Simple Detection Based on Soft-Limiting for Binary Transmission in a Mixture of Generalized Normal-Laplace Distributed Noise and Gaussian Noise

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In this letter, a simplified suboptimum receiver based on soft-limiting for the detection of binary antipodal signals in non-Gaussian noise modeled as a generalized normal-Laplace (GNL) distribution combined with Gaussian noise is presented. The suboptimum receiver has low computational complexity. Furthermore, when the number of diversity branches is small, its performance is very close to that of the Neyman-Pearson optimum receiver based on the probability density function obtained by the Fourier inversion of the characteristic function of the GNL-plus-Gaussian distribution.

Keywords: Log-likelihood ratio (LLR), Neyman-Pearson optimum receiver, non-Gaussian noise, normal-Laplace distribution, generalized normal-Laplace (GNL) distribution.

I. Introduction

Detection of signals in non-Gaussian noise is an important problem that arises in ultra-wideband (UWB) and radar applications. Nonlinear detection approaches in non-Gaussian distributed noise modeled in generalizations of the Gaussian, Cauchy, and beta distributions have been discussed in [1]. Various non-Gaussian distributions [2] including symmetric alpha-stable [3], [4], generalized Gaussian [1], and Gaussian-Laplace mixture [5] distributions have been considered for multiuser UWB applications. Recently, a normal-Laplace (NL) distribution with four parameters was generalized to a new probability distribution called the generalized normal-Laplace (GNL) with five parameters [6]. The detection technique for signals in GNL noise has been presented in [7].

Most of the signal detection schemes in non-Gaussian noise use a closed-form expression for the noise probability density function (PDF). Since the GNL distribution does not have the explicit analytical form of the PDF, the receiver design in the GNL distributed noise employs the approximated PDF as in [7]. The purpose of this letter is to develop a simplified receiver with low computational complexity in GNL-plus-additive white Gaussian noise (AWGN). The design of the suboptimum receiver is based on soft-limiting for detecting binary signals in GNL-plus-AWGN distributed noise. The suggested suboptimum receiver is much simpler than the near-optimum receiver introduced in [7], with almost the same performance.

II. Problem Formulation

Assuming that the constant signal is transmitted, the received signal in GNL noise combined with AWGN can be written as

$$H_i \ : y_k = d_k^{(i)} + m_k + a_k, \tag{1}$$

where $H_i \triangleq \left\{ d_k^{(i)} \text{ sent} \,\middle|\, y_k \text{ observed} \right\}$, $i \in \{1, 0\}$, $k = 1, 2, \cdots, K$. The received sequence is represented by K independent copies of the received signal from different diversity channels. Assuming $d_1^{(i)} = d_2^{(i)} = \cdots = d_K^{(i)}$, $d_k^{(1)} = -d_k^{(0)}$ is the transmitted antipodal signal during one symbol interval. The GNL and zero-mean AWGN components are m_k and a_k, respectively. The transmitted symbols with equal probability are identical and independently distributed (i.i.d.),

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and thus signal detection is performed on a symbol-by-symbol basis.

The characteristic function (CF) of a mixture of GNL noise and AWGN is given by $\phi_{\mathrm{MIX}}(s)=\phi_{\mathrm{GNL}}(s)\phi_{\mathrm{AWGN}}(s)$, where $\phi_{\mathrm{AWGN}}(s)=\exp(-\sigma_a^2 s^2/2)$ is the AWGN's CF and the GNL distribution's CF is given by [6]

$$\phi_{\mathrm{GNL}}(s)=\exp\left(i\rho\mu s-\frac{\rho\sigma_m^2 s^2}{2}\right)\left[\frac{\alpha}{\alpha-is}\right]^{\rho}\left[\frac{\beta}{\beta+is}\right]^{\rho}, \quad (2)$$

where α, β, ρ, and σ_m are positive parameters, $-\infty<\mu<\infty$. Thus, the GNL-plus-AWGN random variable x can be represented as $x=m+a$, where $m\sim GNL(\mu,\sigma_m^2,\alpha,\beta,\rho)$ is the GNL random variable represented as $m=\rho\mu+\sigma_m\sqrt{\rho}w+g_1/\alpha-g_2/\beta$ [6] and $a\sim N(0,\sigma_a^2)$ is the AWGN random variable. Here a, m, $w\sim N(0,1)$, g_1, and g_2 are independent of each other and the signal. The gamma random variables with scale parameter 1 and a shape parameter are ρ, g_1, and g_2.

Since the closed-form for the PDF of the GNL distribution has not been known except for special cases, the PDF of the GNL-plus-AWGN is obtained by the numerical inversion of the corresponding CF of the GNL-plus-AWGN as

$$f_{\mathrm{MIX}}(x)=\left(1/2\pi\right)\int_{-\infty}^{\infty}e^{-isx}\phi_{\mathrm{MIX}}(s)\,ds. \quad (3)$$

In the $GNL(\mu,\sigma_m^2,\alpha,\beta,\rho)$ distribution, μ is a location parameter and σ_m^2 is a scale parameter for the normal component which affects the spread of the distribution. The parameters α and β, respectively, influence the behavior of upper and lower tails. The shape parameter ρ also affects the tail shape. This work considers the symmetric GNL-plus-AWGN distribution with $\alpha=\beta$. Then, the GNL-plus-AWGN random variable x can be rewritten as $x=\delta+u+z/\alpha+a$, where $\delta=\rho\mu$, $u=cw\sim N(0,c^2)$ with $c=\sigma_m\sqrt{\rho}$, and $z=g_1-g_2$. The analytic representation of the PDF of the random variable x can be approximated by a sum of a finite number of random samples as in the following expression [7]:

$$f_{\mathrm{MIX}}^{\mathrm{approx}}(x)\approx\frac{1}{\sqrt{2\pi\left(\sigma_a^2+c^2\right)}}\frac{1}{J}\sum_{j=1}^{J}\exp\left\{-\frac{1}{2\left(\sigma_a^2+c^2\right)}\left(x-\delta-\frac{z_j}{\alpha}\right)^2\right\}, \quad (4)$$

where z_j, j=1, 2,..., J, of z is an i.i.d. sample of the difference of two independent gamma random variables.

III. Near-Optimum and Suboptimum Detectors

The detection scheme for signals in GNL-plus-AWGN consists of a nonlinearity function followed by an accumulator whose output is compared with the threshold 0. From the Neyman-Pearson (NP) lemma, the optimum and near-optimum detectors for the hypothesis for (1) are given by the log-likelihood ratio (LLR) test

$$\Lambda_{\mathrm{NP}}(\mathbf{y})=\sum_{k=1}^{K}v_{np}(y_k)\begin{cases}>0\Rightarrow H_1,\\<0\Rightarrow H_0,\end{cases} \quad (5)$$

where the nonlinearity function $v_{np}(y_k)$ is defined by

$$v_{np}(y_k)=\ln\left[f\left(y_k-d_k^{(1)}\right)/f\left(y_k-d_k^{(0)}\right)\right], \quad (6)$$

and $f(\cdot)$ represents the PDF of the GNL-plus-AWGN vector $\mathbf{y}=[y_1\ y_2\ \cdots\ y_K]$ under the hypothesis. The NP optimum detector is based on the PDF of the GNL-plus-AWGN numerically obtained by an inverse Fourier transform of the CF while the near-optimum detector employs the approximated GNL-plus-AWGN PDF of (4). Note that the optimum detector requires the numerical inversion of the CF, which is computationally expensive. Moreover, the CF's computation is approximately $O\left((\log n)^2 M(n)\right)$ due to the exponential function for n-digit precision where $M(n)$ covers for the complexity of the multiplication algorithm. The near-optimum detector needs the approximated complexity of $O\left((\log n)^2 M(n)\right)$ for computation of (4). Both detectors additionally require the division and ln operation in (6), whose complexity is approximated by $O\left((\log n)^2 M(n)+n^2\right)$.

To derive a simple suboptimum receiver, the approximated PDF of (4) based on only one sample is substituted into (6). Then, the nonlinearity function of a suboptimum detector based on a single sample is obtained as

$$v_{\mathrm{sub}}(y_k)=\left(y_k-d_k^{(0)}-\delta-\frac{z_1}{\alpha}\right)^2-\left(y_k-d_k^{(1)}-\delta-\frac{z_1}{\alpha}\right)^2. \quad (7)$$

The nonlinearity function of (7) can be accumulated as in (5) and compared with the threshold 0. Thus, the expression of (7) can be used alternatively as

$$v_{\mathrm{sub}}(y_k)=\left|y_k-d_k^{(0)}-\delta-\frac{z_1}{\alpha}\right|-\left|y_k-d_k^{(1)}-\delta-\frac{z_1}{\alpha}\right|. \quad (8)$$

Then, taking an average on J multiple samples of the difference of two independent gamma random variables in the nonlinearity function given by (8), it can be rewritten as

$$v_{\mathrm{sub}}(y_k)=\left|y_k-d_k^{(0)}-\delta-\frac{1}{J}\sum_{j=1}^{J}\frac{z_j}{\alpha}\right|-\left|y_k-d_k^{(1)}-\delta-\frac{1}{J}\sum_{j=1}^{J}\frac{z_j}{\alpha}\right|. \quad (9)$$

Here, the average of finite multiple samples goes to zero as J increases to infinity. Thus, based on the observed signals, the LLR test of a suboptimum soft-limiting detector, which is used for a simplified suboptimum receiver with much lower computational complexity $O(n)$, can be rewritten as

$$\sum_{k=1}^{K} \left| y_k - d_k^{(1)} - \delta \right| \underset{H_1}{\overset{H_0}{\gtrless}} \sum_{k=1}^{K} \left| y_k - d_k^{(0)} - \delta \right|. \quad (10)$$

Here, the nonlinearity function used is given by

$$v_{\text{sub}}(y_k) = \left| y_k - d_k^{(0)} - \delta \right| - \left| y_k - d_k^{(1)} - \delta \right|. \quad (11)$$

IV. Simulation Results

The signal-to-noise ratio (SNR) is defined as

$$SNR = \sum_{k=1}^{K} d_k^{(i)^2} \Big/ K\left(\sigma_a^2 + c^2\right) = \left(\frac{1}{SNR_a} + \frac{1}{SNR_m}\right)^{-1}, \quad (12)$$

where

$$SNR_a = \sum_{k=1}^{K} \left(d_k^{(i)}\right)^2 \Big/ K\sigma_a^2, \quad SNR_m = \sum_{k=1}^{K} \left(d_k^{(i)}\right)^2 \Big/ Kc^2. \quad (13)$$

The simulation results are based on 100,000 realizations of $\mathbf{y}$ with vector size K=1, 5, and 10. A rectangular pulse is assumed to be employed. The values of the parameters, σ_m^2 and ρ, used for the GNL distribution, depend on the value of SNR_m. The parameters of μ=0 and α=β=0.4 are fixed. The number of samples used in the PDF approximation of GNL-plus-AWGN is J=2,000.

Figure 1 shows the nonlinearity functions for the optimum detector, near-optimum detector, and suboptimum detector using (9) and (11). The nonlinearity function of (6) using the PDF of GNL-plus-AWGN is obtained for optimum detection by the numerical inversion of the CF (2). It includes the nonlinearity function based on the approximated PDF of (4). The nonlinearity curve of the proposed suboptimum detector based on soft-limiting is also plotted using (11). It is observed

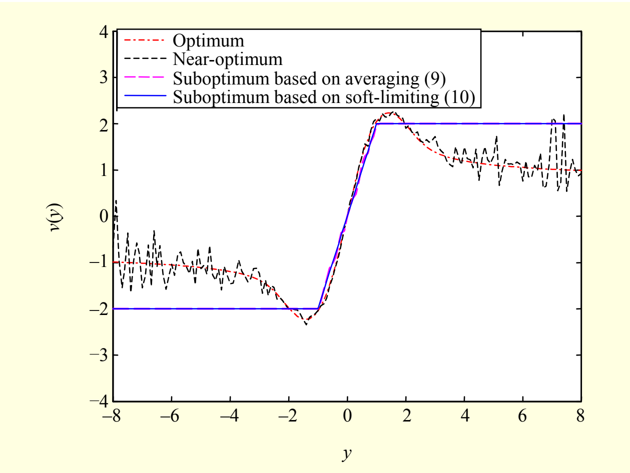

Fig. 1. Nonlinearity functions.

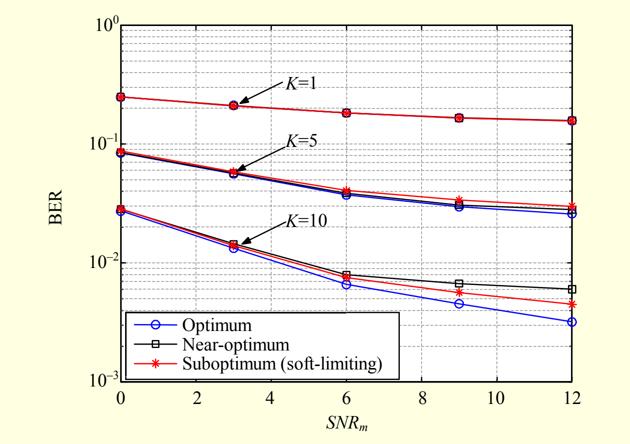

Fig. 2. BER of optimum, near-optimum, and suboptimum detectors.

that it almost overlaps the nonlinearity function of (9). Figure 2 shows the performance of the NP optimum, near-optimum, and suboptimum detectors as a function of the SNR_m for different K with $\sigma_a^2 = 0.1$, ρ=0.25, and SNR_a=10 dB. Here, the NP optimum detector is based on the numerical inversion of the CF (3). The NP near-optimum detector employs the approximated PDF of the expression (4). The suboptimum detector proposed in this work is based on the LLR test (10). It is seen that the better performance for larger K is obtained. As the SNR_m increases, the BER performance is improved. The suboptimum detector's performance is close to the NP optimum detector as well as the near-optimum detector. Note that all detectors have the same performance for K=1, and the much less complex suboptimum detector shows a small loss of performance compared with the optimum one for K=10 over high SNR_m ranges. Finally, it is pointed out that the effect of a restricted filter bandwidth is not considered in this work. If the available bandwidth is limited, the smallest possible sampling rate of the received signal is fixed. Thus, the assumption of independent noise samples during one symbol may be broken, and the detectors could have different schemes.

V. Conclusion

A simple receiver based on soft-limiting for the detection of signals in GNL-plus-AWGN has been presented. The simulation results show that the proposed simple suboptimum detector has almost the same performance as the NP optimum and near-optimum detectors for small K. Since the GNL noise could be a good model for non-Gaussian noise, such as multiple user interference in UWB applications, the suboptimum detection scheme would be practically useful in designing UWB receiver in multiuser interference environments.

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