

# Performance of Two-User Two-Way Amplify-and-Forward Relaying Systems with Scheduling

Zhaoxi Fang, Guosheng Li, and Jun Li

**In this paper, we study scheduling schemes for two-user two-way wireless relaying systems. Two transmission modes are considered: point-to-point direct transmission and two-way amplify-and-forward relaying. An optimal scheduling scheme that opportunistically selects the best transmission mode for each user is proposed to minimize the sum bit error rate (BER). The performance lower bound of the optimal scheduling scheme is analyzed, and closed-form expression of the lower-bound BER is derived. However, for optimal scheduling, the scheduler requires the knowledge of channel state information (CSI) of all links. To reduce the feedback information of CSI, we also propose a suboptimal scheduling scheme that selects the transmission mode using only the CSI of two direct links. Simulation results show that there are 4 dB to 8 dB gains for the proposed optimal and suboptimal schemes over the fixed direct transmission and fixed two-way relayed transmission scheme. The performance gap between the optimal and suboptimal scheduling schemes is small, which implies a good trade-off between implementation complexity and system performance.**

**Keywords: Two-way relaying, amplify-and-forward, scheduling, performance lower bound.**

## I. Introduction

A relay transmission technique can increase signal reliability and extend the coverage [1]. A lot of relaying schemes have been proposed, and their performances have been analyzed [2]-[5]. The amplify-and-forward (AF) relaying scheme is one of the most attractive since the relays only linearly process the received signal, which leads to a low-complexity transceiver design.

In a practical wireless system, the relays operate in half-duplex mode, that is, they cannot receive and transmit simultaneously. Hence, there is spectrum loss in conventional one-way AF relaying as compared to point-to-point direct transmission. Also, the channel quality of the relay link may be even worse than the direct link in time-varying wireless fading channels. As a result, one-way AF relaying cannot always guarantee a better system performance as compared to direct transmission. Hence, a proper scheduling scheme that switches between direct transmission and relayed transmission can be adopted to enhance the system performance [6]-[8]. In [7], the authors consider a simple three-node wireless network, where each node can be a source, relay, or destination. It is shown that by properly selecting the source and the relay according to the channel condition, the bit error rate (BER) performance as well as the system throughput can be greatly improved. In [8], the authors propose a two-step scheduling method for multiuser single-relay systems. The multiuser scheduling method opportunistically selects both the transmission mode and the desired mobile station. Simulation results in [8] show that the proposed multiuser scheduling scheme outperforms either direct transmission or fixed AF relaying in terms of average achievable rates. These results show that scheduling is essential in conventional one-way AF relaying networks.

---

Manuscript received Oct. 22, 2010; revised Feb. 21, 2011; accepted Mar. 8, 2011.

This work was supported by the Natural Science Foundation of Zhejiang Province under Grant No. Y1101123 and the Natural Science Foundation of Ningbo under Grant No. 2010A610173 and No. 2010A610121.

Zhaoxi Fang (phone: +86 574 88223268, email: zhaoxifang@gmail.com), Guosheng Li (email: guoshengli@zwu.edu.cn), and Jun Li (email: junli@zwu.edu.cn) are with the School of Electronic and Information Engineering, Zhejiang Wanli University, Ningbo, China.

<http://dx.doi.org/10.4218/etrij.11.0110.0622>

To reduce the spectrum loss in one-way relaying systems, two-way relaying or bidirectional relaying has been proposed in recent years [9]-[12]. The key idea behind two-way relaying is that each user can cancel the self-interference from its received signal to help decode the information from the other user [9]. By allowing concurrent transmission of two information flows, two-way relaying achieves the same transmission efficiency as direct transmission.

However, due to the time-varying nature of wireless fading channels, the end-to-end signal-to-noise ratio (SNR) of two-way relaying may not be always higher than the direct transmission. Proper scheduling can also be adopted for two-way relaying networks. In this paper, we study the scheduling schemes in a two-user two-way relaying network, where each user can directly communicate with the base station (BS) or exchange information with the help of the other user in a two-way relaying mode. With the knowledge of instantaneous channel state information (CSI), an optimal scheduling method is proposed by selecting the best transmission mode for each user to minimize the sum BER of the downlink and uplink. We also analyze the BER performance lower bound of the proposed optimal scheduling scheme, and closed-form average BER expression of the lower bound is derived. However, the optimal scheme requires channel knowledge between two users and that between the users and the BS, which may lead to considerable signaling overhead. To reduce the CSI feedback information, we also propose a suboptimal scheduling scheme that selects the transmission mode based on the CSI of direct links only.

The remainder of this paper is organized as follows. Section II describes the system model. The optimal and suboptimal scheduling schemes are shown in sections III and IV, respectively. Section V presents some simulation results, and section VI provides conclusions.

## II. System Model

Figure 1 shows a time-division duplex wireless network that consists of one BS and two users. Each node in the network is equipped with a single antenna. We assume that both the users

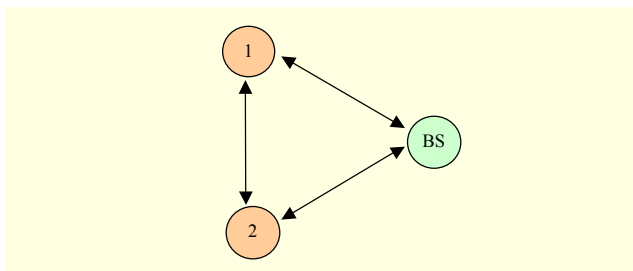


Fig. 1. Two-user two-way relay network.

and the BS operate in half-duplex mode, that is, all nodes cannot transmit and receive simultaneously. The channels between each user and the destination and those between two users are flat fading. Each user is designed to exchange information with the BS. There are two possible transmission modes for each node, that is, direct transmission and two-way AF relaying.

### 1. Direct Transmission

For direct transmission, the received SNR at node  $j$  corresponding to node  $i$  can be expressed as

$$\gamma_{i,j} = \frac{P_i |h_{i,j}|^2}{N_0}, \quad (1)$$

where  $i, j \in \{1, 2, \text{BS}\}$ ,  $P_i$  is the transmit power of node  $i$ ,  $h_{ij}$  denotes the channel impulse response from node  $i$  to node  $j$ , and  $N_0$  denotes the variance of additive white Gaussian noise. In this paper, we assume that the transmit power of the two users and the BS are the same:  $P_i = P$ , and the channels are symmetric, that is,  $h_{ij} = h_{j,i}$ . Under these assumptions, we have  $\gamma_{i,j} = \gamma_{j,i}$ .

### 2. Two-Way Relaying

In this transmission mode, user  $i$  and the BS exchange information with the help of user  $j$ ,  $i, j \in \{1, 2\}$ , that is, user  $j$  acts as user  $i$ 's relay. The data transmission is performed in two time slots as in [10]. In the first time slot, user  $i$  and the BS send information to user  $j$  simultaneously. In the second time slot, user  $j$  amplifies the received signal and forwards it to user  $i$  and the BS. Both user  $i$  and the BS can cancel the self-interference from its received signal in the second time slot. The received SNR at user  $i$  and the BS are given by [10]

$$\gamma_{\text{BS},j,i} = \frac{\gamma_{\text{BS},j} \gamma_{j,i}}{2\gamma_{\text{BS},j} + \gamma_{j,i} + 1} \quad (2)$$

and

$$\gamma_{i,j,\text{BS}} = \frac{\gamma_{i,j} \gamma_{j,\text{BS}}}{2\gamma_{i,j} + \gamma_{j,\text{BS}} + 1}, \quad (3)$$

respectively.

## III. Optimal Scheduling and Performance Analysis

As mentioned, the channel quality of two-way relay links cannot be always better than the direct links in fading channels. From (1) to (3), it can be seen that the received SNR of two-way relaying transmission mode may be lower than that of direct transmission mode if the channels between the user and

the relay or that between the BS and the relay are undergoing deep fading. Hence, a proper scheduling scheme can be adopted to improve the system performance, provided that perfect knowledge of CSI is available. In this section, we study the optimal scheduling scheme that aims at minimizing the sum BER. However, the method presented in this section can also be readily extended to other performance metrics, for example, minimizing the sum outage probability or maximizing the sum capacity. In the following, we will take user 1 as an example to illustrate the optimal scheduling scheme.

## 1. Optimal Scheduling

The sum BER of user 1 in direct transmission mode can be expressed as

$$P_1^{\text{direct}} = f(\gamma_{1,\text{BS}}) + f(\gamma_{\text{BS},1}), \quad (4)$$

where the first and second terms on the right hand side of (4) denote the uplink and downlink BERs of user 1, respectively, and  $f(x)$  is the BER function. For M-ary phase shift keying modulation,  $f(x)$  is given by [13]

$$f(x) = \frac{1}{\pi \log_2^M} \int_0^{(M-1)\pi/M} \exp\left(-\frac{g_{\text{MPSK}}}{\sin^2 \theta} x\right) d\theta, \quad (5)$$

where  $g_{\text{MPSK}} = \sin^2(\pi/M)$ .

In two-way relaying mode, the sum BER of user 1 is given by

$$P_1^{\text{two-way}} = f(\gamma_{1,2,\text{BS}}) + f(\gamma_{2,1,\text{BS}}). \quad (6)$$

To enhance the system performance, the transmission mode with the minimum sum BER is selected:

$$\text{transmission mode} = \arg \min \{P_1^{\text{direct}}, P_1^{\text{two-way}}\}. \quad (7)$$

## 2. Performance Lower Bound

With the above optimal scheduling, the sum BER of user 1 is

$$P_1 = \min \{P_1^{\text{direct}}, P_1^{\text{two-way}}\}. \quad (8)$$

It is quite difficult to derive the exact average sum BER in (8). Hence, we resort to a lower bound of this BER. Substituting (4) and (6) into (8), we have

$$P_1 = \min \{f(\gamma_{1,\text{BS}}) + f(\gamma_{\text{BS},1}), f(\gamma_{1,2,\text{BS}}) + f(\gamma_{\text{BS},2,1})\} \\ \geq \min \{f(\gamma_{1,\text{BS}}), f(\gamma_{1,2,\text{BS}})\} + \min \{f(\gamma_{\text{BS},1}), f(\gamma_{\text{BS},2,1})\}. \quad (9)$$

Since  $f(x)$  is monotonously decreasing, (9) can also be expressed as

$$P_1 \geq f(\max(\gamma_{1,\text{BS}}, \gamma_{1,2,\text{BS}})) + f(\max(\gamma_{\text{BS},1}, \gamma_{\text{BS},2,1})). \quad (10)$$

From (3), we have

$$\gamma_{1,2,\text{BS}} = \frac{\gamma_{1,2}\gamma_{2,\text{BS}}}{2\gamma_{1,2} + \gamma_{2,\text{BS}} + 1} \leq \min(\gamma_{1,2}, \frac{\gamma_{2,\text{BS}}}{2}). \quad (11)$$

Hence,

$$\max(\gamma_{1,\text{BS}}, \gamma_{1,2,\text{BS}}) = \max(\gamma_{1,\text{BS}}, \frac{\gamma_{1,2}\gamma_{2,\text{BS}}}{2\gamma_{1,2} + \gamma_{2,\text{BS}} + 1}) \\ \leq \max(\gamma_{1,\text{BS}}, \min(\gamma_{1,2}, \frac{\gamma_{2,\text{BS}}}{2})). \quad (12)$$

Similarly, we have

$$\max(\gamma_{\text{BS},1}, \gamma_{\text{BS},2,1}) = \max(\gamma_{\text{BS},1}, \frac{\gamma_{\text{BS},2}\gamma_{2,1}}{2\gamma_{\text{BS},2} + \gamma_{2,1} + 1}) \\ \leq \max(\gamma_{\text{BS},1}, \min(\gamma_{\text{BS},2}, \frac{\gamma_{2,1}}{2})). \quad (13)$$

Define  $g_1 = \max(\gamma_{1,\text{BS}}, \min(\gamma_{1,2}, \gamma_{2,\text{BS}}/2))$  and  $g_2 = \max(\gamma_{\text{BS},1}, \min(\gamma_{\text{BS},2}, \gamma_{2,1}/2))$ , then the sum BER of user 1 is lower bounded by

$$P_1 \geq f(g_1) + f(g_2). \quad (14)$$

Hence, the average sum BER of user 1 is lower bounded by

$$P_1^{\text{avg}} = E\{P_1\} \geq E\{f(g_1)\} + E\{f(g_2)\}. \quad (15)$$

From [13],

$$E\{f(g_i)\} = \frac{1}{\pi \log_2^M} \int_0^{(M-1)\pi/M} M_i\left(\frac{g_{\text{MPSK}}}{\sin^2 \theta}\right) d\theta, \quad i=1, 2, \quad (16)$$

where  $M_i(s)$  is the moment generating function (MGF) of  $g_i$ . According to the definition of  $g_1$ , the probability distribution function of  $g_1$  can be calculated as

$$f_{g_1}(\gamma) = \frac{1}{\gamma_{1,\text{BS}}} \exp\left(-\frac{\gamma}{\gamma_{1,\text{BS}}}\right) \left(1 - \exp\left(-\frac{\gamma}{\gamma_1}\right)\right) \\ + \frac{1}{\gamma_1} \exp\left(-\frac{\gamma}{\gamma_1}\right) \left(1 - \exp\left(-\frac{\gamma}{\gamma_{1,\text{BS}}}\right)\right), \quad (17)$$

where  $\bar{\gamma}_1 = \frac{\bar{\gamma}_{1,2}\bar{\gamma}_{2,\text{BS}}}{2\bar{\gamma}_{1,2} + \bar{\gamma}_{2,\text{BS}}}$  and  $\bar{\gamma}_{1,\text{BS}}$  is the average SNR of

the channel between user 1 and the BS. Then, the MGF of  $g_1$  is given by

$$M_1(s) = \frac{1}{1 + s\bar{\gamma}_{1,\text{BS}}} + \frac{1}{1 + s\bar{\gamma}_1} - \frac{\bar{\gamma}_1 + \bar{\gamma}_{1,\text{BS}}}{\bar{\gamma}_1 + \bar{\gamma}_{1,\text{BS}} + s\bar{\gamma}_1\bar{\gamma}_{1,\text{BS}}}. \quad (18)$$

Similarly, the MGF of  $g_2$  is given by

$$M_2(s) = \frac{1}{1 + s\bar{\gamma}_{\text{BS},1}} + \frac{1}{1 + s\bar{\gamma}_2} - \frac{\bar{\gamma}_2 + \bar{\gamma}_{\text{BS},1}}{\bar{\gamma}_2 + \bar{\gamma}_{\text{BS},1} + s\bar{\gamma}_2\bar{\gamma}_{\text{BS},1}}, \quad (19)$$

where  $\bar{\gamma}_2 = \frac{\bar{\gamma}_{BS,2}\bar{\gamma}_{2,1}}{2\bar{\gamma}_{BS,2} + \bar{\gamma}_{2,1}}$ . Substituting (18) and (19) into

(15) and (16), we have that the sum BER of user 1 with the optimal scheduling is lower bounded by

$$P_1^{\text{avg}} \geq \frac{1}{\pi \log_2^M} \int_0^{(M-1)\pi/M} \left[ M_1 \left( \frac{\mathcal{G}_{\text{MPSK}}}{\sin^2 \theta} \right) + M_2 \left( \frac{\mathcal{G}_{\text{MPSK}}}{\sin^2 \theta} \right) \right] d\theta. \quad (20)$$

The performance lower bound of user 2 can be derived similarly.

#### IV. Suboptimal Scheduling

In the proposed optimal scheduling scheme, global CSI is required to select the transmission mode. The scheduler must have the instantaneous information of  $\gamma_{1,BS}$ ,  $\gamma_{2,BS}$ , and  $\gamma_{1,2}$  to make proper decision. Generally, the scheduling scheme is employed at the BS. These CSI must be estimated and then feedback to the BS, which may result in considerable signaling overhead. Hence, it is necessary to consider a low complexity scheduling scheme with partial channel information. Since it is quite difficult for the BS to have knowledge of the inter-user channels, that is,  $\gamma_{1,2}$ , we here propose a suboptimal scheme based on the knowledge of the direct links only, that is,  $\gamma_{1,BS}$  and  $\gamma_{2,BS}$ .

Take user 1 as an example. If user 1 and the BS use a two-way relaying transmission mode, then the received SNR at the BS is given by

$$\gamma_{1,2,BS} = \frac{\gamma_{1,2}\gamma_{2,BS}}{2\gamma_{1,2} + \gamma_{2,BS} + 1}. \quad (21)$$

If  $\gamma_{2,BS} < 2\gamma_{1,BS}$ , then we have

$$\gamma_{1,2,BS} < \frac{\gamma_{2,BS}}{2} < \gamma_{1,BS}. \quad (22)$$

From (22), it can be seen that if  $\gamma_{2,BS} < 2\gamma_{1,BS}$ , then the received SNR at the BS in two-way relaying mode will be lower than

Table 1. Suboptimal scheduling scheme.

| R                   | Transmission mode   |                     |
|---------------------|---------------------|---------------------|
|                     | User 1              | User 2              |
| $R < 0.5$           | Two-way relaying    | Direct transmission |
| $0.5 \leq R \leq 2$ | Direct transmission | Direct transmission |
| $R > 2$             | Direct transmission | Two-way relaying    |

the SNR in direct transmission mode, regardless of the SNR of the inter-user channel, that is,  $\gamma_{1,2}$ . Hence, once  $\gamma_{2,BS} < 2\gamma_{1,BS}$ , the scheduler can decide that direct transmission mode is more suitable for user 1.

Similarly, for user 2, if  $\gamma_{1,BS} < 2\gamma_{2,BS}$ , then

$$\gamma_{2,1,BS} < \gamma_{2,BS}, \quad (23)$$

and the two-way relaying mode should not be adopted.

From (22) and (23), we can define a cooperative ratio  $R$  as

$$R = \frac{\gamma_{1,BS}}{\gamma_{2,BS}}. \quad (24)$$

Depending on the value of  $R$ , the scheduler can select the proper transmission mode for user 1 and user 2 as summarized in Table 1.

The suboptimal scheme discussed above only requires the knowledge of  $\gamma_{1,BS}$  and  $\gamma_{2,BS}$ , and hence can reduce the feedback CSI as compared to the optimal scheme in section III.

#### V. Simulation Results

In this section, we perform computer simulations to demonstrate the advantage of the proposed scheduling method. The transmit power of the two users and the BS are the same, and quadrature phase shift keying modulation is used. We consider two system models. In system A, the two users are far away from the BS, but are close to each other, and the average SNR of the inter-user channel is 9 dB higher than the two direct channels, that is,  $SNR_{1,BS} = SNR_{2,BS}$ , and  $SNR_{1,2} = SNR_{2,BS} + 9$  dB. In system B, user 1 is close to the BS while user 2 is far away from the BS, and the average SNRs are set to be  $SNR_{1,BS} = SNR_{2,BS} + 9$  dB, and  $SNR_{1,2} = SNR_{2,BS} + 9$  dB.

Figure 2 shows the average sum BER of user 1 with optimal and suboptimal scheduling schemes in system A. Note that the average sum BER of user 2 in this case is the same as user 1 since system A is symmetric. We also plot the average sum

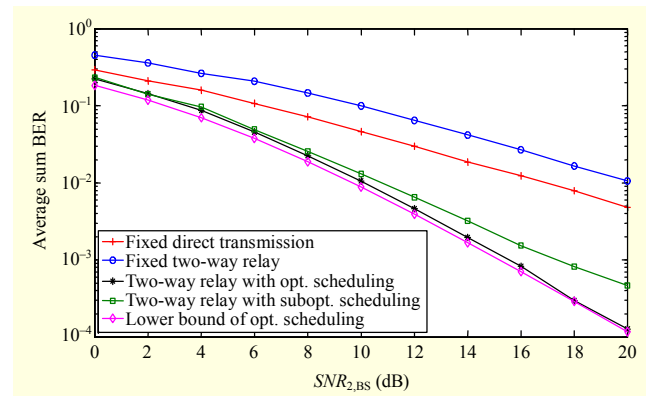


Fig. 2. Average sum BER of user 1 in system A.

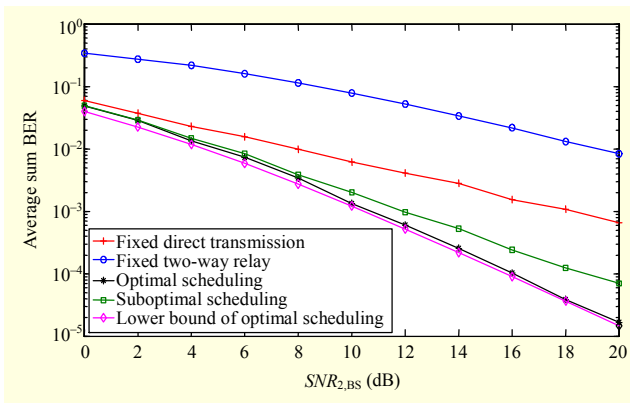


Fig. 3. Average sum BER of user 1 in system B.

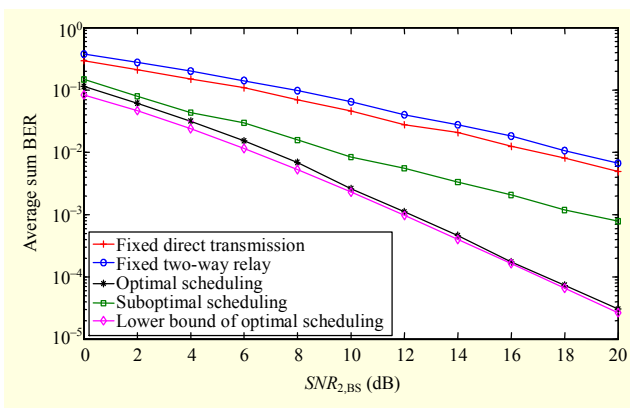


Fig. 4. Average sum BER of user 2 in system B.

BER of fixed direct transmission and that of fixed two-way relaying for comparison. From this figure, we can see that both the optimal and suboptimal scheduling schemes significantly outperform the fixed direct transmission and fixed two-way relay schemes, especially in the high SNR range. Also, the performance gap between the optimal and suboptimal schemes is small in the low SNR region but enlarges in the high SNR region. This is due to the lack of knowledge of the inter-user channel information for the suboptimal scheduling scheme. It can also be seen that the lower bound in (20) is shown to be very tight in the whole SNR region.

Figures 3 and 4 show the average sum BER of user 1 and user 2 in system B, respectively. Since user 2 is far away from the BS, the BER performance of user 2 is 9 dB worse than that of user 1 in fixed direct transmission mode. When scheduling schemes are adopted, the performance of user 2 can be greatly improved as can be seen from Fig. 4. Though the performance gap between the optimal and suboptimal scheduling schemes enlarges as the SNR increases, the performance of the suboptimal scheduling scheme still outperforms the fixed direct transmission and fixed two-way relaying. For instance, there is 8 dB gain for the suboptimal schemes at a BER of  $10^{-2}$ .

## VI. Conclusion

In this paper, we studied the scheduling scheme for two-user two-way amplify-and-forward relaying systems. An optimal scheduling scheme that selects the best transmission mode with instantaneous global CSI was proposed, aiming at minimizing the sum BER for each user. The achievable performance lower bound of the proposed scheduling scheme was analyzed. To reduce the signaling overhead, we also proposed a suboptimal scheme based on the CSI of two direct links. Simulation results showed that the proposed schemes can greatly improve the BER performance, especially for the weak-channel user, and the derived lower bound was shown to be tight in various cases.

## References

- [1] R. Pabst, "Relay-Based Deployment Concepts for Wireless and Mobile Broadband Radio," *IEEE Trans. Wireless Commun.*, vol. 42, Sept. 2004, pp. 80-89.
- [2] J.N. Laneman, D.N.C. Tse, and G.W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Trans. Inf. Theory*, vol. 50, Dec. 2004, pp. 3062-3080.
- [3] Y. Jing and B. Hassibi, "Distributed Space-Time Coding in Wireless Relay Networks," *IEEE Trans. Wireless Commun.*, vol. 5, Dec. 2006, pp. 3524-3536.
- [4] P.A. Anghel and M. Kaveh, "On the Performance of Distributed Space Time Coding Systems with One and Two Non-regenerative Relays," *IEEE Trans. Wireless Commun.*, vol. 5, Mar. 2006, pp. 682-692.
- [5] Y. Zhao, R. Adve, and T.J. Lim, "Improving Amplify-and-Forward Relay Networks: Optimal Power Allocation versus Selection," *IEEE Trans. Wireless Commun.*, vol. 6, Aug. 2007, pp. 3114-3123.
- [6] O. Oyman, "Opportunism in Multiuser Relay Channel: Scheduling, Routing and Spectrum Reuse," in *Proc. IEEE Int. Symp. Inform. Theory (ISIT)*, Nice, France, June 2007, pp. 286-290.
- [7] I. Krikidis and J.C. Belfiore, "Three Scheduling Schemes for Amplify-and-Forward Relay Environments," *IEEE Commun. Lett.*, vol. 11, no. 5, May 2007, pp. 414-416.
- [8] Y.-U. Jang, W.-Y. Shin, and Y.H. Lee, "Multiuser Scheduling Based on Reduced Feedback Information in Cooperative Communications," *VTC Spring*, Barcelona, Apr. 2009, pp. 1-5.
- [9] B. Rankov and A. Wittneben, "Spectral Efficient Protocols for Half-Duplex Fading Relay Channels," *IEEE J. Select. Areas Commun.*, vol. 25, Feb. 2007, pp. 379-389.
- [10] P. Popovski and H. Yomo, "Wireless Network Coding by Amplify-and-Forward for Bi-directional Traffic Flows," *IEEE Commun. Lett.*, vol. 11, no. 1, Jan. 2007, pp. 16-18.

- [11] D. Gündüz, A. Goldsmith, and H.V. Poor, "MIMO Two-Way Relay Channel: Diversity-Multiplexing Trade-Off Analysis," *Proc. Asilomar Conf. Signals, Syst. Comput.*, Pacific Grove, CA, Oct. 2008, pp. 1474-1478.
- [12] W. Nam, S.-Y. Chung, and Y.H. Lee, "Capacity Bounds for Two-Way Relay Channel," *Proc. Int. Zurich Seminar Commun.*, 2008, pp. 144-147.
- [13] M.K. Simon and M.-S. Alouini, "A Unified Approach to the Performance Analysis of Digital Communication over Generalized Fading Channels," *Proc. IEEE*, vol. 86, Sept. 1998, pp. 1860-1877.



**Zhaoxi Fang** received the BEng in communication engineering and the PhD in electrical engineering from Fudan University, Shanghai, China, in 2004 and 2009, respectively. Since June 2009, he has been an assistant professor in the School of Electronic and Information Engineering, Zhejiang Wanli

University, Ningbo, China. His research interests include iterative detection, frequency domain equalization, and cooperative communications.



**Guosheng Li** received the BS in computer science from Hubei University of Technology, Wuhan, China, in 2000. Since September 2000, he has been an assistant professor in the School of Electronic and Information Engineering, Zhejiang Wanli University, Ningbo, China. His current research interest is wireless

communication software development based on android and j2me.



**Jun Li** is an associate professor of Communications Engineering Department at Zhejiang Wanli University. She received her BS and MS in computer science and engineering from Zhejiang University in 1993 and 2001, and her PhD in computer networks from Nanjing University of Posts and

Telecommunications in 2009. Her research interests include peer-to-peer networks, network performance monitoring and optimizing, network traffic classification, and network management.