Distribution of Path Loss for Wireless Personal Networks Operating in a Square Region

Rumin Yang, Bin Shen, and Kyung Sup Kwak

Path loss plays fundamental roles in system design, spectrum management, and performance evaluation. The traditional path loss model has a slight inconvenience; it depends on the unknown distance. In this letter, we explore the probability distribution function (PDF) of path loss in an indoor office environment by randomizing out the distance variable. It is shown that the resulting PDF is not Gaussian-like but is skewed to the right, and both the PDF and the moments are related to the size of the office instead of the unknown distance. To be specific, we incorporate the IEEE 802.15.4a channel parameters into our model and tabulate the cumulative distribution function with respect to different room sizes. Through a simple example, we show how our model helps a cognitive spectrum user to infer path loss information of primary users without necessarily knowing their transmitter-receiver distance.

Keywords: Path loss, PDF of path loss, WPAN.

I. Introduction

Path loss is a physical phenomenon in wireless transmission, indicating the power loss after a signal passes through a wireless channel. In the design of a conventional system, path loss is used to budget the transmit power, determine the signal coverage, and evaluate the bit-error performance. In the state-of-the-art cognitive radio network, path loss is used in optimal resource allocation and dynamic spectrum management [1]. However, the conventional path loss model has a remarkable shortcoming. It depends on the path distance, which can be measured by the communicating partners but is unknown to other interested onlookers, such as cognitive secondary spectrum users, who are obliged to be aware of the primary users' parameters, including the distance and path loss, before making spectrum-access decisions. In cases where the provision of such information by the primary users is impractical, either because an older-type device is incapable or a broadcasting channel is unavailable, the path loss statistics become most desirable.

Manuscript received June 22, 2010; accepted Aug. 23, 2010.
This work was supported by Inha University Research Grant.
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doi:10.4218/etrij.11.0210.0218.

II. Probability Distribution of Indoor Path Loss

1. Path Loss Model in Literature

Path loss is defined as the transmit power divided by received power and is typically characterized in decibel scales as

$$L = l_0 + 10n\log_{10} D + S, \qquad (1)$$

where l_0 is the path loss at a reference distance (d_0=1 m); n is the path loss exponent; D is the transmitter-receiver separation, defined as link distance or path distance; and S is the shadowing, known as Log-normal, $S \sim N(0, \sigma_S^2)$.

Also, typical in these models is that the distance D is invariably treated as deterministic. In a cognitive radio network, radio devices are confined within a geographical region, such as an indoor office environment, where the distance itself is at random. Then, we can randomize the distance by relating it to the size of the office environment. In what follows, we refer to the office environment as a room and define the size of the office as 'room size.' We seek to derive the probability distribution function (PDF) of the path loss with the randomized distance.

2. PDF of Path Loss L in a Square Region

Before proceeding, we examine the dependency of channel parameters on distance. Obviously, l_0, n, and σ_S are site specific and may vary from room to room [2]. In our current study, we focus on a single room, not on many rooms. As a result, it is plausible to assume that the above three parameters are independent of D, and to view the path loss L as a function of two random variables.

For simplicity, let us normalize L as

$$U = \frac{L - l_0}{\sigma_S} = \frac{10n\log_{10} D}{\sigma_S} + \frac{S}{\sigma_S}. \quad (2)$$

Then the normalized shadowing parameter is $S/\sigma_S \sim N(0,1)$.

Next, let us elaborate on the PDF of D. To save space, we focus on a square office room with side length denoted by a. Suppose $A(X_1, Y_1)$ and $B(X_2, Y_2)$ represent a pair including a transmitter and receiver, which are uniformly distributed over the interior of the region. The coordinates X_1, Y_1, X_2, and Y_2 are assumed independently and uniformly distributed over the interval $[0, a]$. The distance between A and B is

$$D = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2},$$

which has a PDF given as a piecewise function [3]:

$$f_D(d) = \begin{cases} \frac{2d^3}{a^4} - \frac{8d^2}{a^3} + \frac{2\pi d}{a^2}, & 0 \le d < a, \\ -\frac{2d^3}{a^4} + \left[-\frac{4}{a^2} + \frac{8\arcsin(a/d)}{a^2} + \frac{8\sqrt{d^2 - a^2}}{a^3} - \frac{2\pi}{a^2} \right] d, & a \le d \le \sqrt{2}a \end{cases}$$

$$\approx \begin{cases} \frac{2d^3}{a^4} - \frac{8d^2}{a^3} + \frac{2\pi d}{a^2}, & 0 \le d < a, \\ \frac{-8(d - \sqrt{2}a)^3}{3a^4}, & a \le d \le \sqrt{2}a, \end{cases} \quad (3)$$

where the approximation results from Taylor series expansion applied to the second part of $f_D(d)$ on the interval $[a, \sqrt{2}a]$. The underlying consideration for the approximation is twofold: first, to reduce the computational complexity; second, to keep the approximation error, measured in the area under the graph of $f_D(d)$, well below 0.5%. This requires that the Taylor series be centered at the right endpoint $d = \sqrt{2}a$. Also, the expansion order should be three.

Let $Y = 10n\log_{10}(D)/\sigma_S$. Then, the PDF of Y can be transformed from (3) as

$$f_Y(y) = \begin{cases} \frac{2\xi e^{4\xi y}}{a^4} - \frac{8\xi e^{3\xi y}}{a^3} + \frac{2\pi\xi e^{2\xi y}}{a^2}, & -\infty < y < \frac{\ln a}{\xi}, \\ -\frac{8\xi e^{4\xi y}}{3a^4} + \frac{8\sqrt{2}\xi e^{3\xi y}}{a^3} - \frac{16\xi e^{2\xi y}}{a^2} + \frac{16\sqrt{2}\xi e^{\xi y}}{3a}, & \frac{\ln a}{\xi} \le y \le \frac{\ln(\sqrt{2}a)}{\xi}, \end{cases} \quad (4)$$

where $\xi = \ln(10)\sigma_S/(10n)$.

Furthermore, as S and D are independent, the PDF of U is the convolution integration of $f_Y(y)$ with the Gaussian normal function:

$$\begin{aligned} f_U(u) &= \int_{-\infty}^{\infty} f_Y(y) \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(u-y)^2}{2} \right] dy \\ &= \frac{\xi\pi e^{2\xi u} e^{2\xi^2}}{a^2} erfc\left(\frac{\xi u - \ln a + 2\xi^2}{\sqrt{2}\xi} \right) \\ &- \frac{4\xi e^{3\xi u} e^{\frac{9}{2}\xi^2}}{a^3} erfc\left(\frac{\xi u - \ln a + 2\xi^2}{\sqrt{2}\xi} \right) \\ &+ \frac{\xi e^{4\xi u} e^{8\xi^2}}{a^4} erfc\left(\frac{\xi u - \ln a + 2\xi^2}{\sqrt{2}\xi} \right) \\ &- \frac{4\xi e^{4\xi u} e^{8\xi^2}}{3a^4} \left[erfc\left(\frac{\xi u - \ln(\sqrt{2}a) + 2\xi^2}{\sqrt{2}\xi} \right) - erfc\left(\frac{\xi u - \ln a + 2\xi^2}{\sqrt{2}\xi} \right) \right] \\ &+ \frac{4\sqrt{2}\xi e^{3\xi u} e^{\frac{9}{2}\xi^2}}{a^3} \left[erfc\left(\frac{\xi u - \ln(\sqrt{2}a) + 2\xi^2}{\sqrt{2}\xi} \right) - erfc\left(\frac{\xi u - \ln a + 2\xi^2}{\sqrt{2}\xi} \right) \right] \\ &- \frac{8\xi e^{2\xi u} e^{2\xi^2}}{a^2} \left[erfc\left(\frac{\xi u - \ln(\sqrt{2}a) + 2\xi^2}{\sqrt{2}\xi} \right) - erfc\left(\frac{\xi u - \ln a + 2\xi^2}{\sqrt{2}\xi} \right) \right] \\ &+ \frac{8\sqrt{2}\xi e^{\xi u} e^{\frac{1}{2}\xi^2}}{3a} \left[erfc\left(\frac{\xi u - \ln(\sqrt{2}a) + 2\xi^2}{\sqrt{2}\xi} \right) - erfc\left(\frac{\xi u - \ln a + 2\xi^2}{\sqrt{2}\xi} \right) \right], \end{aligned}$$

$$\text{for } u \le \ln(\sqrt{2}a)/\xi + 3, \quad (5)$$

where $erfc(\cdot)$ is the complementary error function.

The domain of u is determined as follows. The normal curve can be thought of as effectively supported on [–3, 3], which is the 99.9% confidence interval. Whereas, the support of $f_Y(y)$ is $(-\infty, \ln(\sqrt{2}a)/\xi]$. Obviously, if $u > [\ln(\sqrt{2}a)]/\xi + 3$, the graph of $f_Y(y)$ and the right-shifted-by-u graph of the normal curve will no longer overlap, leading the convolution integration in (5) to zero. Therefore, $u \le [\ln(\sqrt{2}a)]/\xi + 3$.

Finally, the PDF of L is transformed from (5) as

$$f_L(l) = \frac{1}{\sigma_S} f_U\left(\frac{l - l_0}{\sigma_S}\right), \text{ for } l \le l_0 + \frac{\sigma_S \ln(\sqrt{2}a)}{\xi} + 3\sigma_S. \quad (6)$$

3. CDF and Moments of L

From (5), (6), and (1), we work out the moments of L as

$$E(L) \approx l_0 + 10n[\ln(a) - 0.81]/\ln(10), \quad (7)$$

$$\sigma_L^2 = E[L - E(L)]^2 = \sigma_S^2 + \frac{(10n)^2}{[\ln(10)]^2} E\left[\ln\left(\frac{D}{a}\right) + 0.81\right]^2$$
$$\approx \sigma_S^2 + 0.39(10n)^2/[\ln(10)]^2. \quad (8)$$

The PDF and the mean of L are no longer dependent on the distance but on the room size a. The variance of L is even independent of a due to the fact that the ratio D/a in (8) is indeed the distance normalized to the size a. As the distribution density of D/a is $af_D(ad)$, which is no longer dependent on a, so is $E[\ln(D/a)+0.81]^2$. The cumulative distribution function (CDF) can also be explicitly given, but the resulting formula is too lengthy. To save space and also to aid application, we prefer to tabulate the CDF in a later section (see Fig. 2)

4. Suitability

Until this point, our modeling has been in two dimensions and applicable to square regions. Due to the earth's gravity, however, mobile devices are generally not random at a vertical z-coordinate in three dimensions. Most of the devices are vertically fixed at a height whose difference is negligible compared with the difference in horizontal coordinates. In such a case, the two-dimensional model suffices. Otherwise, the PDFs of d and the logarithm of d in three dimensions should be adopted instead of (3) and (4). Our future research work will tackle these issues together with the cases of non-square operating regions.

5. IEEE 802.15.4a Specific

The above PDF, CDF, and moments are applicable to a large variety of channel models prevailing in current literature which takes the form as given by (1). In what follows, we will demonstrate the combination of our model with a state-of-the-art channel model in the literature, the IEEE 802.15.4a channel

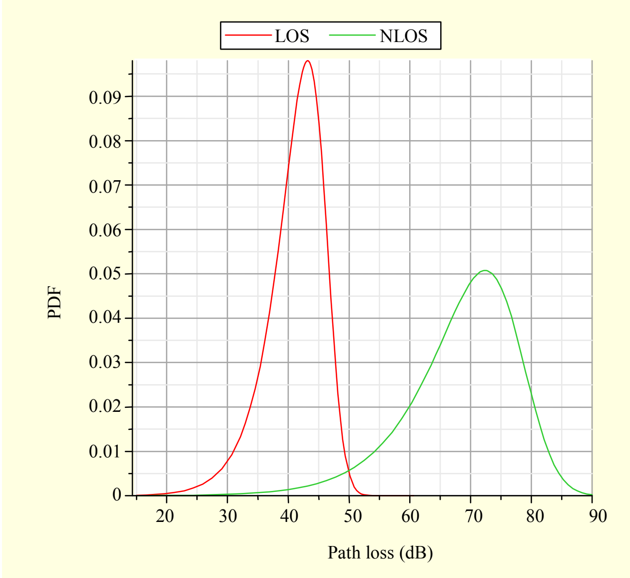

Fig. 1. PDF of path loss in a square region.

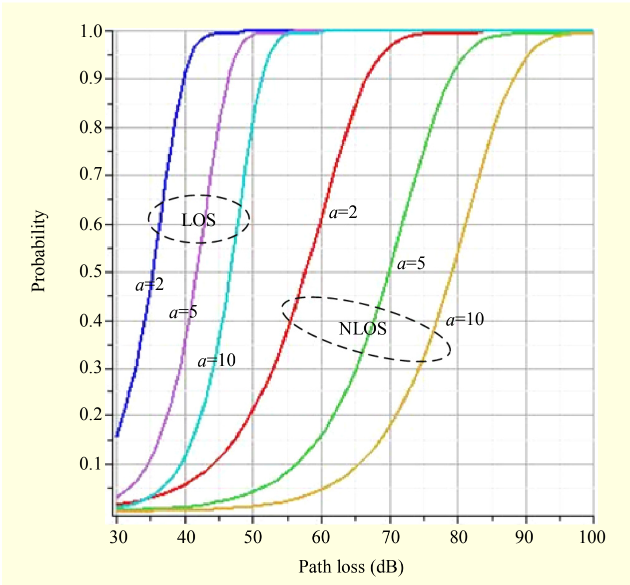

Fig. 2. CDF of path loss in indoor square region.

model [4], which features the following parameters:

$$\begin{aligned} &\text{LOS}: \quad l_0 = 35.4,\ n = 1.63,\ \sigma_S = 1.9, \\ &\text{NLOS}: \quad l_0 = 57.9,\ n = 3.27,\ \sigma_S = 3.9. \end{aligned} \quad (9)$$

The resulting PDFs (when a=5 m) are illustrated in Fig.1. The PDF under LOS is shown on the left, and the PDF under NLOS is on the right. The figure shows that the PDF of path loss is not Gaussian-like. It is asymmetric, slightly skewed to the right.

The CDFs are tabulated in Fig. 2. In the figure, three room sizes are included in which a is 2 m, 5 m, and 10 m. It is shown that with the increase of the room size, the CDF curves are shifted towards right, suggesting that, in statistical sense, a larger room size leads to a more severe path loss.

III. Example

We provide a simple example on how to use our model in practice.

Suppose the room size is a=5 m, which is either given as a design parameter or measured by cognitive devices. Also, suppose that a 95%-confidence path-loss interval is guaranteed to the primary user, namely, $P(L \le \hat{L}) = 95\%$. Now, cognitive secondary spectrum users are going to re-use the spectrum in an opportunistic manner. To do so, they are obliged to predict, in a statistical manner and without knowing the distance between the primary transmitter and receiver, what the primary user's maximum path loss is.

As an NLOS path causes larger path loss for the primary user, by consulting the second curve within the NLOS group in Fig. 2, the cognitive users know that probability of 95% at a=5 corresponds to a path loss:

$$\hat{L} = 82 \text{ dB}. \tag{10}$$

Meanwhile, if the secondary users know the primary user's nominal effectively-radiated power (ERP), they can estimate the latter's average received signal power by subtracting the ERP with the predicted path loss $(\hat{L} = 82 \text{ dB})$ without measuring the path distance first.

IV. Conclusion

By randomizing out the path distance, we derive the probability distribution function (PDF), as well as the moments, of path loss for wireless networks operating in an indoor square region. The resulting PDF and the mean of the path loss are related to the room size instead of the path distance. The PDF is not Gaussian-like but is asymmetric and skewed to the right. Our model can be combined with a large variety of the traditional channel models in the literature. To aid application, we tabulate the cumulative distribution functions by referencing to the prevailing IEEE 802.15.4a channel parameters.

Through a simple example, we show how our model helps cognitive secondary users to predict the path loss information of a primary spectrum user without knowing the latter path distance first.

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