2. The transmission is more liable to fail under Rayleigh fading channels than Rician ones with the same average SNR per hop, especially when the K-factor is large. This conclusion can also be explained through (13) and (15), where the coefficient $(K+1)e^{-K}$ is much less than 1 (approaching 0) with a larger K, which makes case 1 and case 2 be more sensitive to the first and second hop (both are under Rayleigh fading), respectively. For both cases, the average SNR of the second hop are

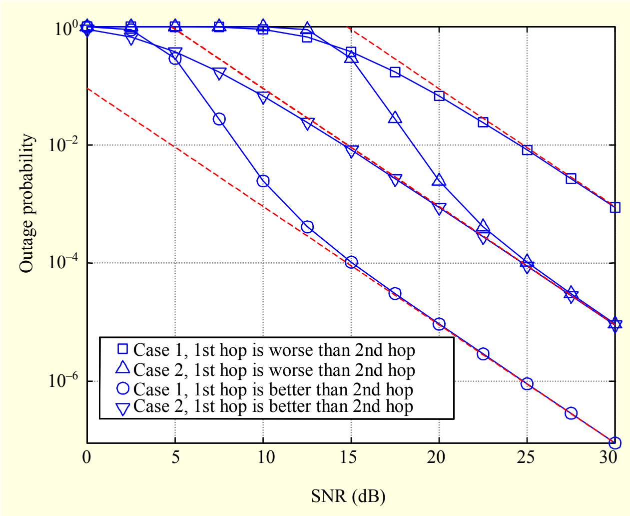

Fig. 1. Outage probability vs. average SNR of second hop (dB) with different average SNR of first hop. N=2, K=10 dB, R_{th}=1 bps/Hz, $\bar{\xi}_j = 10\bar{\zeta}_j$ when 1st hop is better than 2nd hop. $\bar{\xi}_j = 0.1\bar{\zeta}_j$ when 1st hop is worse than 2nd hop.

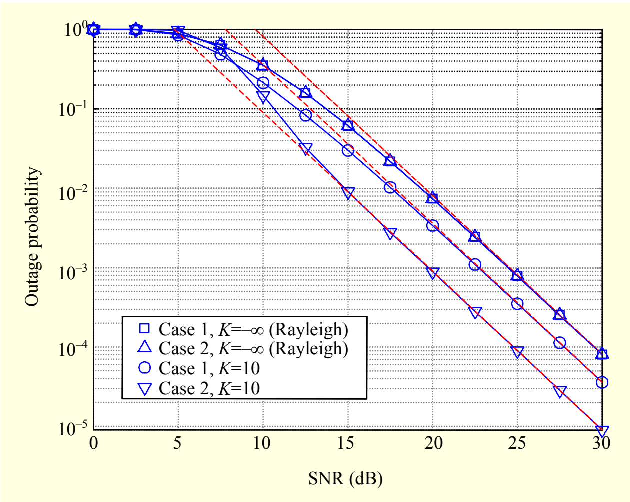

Fig. 2. Outage probability vs. average SNR of second hop (dB) with different K-factors (dB) when 1st hop is worse than 2nd hop. N=2, R_{th}=1 bps/Hz, $\bar{\xi}_j = 0.5\bar{\zeta}_j$.

identical, which makes case 1 change rapidly according to the varying average SNR of the first hop.

Figure 2 shows the outage probability for different K-factors. When K-factor approaches negative infinity (dB), Rician fading channels are equivalent to Rayleigh fading ones, so the two curves overlap in the plot. When the K-factor becomes larger, the two curves separate from each other gradually. In an extreme case, the K approaches positive infinity (dB). Then, $(K+1)e^{-K}$ approaches 0, and the approximate expressions of outage probability for two cases are $(\eta/\bar{\xi})^N$ and $(\eta/\bar{\zeta})^N$, respectively (see (13) and (15)).

V. Conclusion

We have derived the exact expressions of the outage probability for opportunistic DF cooperation in mixed Rayleigh and Rician fading channels and analyzed the asymptotic performance in a high SNR region. Even in different mixed fading environments (case 1 and case 2), the same diversity can be achieved, and that order is equal to the relay number. The analysis is useful for system design engineer on network planning and optimization when LoS and non-LoS environments should be taken into consideration simultaneously.

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