Least Square Channel Estimation for Two-Way Relay MIMO OFDM Systems

Zhaoxi Fang and Jiong Shi

This letter considers the channel estimation for two-way relay MIMO OFDM systems. A least square (LS) channel estimation algorithm under block-based training is proposed. The mean square error (MSE) of the LS channel estimate is computed, and the optimal training sequences with respect to this MSE are derived. Some numerical examples are presented to evaluate the performance of the proposed channel estimation method.

Keywords: Two-way relay, channel estimation, MIMO, OFDM.

I. Introduction

Two-way relaying is an effective means to improve the spectral efficiency [1], where two source nodes can exchange information simultaneously with the help of a relay node. The main idea with two-way relaying is that each node can cancel the self-interference from its received signal to help decode the information from the other node.

To remove the self-interference and for data detection, accurate channel state information (CSI) is required for each node. Compared with point-to-point transmission, channel estimation in two-way relay systems becomes more complicated. Each node must estimate the channel between the relay and itself as well as the channel between the relay and the other node. In [2], Gao and others proposed maximum likelihood and linear maximum SNR channel estimation schemes for two-way amplify-and-forward (AF) relaying systems over flat fading channels. For broadband transmissions, the authors in [3] developed channel estimation algorithms for block-based training and pilot-tone-based training methods for two-way relay OFDM systems.

However, all of the above works assume that both the source nodes and the relay node only have a single antenna. It is reported in [4] that when each node is equipped with multiple antennas, similar gains as in point-to-point MIMO links can be obtained in two-way relay MIMO systems. Similar results can be found in [5]. In this letter, we consider the channel estimation in two-way relay MIMO OFDM systems, where both the source nodes and the relay node are equipped with multiple antennas to enhance the system performance. A least square (LS) channel estimation algorithm under block-based training is proposed. The mean square error (MSE) of the LS channel estimate is computed. We also discuss the optimal training sequence design with respect to this MSE.

II. System Model

Figure 1 shows a typical two-way relay MIMO OFDM system with two source nodes, A, B, and one relay node, R. The two source nodes exchange information with the help of relay node. Nodes A and B are equipped with N_T antennas, while R is equipped with N_R antennas. The average transmit power of A, B, and R are denoted as P_A, P_B, and P_R, respectively. All the channels are assumed to be quasi-statistic frequency-selective fading.

The bidirectional communication is performed in two phases as shown in Fig. 1. In phase I, nodes A and B transmit simultaneously to the relay node R. In phase II, the relay processes the received signal and broadcasts it to the source nodes A and B.

Suppose the OFDM training symbol that is transmitted from

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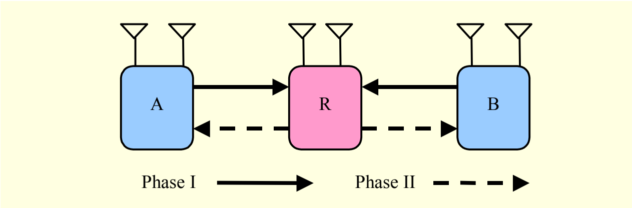

Fig. 1. Two-way relay MIMO OFDM system.

the m-th antenna at node i is denoted by the $K\times 1$ vector $\mathbf{X}_i^m$, i=A, B, m=1, 2,…, N_T. In phase I, this symbol vector is processed by an inverse fast Fourier transform, and a cyclic prefix (CP) is added to avoid inter-symbol interference. Then, the received signal at the r-th antenna of node R after removing CP can be expressed as

$$\mathbf{y}_\mathrm{R}^r = \sum_{m=1}^{N_\mathrm{T}} \left(\tilde{\mathbf{H}}_{\mathrm{A,R}}^{m,r} \mathbf{F}^\mathrm{H} \mathbf{X}_\mathrm{A}^m + \tilde{\mathbf{H}}_{\mathrm{B,R}}^{m,r} \mathbf{F}^\mathrm{H} \mathbf{X}_\mathrm{B}^m \right) + \mathbf{n}_\mathrm{R}^r, \tag{1}$$

where $\mathbf{F}$ is the $K\times K$ unitary discrete Fourier transform (DFT) matrix, $\tilde{\mathbf{H}}_{i,\mathrm{R}}^{m,r}$, i=A, B, is a circulant matrix with the first column given by $[\mathbf{h}_{i,\mathrm{R}}^{m,r\mathrm{T}}, 0_{1\times(K-L)}]^\mathrm{T}$, and $\mathbf{h}_{i,\mathrm{R}}^{m,r}$ is the $L\times 1$ vector representing the L-tap channel impulse response from the m-th antenna of node i to the r-th antenna of the relay node R. $\tilde{\mathbf{H}}_{i,\mathrm{R}}^{m,r}$ can also be expressed as $\tilde{\mathbf{H}}_{\mathrm{A,R}}^{m,r} = \mathbf{F}^\mathrm{H} \mathbf{\Lambda}_{\mathrm{A,R}}^{m,r} \mathbf{F}$, where $\mathbf{\Lambda}_{\mathrm{A,R}}^{m,r}$ is an diagonal matrix given by $\mathbf{\Lambda}_{\mathrm{A,R}}^{m,r} = \mathrm{diag}\{\mathbf{F}_L \mathbf{h}_{\mathrm{A,R}}^{m,r}\}$, and $\mathbf{F}_L$ denotes the first L columns of the $K\times K$ DFT matrix. In addition, $\mathbf{n}_\mathrm{R}^r$ denotes the zero-mean additive white Gaussian noise vector.

In phase II, the relay amplifies the received signal as

$$\mathbf{X}_\mathrm{R}^r = \alpha \mathbf{y}_\mathrm{R}^r, \tag{2}$$

where α is a real factor. To ensure an average transmit power of P_R at the relay node R, α is chosen as

$$\alpha = \sqrt{\frac{P_\mathrm{R}}{\sum_{r=1}^{N_\mathrm{R}} \sum_{m=1}^{N_\mathrm{T}} \sum_{l=1}^{L} \left[P_\mathrm{A} G_{\mathrm{A,R}}^{m,r}(l) + P_\mathrm{B} G_{\mathrm{B,R}}^{m,r}(l) \right] + N_\mathrm{R} N_0}} \tag{3}$$

with $G_{\mathrm{A,R}}^{m,r}(l) = E\left[\left| h_{\mathrm{A,R}}^{m,r}(l) \right|^2 \right]$, and $G_{\mathrm{B,R}}^{m,r}(l) = E\left[\left| h_{\mathrm{B,R}}^{m,r}(l) \right|^2 \right]$.

Node R then adds new CP to $\mathbf{X}_\mathrm{R}^r$ and broadcasts the signal to the two source nodes. In the following, we will take node A as an example to illustrate the LS channel estimate method. The received signal at the q-th antenna of node A after removing CP is given by

$$\mathbf{y}_\mathrm{A}^q = \sum_{r=1}^{N_\mathrm{R}} \tilde{\mathbf{H}}_{\mathrm{A,R}}^{q,r} \mathbf{X}_\mathrm{R}^r + \mathbf{n}_\mathrm{A}^q. \tag{4}$$

Substituting (1) and (2) into (4), we have

$$\mathbf{y}_\mathrm{A}^q = \sum_{r=1}^{N_\mathrm{R}} \sum_{m=1}^{N_\mathrm{T}} \alpha \tilde{\mathbf{H}}_{\mathrm{A,R}}^{q,r} \left(\tilde{\mathbf{H}}_{\mathrm{A,R}}^{m,r} \mathbf{F}^\mathrm{H} \mathbf{X}_\mathrm{A}^m + \tilde{\mathbf{H}}_{\mathrm{B,R}}^{m,r} \mathbf{F}^\mathrm{H} \mathbf{X}_\mathrm{B}^m \right) + \tilde{\mathbf{n}}_\mathrm{A}^q, \tag{5}$$

where

$$\tilde{\mathbf{n}}_\mathrm{A}^q = \sum_{r=1}^{N_\mathrm{R}} \alpha \tilde{\mathbf{H}}_{\mathrm{A,R}}^{q,r} \mathbf{n}_\mathrm{R}^r + \mathbf{n}_\mathrm{A}^q. \tag{6}$$

The normalized DFT of $\mathbf{y}_\mathrm{A}^q$ is

$$\mathbf{Y}_\mathrm{A}^q = \sum_{r=1}^{N_\mathrm{R}} \sum_{m=1}^{N_\mathrm{T}} \alpha \mathbf{\Lambda}_{\mathrm{A,R}}^{q,r} \left(\mathbf{\Lambda}_{\mathrm{A,R}}^{m,r} \mathbf{X}_\mathrm{A}^m + \mathbf{\Lambda}_{\mathrm{B,R}}^{m,r} \mathbf{X}_\mathrm{B}^m \right) + \mathbf{F} \tilde{\mathbf{n}}_\mathrm{A}^q. \tag{7}$$

Using the DFT theory and assuming $N\geq(2L-1)$, we have

$$\alpha \mathbf{\Lambda}_{\mathrm{A,R}}^{q,r} \mathbf{\Lambda}_{\mathrm{A,R}}^{m,r} = \mathrm{diag}[\mathbf{F}_{2L-1} \mathbf{g}_1^{q,m,r}], \tag{8}$$

where $\mathbf{F}_{2L-1}$ is the first $2L-1$ columns of the $K\times K$ DFT matrix, and

$$\mathbf{g}_1^{q,m,r} = \alpha \left(\mathbf{h}_{\mathrm{A,R}}^{q,r\,\mathrm{T}} \otimes \mathbf{h}_{\mathrm{A,R}}^{m,r\,\mathrm{T}} \right)^\mathrm{T}. \tag{9}$$

Similarly,

$$\alpha \mathbf{\Lambda}_{\mathrm{A,R}}^{q,r} \mathbf{\Lambda}_{\mathrm{B,R}}^{m,r} = \mathrm{diag}[\mathbf{F}_{2L-1} \mathbf{g}_2^{q,m,r}] \tag{10}$$

with

$$\mathbf{g}_2^{q,m,r} = \alpha \left(\mathbf{h}_{\mathrm{A,R}}^{q,r\,\mathrm{T}} \otimes \mathbf{h}_{\mathrm{B,R}}^{m,r\,\mathrm{T}} \right)^\mathrm{T}. \tag{11}$$

Then, (7) can be expressed as

$$\mathbf{Y}_\mathrm{A}^q = \sum_{r=1}^{N_\mathrm{R}} \sum_{m=1}^{N_\mathrm{T}} \mathrm{diag}[\mathbf{X}_\mathrm{A}^m] \mathbf{F}_{2L-1} \mathbf{g}_1^{q,m,r} + \sum_{r=1}^{N_\mathrm{R}} \sum_{m=1}^{N_\mathrm{T}} \mathrm{diag}[\mathbf{X}_\mathrm{B}^m] \mathbf{F}_{2L-1} \mathbf{g}_2^{q,m,r} + \mathbf{F} \tilde{\mathbf{n}}_\mathrm{A}^q, \tag{12}$$

which can also be written as

$$\mathbf{Y}_\mathrm{A}^q = \mathbf{S}_\mathrm{A} \mathbf{g}_1^q + \mathbf{S}_\mathrm{B} \mathbf{g}_2^q + \mathbf{F} \tilde{\mathbf{n}}_\mathrm{A}^q, \tag{13}$$

where

$$\mathbf{g}_j^q = \begin{bmatrix} \sum_{r=1}^{N_\mathrm{R}} \mathbf{g}_j^{q,1,r} \\ \cdots \\ \sum_{r=1}^{N_\mathrm{R}} \mathbf{g}_j^{q,N_\mathrm{T},r} \end{bmatrix}, \quad j = 1, 2, \tag{14}$$

$$\mathbf{S}_i = \left[\mathrm{diag}\left(\mathbf{X}_i^1 \right) \mathbf{F}_{2L-1}, \cdots, \mathrm{diag}\left(\mathbf{X}_i^{N_\mathrm{T}} \right) \mathbf{F}_{2L-1} \right]. \tag{15}$$

Considering all the N_T antennas at node A, the received signal can be expressed as

$$\mathbf{Y}_\mathrm{A} = \mathbf{S}_\mathrm{A} \mathbf{g}_1 + \mathbf{S}_\mathrm{B} \mathbf{g}_2 + \mathbf{F} \tilde{\mathbf{n}}_\mathrm{A}, \tag{16}$$

where

$$\mathbf{Y}_\mathrm{A} = \left[\mathbf{Y}_\mathrm{A}^1, \cdots, \mathbf{Y}_\mathrm{A}^{N_\mathrm{T}} \right], \mathbf{g}_j = \left[\mathbf{g}_j^1, \cdots, \mathbf{g}_j^{N_\mathrm{T}} \right], \text{ and } \tilde{\mathbf{n}}_\mathrm{A} = \left[\tilde{\mathbf{n}}_\mathrm{A}^1, \cdots, \tilde{\mathbf{n}}_\mathrm{A}^{N_\mathrm{T}} \right]. \tag{17}$$

III. Channel Estimation and Training Design

1. LS Channel Estimation

The pilot matrices $\mathbf{S}_A$ and $\mathbf{S}_B$ in (16) are known to the receiver and can be exploited to estimate the channel matrices $\mathbf{g}_1$ and $\mathbf{g}_2$. By rewriting (16), we have

$$\mathbf{Y}_A = \mathbf{S}\mathbf{g} + \mathbf{F}\tilde{\mathbf{n}}_A \quad (18)$$

with $\mathbf{S} = [\mathbf{S}_A, \mathbf{S}_B]$, $\mathbf{g} = [\mathbf{g}_1^T, \mathbf{g}_2^T]^T$. Then, (18) has the same signal structure with a traditional point-to-point MIMO OFDM system [6]. The LS estimation of the 2 N_T $(2L-1)\times N_T$ channel matrix $\mathbf{g}$ is given by

$$\hat{\mathbf{g}} = \mathbf{S}^+\mathbf{Y}_A = \mathbf{g} + \mathbf{S}^+\mathbf{F}\tilde{\mathbf{n}}_A, \quad (19)$$

where $\mathbf{S}^+$ is the pseudo-inverse of $\mathbf{S}$ and is given by

$$\mathbf{S}^+ = \left(\mathbf{S}^H\mathbf{S}\right)^{-1}\mathbf{S}^H.$$

From (19), the MSE of the LS channel estimate is given by

$$\text{MSE} = E\left\{\left\|\hat{\mathbf{g}} - \mathbf{g}\right\|^2\right\} = E\left\{\left\|\mathbf{S}^+\mathbf{F}\tilde{\mathbf{n}}_A\right\|^2\right\}. \quad (20)$$

From (6) and (17), we have

$$E\left\{\tilde{\mathbf{n}}_A\tilde{\mathbf{n}}_A^H\right\} = \alpha^2 N_0 \sum_{m=1}^{N_T}\sum_{r=1}^{N_R} E\left[\mathbf{\Lambda}_{A,R}^{m,r}\left(\mathbf{\Lambda}_{A,R}^{m,r}\right)^H\right] + N_T N_0 \mathbf{I}_K. \quad (21)$$

Then, the MSE in (20) can be written as

$$\text{MSE} = \lambda tr\left\{\left(\mathbf{S}^H\mathbf{S}\right)^{-1}\right\} \quad (22)$$

with $\lambda = \alpha^2 N_0 \sum_{m=1}^{N_T}\sum_{r=1}^{N_R}\sum_{l=0}^{L-1} G_{A,R}^{m,r}(l) + KN_T N_0$. It should be noted that nodes A and B have their individual transmit power constraints. The traditional method for point-to-point MIMO OFDM systems cannot be directly applied to two-way relay MIMO OFDM systems. Using the results in [3], the MSE in (22) is lower bounded as

$$\text{MSE}=\lambda tr\left\{\left(\mathbf{S}^H\mathbf{S}\right)^{-1}\right\} \ge \lambda\left[tr\left\{\left(\mathbf{S}_A^H\mathbf{S}_A\right)^{-1}\right\} + tr\left\{\left(\mathbf{S}_B^H\mathbf{S}_B\right)^{-1}\right\}\right], \quad (23)$$

where the equality holds if and only if

$$\mathbf{S}_A^H\mathbf{S}_B = 0. \quad (24)$$

Furthermore, it is easy to show that $tr\left\{\left(\mathbf{S}_i^H\mathbf{S}_i\right)^{-1}\right\}$ achieves its minimal when $\mathbf{S}_i^H\mathbf{S}_i$ is a diagonal matrix with equal diagonal elements [6]:

$$\mathbf{S}_i^H\mathbf{S}_i = KP_i\mathbf{I}_{N_T(2L-1)}. \quad (25)$$

The corresponding minimum value of $tr\left\{\left(\mathbf{S}_i^H\mathbf{S}_i\right)^{-1}\right\}$ is

$$tr\left\{\left(\mathbf{S}_i^H\mathbf{S}_i\right)^{-1}\right\}_{\min} = \frac{N_T(2L-1)}{KP_i}. \quad (26)$$

Hence, the minimum MSE is given by

$$\text{MSE}_{\min} = \lambda\frac{N_T(2L-1)}{K}\left(\frac{1}{P_A} + \frac{1}{P_B}\right). \quad (27)$$

Substituting λ into (27), it is easy to show that the minimal MSE is proportional to the square of the number of transmit antennas N_T, which is different from traditional MIMO OFDM systems [6].

2. Optimal Training Sequence

To achieve the minimal MSE, the pilot matrices $\mathbf{S}_A$ and $\mathbf{S}_B$ should both satisfy (24) and (25). From (24), it can be shown that $\mathbf{X}_A$ and $\mathbf{X}_B$ should satisfy the following

$$\sum_{k=0}^{K-1}\left[\text{diag}(\mathbf{X}_A^p)^H\mathbf{D}_\varphi\text{diag}(\mathbf{X}_B^q)\right]_{k,k} = 0. \quad (28)$$

For $\forall\varphi \in \{-2L+2,\cdots,2L-2\}$ and p, $q\in\{1,\ldots, N_T\}$, where $\mathbf{D}_\varphi$ represents the $K\times K$ phase shift matrix with phase ϕ:

$$\mathbf{D}_\varphi = \text{diag}\{1, e^{-j2\pi\varphi/K},\cdots,e^{-j2\pi\varphi(K-1)/K}\}. \quad (29)$$

Using a similar argument as in [6], we can show that to satisfy (25), $\mathbf{X}_i$, i=A, B, should satisfy the following two equations:

$$\left|\mathbf{X}_i^p(k)\right|^2 = P_i / N_T, k\in\{0,\cdots,K-1\}, p\in\{1,\cdots,N_T\} \quad (30)$$

and

$$\sum_{k=0}^{K-1}\left[\text{diag}(\mathbf{X}_i^p)^H\mathbf{D}_\varphi\text{diag}(\mathbf{X}_i^q)\right]_{k,k} = 0 \quad (31)$$

for $\forall\varphi\in\{-2L+2,\cdots,2L-2\}$ and $\forall p,q\in\{1,\cdots,N_T\}$ with $p\neq q$. Hence, the optimal training sequences to minimize the MSE should satisfy (28), (30), and (31).

A simple example of optimal training sequences is

$$\mathbf{X}_A^p(k) = \sqrt{P_A/N_T}\,e^{-j2\pi(p-1)(2L-1)k/K} \quad (32)$$

and

$$\mathbf{X}_B^p(k) = \sqrt{P_B/N_T}\,e^{-j2\pi(N_T+p-1)(2L-1)k/K} \quad (33)$$

for p=1, 2,…, N_T, k=0, 1,…, K–1.

IV. Simulation Results

In this section, we present computer simulation results to show the performance of the proposed LS channel estimate. We consider K=128 subcarriers with a CP length of 16. The channels are assumed to have L=4 taps, each with unit variance. The SNR in the figures are defined as SNR=P_t/N_0, where $P_t=(P_A+P_B+P_R)$ is the total transmit power of the three nodes.

Figure 2 shows the MSE performance of node A with different numbers of antennas. The transmit power of the three nodes are the same, $P_A=P_B=P_R=P_t/3$. We also plot the MSE of LS estimate with random training sequences. As shown in the figure, there is a 6-dB gain in SNR for the optimal training over random training at an MSE of 10^{-2} for the $N_T=N_R=2$ case. Similar results hold when the number of antennas at R increased from 2 to 4.

Figure 3 studies the benefit of the optimal training sequences

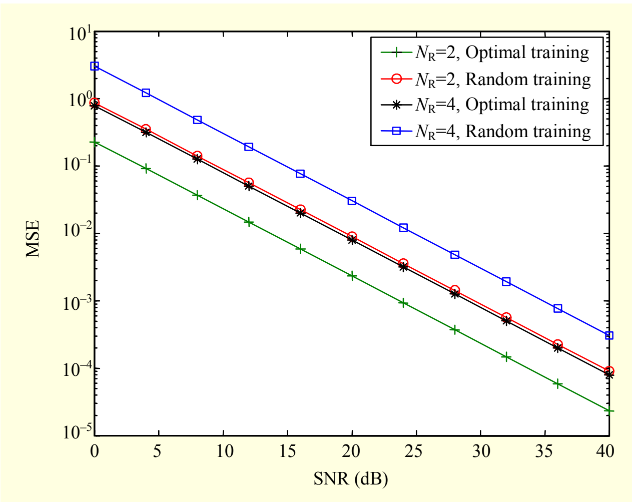

Fig. 2. MSE versus SNR, N_T=2, $P_A=P_B=P_R$.

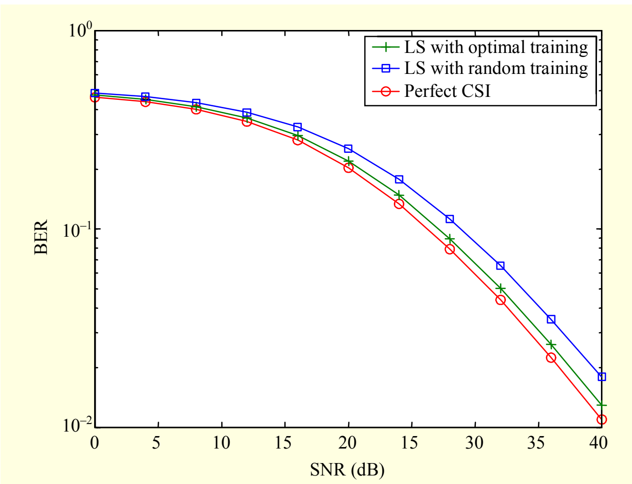

Fig. 3. BER versus SNR, $N_T=N_R$=2, $P_A=P_B=P_R$.

to the BER performance of the two-way relay MIMO OFDM system with quadrature phase shift keying modulation. In the simulation, each node is equipped with two antennas, while other simulation parameters remain the same as those in Fig. 2. A zero-forcing equalizer is applied at the receiver to recover the transmitted signal. For performance comparison, we show the BER performance with random training sequence and that with perfect CSI as well. It can be seen that there is a 2-dB performance gain for the optimal training sequences over random training sequences, and the gap between the LS method with optimal training and the perfect CSI case is within 1 dB for the whole SNR region.

V. Conclusion

In this letter, we proposed an LS channel estimation algorithm for two-way relay MIMO OFDM systems based on block-based training. The MSE of the proposed LS channel estimate is computed. We also investigated the optimal training sequences designed to minimize the MSE. Simulation results show that the proposed LS channel estimate with optimal training sequences outperforms that with random training significantly. We will study the Cramer-Rao lower bound in future work.

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