Development of a Recursive Multinomial Probit Model and its Possible Application for Innovation Studies

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ABSTRACT

This paper develops a recursive multinomial probit model and describes its estimation method. The recursive multinomial probit model is an extension of a recursive bivariate probit model. The main difference between the two models is that a single decision among two or more alternatives can be considered in each choice equation in the proposed model. The recursive multinomial probit model is developed based on a standard framework of the multinomial probit model and a Bayesian approach with a Gibbs sampling is adopted for the estimation. The simulation exercise with artificial data sets is showed that the model performed well. Since the recursive multinomial probit model can be applied to analyze the causal relationship between discrete dependent variables with more than two outcomes, the model can play an important role in extending the methodology of the causal relationship analysis in innovation research.

Classification codes: C11, C35

KEYWORDS: Multinomial probit, Endogenous dummy, Bayesian approach

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1. INTRODUCTION

In working with empirical data, one sometimes face the joint problems of discreteness of dependent variables and simultaneity (Blundell & Smith, 1989; Li, 1998). For example, the dependent variables may be binary and some of these binary dependent variables may be endogenous regressors in other decisions. The literature on a simultaneous equation model with limited dependent variables and endogenous dummy variables was pioneered by Amemiya (1978) and Heckman (1978) and have been used to model the situation in which the joint problems of limited dependent variables and simulta-

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neity/endogeneity should be considered (Angrist, 2001; Blundell & Smith, 1989; Greene, 1998; Li, 1998; Monfardini & Radice, 2008; Wilde, 2000; Ye, Pendyala, & Gottardi, 2007).

A bivariate probit with endogenous dummy (also called a recursive bivariate probit model) is a two-equation probit model in which the disturbances are correlated and the binary dependent variable of the first equation is an endogenous regressor in the second equation for binary choice (Wilde, 2000). This model widely used to address the concurrent problems of discreteness of dependent variables and simultaneity (Greene, 1998; Monfardini & Radice, 2008; Wilde, 2000; Ye et al., 2007). However, the model has an obvious limitation in that only a binary choice (i.e., the choice between two alternatives) can be considered in each equation. Yet we often encounter data comprised of simultaneous discrete choices made from more than two options.

This study addresses the limitation of a recursive bivariate probit model. For this purpose, this study suggests a recursive multinomial probit model that is a multinomial probit model with an endogenous dummy and the method for its estimation. The model is an extension of a recursive probit model and a single decision among two or more alternatives can be considered in each choice equation for the model. The recursive multinomial probit model is developed based on a standard framework of the multinomial probit model, and a Bayesian approach with Gibbs sampling is adopted for the estimation. This study conducts a simulation exercise with artificial data sets to show the performance of the model. From the simulation study, we can see the difference between a recursive bivariate probit model and a recursive multinomial probit model.

The recursive bivariate probit model is commonly used in economic applications to analyze the causal relationship between binary dependent variables. Therefore, from an empirical perspective, the recursive multinomial probit model can be applied to any research that uses a recursive probit model because the recursive multinomial probit model makes it possible to consider a single choice from more than two alternatives. To see the possible applicability of a recursive multinomial probit model in various fields including innovation research, this study shows example how previous studies using a recursive probit model can be extended by using a recursive multinomial probit model.

The rest of the paper is organized as follows. Section 2 discusses the limitation of previous model. Section 3 presents a recursive multinomial probit model and method of estimation. The simulation study that illustrates the performance of the model with artificial data sets is presented in Section 4. Possible applications of the model for innovation study are discussed in Section 5. The last section presents the conclusion and summary.

2. RECURSIVE PROBIT MODEL AND ITS LIMITATION

A bivariate probit with endogenous dummy or recursive probit model is a two equation probit model in which the disturbances are correlated and the binary dependent variable of the first equation is an endogenous regressor in the second equation for a binary choice (Wilde, 2000). The model allows the analysis of one-way causal relationships between two choice behaviors (Ye et al., 2007). The general form of the model is as follows:

$$\begin{aligned} y_{1i}^* &= \beta_1 x_{1i} + u_{1i} \\ y_{2i}^* &= \delta y_{1i} + \beta_2 x_{2i} + u_{2i} \end{aligned}, \quad y_{1i} = \begin{cases} 1 & \text{if } y_{1i}^* > 0 \\ 0 & \text{otherwise} \end{cases}, \quad y_{2i} = \begin{cases} 1 & \text{if } y_{2i}^* > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{where } \begin{pmatrix} u_{1i} \\ u_{2i} \end{pmatrix} \sim \mathrm{BVN}(0_2, \Sigma), \qquad \Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

In the above equation, y_{1i}^* and y_{2i}^* are latent variables for which only the dichotomous variables y_{1i} and y_{2i} can be observed. x_{1i} and x_{2i} are vectors of exogenous variables, β_1, β_2 , and δ are parameters. The error terms (u_{1i}, u_{2i}) is a vector of bivariate normally distributed disturbances with zero mean, unit variance and correlation coefficient, ρ , independently across observations.

The correlation coefficient (ρ), which can be interpreted as the correlations between the unobservable explanatory variables of the two equations, is related to the exogeneity of y_{1i}. When, $\rho=0$, y_{1i} is exogenous for the second equation, then it is sufficient for parameter estimation to fit two univariate probit equations separately. Under $\rho\neq0$ (i.e.,y_{1i} is endogenous for the second equation) a simultaneous estimation is needed to form consistent estimates of the parameters of the second equation. In this case, parameter estimation can be accomplished using readily available software dealing with an estimation of the common bivariate probit model because the likelihood functions of the recursive bivariate probit model and the common bivariate probit model are virtually identical (Greene, 1998; Ye et al., 2007).

The recursive bivariate probit model is commonly used in economic applications where there are good a priori reasons to consider a dependent binary variable as simultaneously determined with a dichotomous regressor (Monfardini & Radice, 2008). The model has been applied in various fields to analyze the causal relationship between binary dependent variables. For example, between: mode choice and the complexity of trip chaining patterns (Ye et al., 2007), women's studies program and gender economics courses (Greene, 1998), graduate education and job success (Bowman & Mehay, 1999), entry mode choice and FDI survival (Shaver, 1998), decision to investment and firm's innovation activity (Skuras, Tsegenidi, & Tsekouras, 2008), financial constraint and firm's innovation activity (Savignac, 2008).

Although a recursive probit model is commonly used in many economic applications, it has an obvious limitation in that only binary outcome (i.e., the choice between two alternatives or options) can be considered for each equation. Yet we often encounter data comprising simultaneous discrete choices made from among more than two options. Unlike a recursive probit model situation, a choice may be made from more than two options in first choice behavior, and the result of first choice may affect a choice from more than two options in second choice behavior.

This paper suggests a new recursive multinomial probit model (a multinomial probit model with an endogenous dummy) and the method of its estimation. The model is an extension of a recursive probit model and belongs to the general class of simultaneous equation models with discrete endogenous variables. In each choice equation in the model, a single decision among two or more alternatives can be considered. The model directly deals with simultaneity and endogeneity problems. The following section describes the recursive multinomial probit model and suggests the method for its estimation in detail.

3. THE MODEL AND THE METHOD OF ESTIMATION

Suppose that a decision maker (that is, individual, group, firm, organization, or any other decision-making unit) $i(=1,\cdots,N)$ has two choices jointly, and the result of the first choice affects the second choice. Let U_{ij} be the latent-variable which are related to a decision maker i's single choice j from among J alternatives (i.e., the first choice) and V_{il} be the latent-variable which are related to a decision maker i's single choice l from among L alternatives (the second choice).

To solve the identification problem in the multinomial probit model (that any location shift will not change the observed choices), this study choose alternatives J and L as the base choices in each choice, respectively. Then, each choice can be represented by a standard latent variable interpretation of the multinomial probit model as follows:

$$U_i^* = Z_i\alpha + v_i \tag{1}$$

$$V_i^* = E_i\theta + R_i\delta + \rho_i \tag{2}$$

$$\begin{aligned}
&y_i = j(=1,\cdots,J-1) && \text{if } \max(U_i^*) = U_{ij}^* > 0 \\
&y_i = 0 && \text{if } \max(U_i^*) < 0 \\
&h_i = l(=1,\cdots,L-1) && \text{if } \max(V_i^*) = V_{il}^* > 0 \\
&h_i = 0 && \text{if } \max(V_i^*) < 0
\end{aligned}$$

where U_i^* is a$(J-1)\times 1$ vector with $U_{ij}^*=U_{ij}-U_{iJ}$; Z_i is a $(J-1)\times k$ matrix of regressor variables; $\boldsymbol{v}_i$ is a$(J-1)\times 1$ vector of regression error; V_i^* is a $(L-1)\times 1$ vector with $V_{il}^*= V_{il} - V_{iL}$; E_i is a$(L-1)\times q$ matrix of regressor variables; ρ_i is a$(L-1)\times 1$ vector of regression error; $R_i=[(d_{i1},\cdots,d_{iJ-1})\otimes I]$is a $(L-1)\times m$ matrix; and d_{ij} is a dummy defined as 1 if decision maker i selects j alternative in first choice, and zero otherwise that captures the systematic effects of the first choices on the second choices. The observed choices made by decision maker i are expressed by the indicator variables y_i and h_i.

To consider two discrete choices jointly, this study combine equations 1 and 2 as follows:

$$W_i^* = X_i\beta + \varepsilon_i$$

where

$$W_i^* = \begin{bmatrix} U_i^* \\ V_i^* \end{bmatrix},\ X_i = \begin{bmatrix} Z_i & & \\ & E_i & R_i \end{bmatrix},\ \beta = \begin{bmatrix} \alpha \\ \theta \\ \delta \end{bmatrix}$$

$$\varepsilon_i = \begin{bmatrix} v_i \\ \rho_i \end{bmatrix},\ \varepsilon_i \sim N(0,\Sigma),\ \Sigma = \begin{bmatrix} \Sigma_{vv} & \Sigma_{v\rho} \\ \Sigma_{\rho v} & \Sigma_{\rho\rho} \end{bmatrix}$$

The potential correlations between the unobserved components of two discrete choices can be reflected by the partition ($\Sigma_{v\rho}$) of the covariance matrix (Σ), and simultaneity between the two choices and endogenous dummy problems are captured by the covariance term $\Sigma_{v\rho}$.

The model specified above still has an additional identification problem in that any scale shift will not change the observed choice in each multinomial probit. To deal with this identification problem, this study make use of the ideas of Nobile (2000) and McCulloch and Rossi (1994). Nobile suggests a way of imposing the normalization constraint in a Bayesian multinomial probit model using direct simulation from the Wishart and inverted Wishart distributions (conditional on one of the elements being on the diagonal). Following Nobile, we can impose identification in the second choice equation by restricting one of the diagonal elements of $\Sigma_{\rho\rho}$ (also, Σ) to unity. However, the identification problem still remains in the first choice equation. To impose identification in the first choice equation, this study follow McCulloch and Rossi (1994). That is, we compute the full posterior over β and Σ, and report the marginal posterior distribution of the identified parameters, $\tilde{\beta}$ and $\tilde{\Sigma}$ by

$$\tilde{\beta} = \begin{bmatrix} \alpha / \sigma_{11} \\ \theta \\ \delta \end{bmatrix}, \quad \tilde{\Sigma} = C\Sigma C \quad \text{where} \quad C = \begin{bmatrix} \frac{1}{\sigma_{11}} \mathbf{I}_{J-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{L-1} \end{bmatrix}, \quad \text{and} \quad \sigma_{11} \quad \text{is the square root of the (1,1)}$$

element of Σ (also, Σ_{vv} in the first choice). By applying the ideas of Nobile (2000) and McCulloch and Rossi (1994) in succession, the model can be fully identified.

In model specification, this study selects the following priors;

$\beta \sim N(b, A)$

$\Sigma^{-1} \sim W(v, Q) I(\sigma^2_{J+L-2, J+L-2} = 1)$

where the indicator function on the Wishart distribution serves to enforce the identification restriction that the ($J+L-2, J+L-2$) element of Σ is unity (Koop, Poirier, & Tobias, 2007).

Then the full Gibbs sampler is as follows;

$\beta \,|\, \Sigma^{-1}, W^*$

$\Sigma^{-1} \,|\, W^*, \beta$

$W_i^* \,|\, \beta, \Sigma^{-1}$

Note that all posterior distributions are conditional on both $t_i = \begin{bmatrix} y_i \\ h_i \end{bmatrix}$ and X_i.

The first conditional distribution of β is normal.

$$\beta \,|\, \Sigma^{-1}, W^* \sim N(\hat{\beta}, \Sigma_\beta) \tag{1}$$

where $\Sigma_\beta = \left(\sum_i X_i' \Sigma^{-1} X_i + A^{-1} \right)^{-1}$, $\hat{\beta} = \Sigma_\beta \left(\sum_i X_i' \Sigma^{-1} W_i^* + A^{-1} b \right)$.

The second conditional distribution of Σ^{-1} is Wishart.

$$\Sigma^{-1} \,|\, W^*, \beta \sim W\left(v + N, Q + \sum_i \varepsilon_i \varepsilon_i' \right) I(\sigma^2_{J+L-2, J+L-2} = 1) \tag{2}$$

This study uses Nobile's (2000) algorithm to generate draws from this Wishart.

The third conditional distribution of W_i^* is multivariate truncated normal.

$$W_i^* \,|\, \beta, \Sigma^{-1} \overset{ind}{\sim} MTN_{R_i(t_i)}(X_i \beta, \Sigma) \tag{3}$$

where $MTN_{R_i(t_i)}(X_i \beta, \Sigma)$ denotes multivariate normal distribution with mean $X_i\beta$ and covariance matrix Σ that is truncated to the region R_i. R_i is dependent on t_i. This study uses Geweke's (1991) procedure for drawing from this multivariate truncated normal distribution.

4. SIMULATION STUDY

This section illustrates the performance of the model with artificial data sets. The purpose of simulation exercise is to see how the inference method developed in the previous section performs when the model is correctly specified and to investigate the existence of an estimation bias when the endogeneity of dummy variable is ignored. To do that, this study compared the two models using generated data: (1) a full model that is correctly specified and (2) a restricted model that is incorrectly specified, accordingly has an endogeneity problem. This kind of simulation exercise that compares two models (correctly specified model and incorrectly specified model) is a general methodology in a Bayesian approach to the investigation of the performance of a newly proposed model (Chib & Jacobi, 2007; McCulloch & Rossi, 1994; Munkin & Trivedi, 2003).

The data sets are generated from a recursive multinomial probit model with two joint discrete choice behaviors (the first choice: three alternatives and the second choice: three alternatives) as follows:

$$U_{ij}^{*} = z_{ij1}\alpha_1 + z_{ij2}\alpha_2 + \nu_{ij}, \quad j = 1,2$$

$$V_{il}^{*} = e_{il1}\theta_1 + e_{il2}\theta_2 + d_{l1}\delta_{l1} + d_{l2}\delta_{l2} + \rho_{il}, \quad l = 1,2$$

1,000 observations (i=1,⋯,1000) are generated. A detail generating process is shown in Table 1.

TABLE 1 Generating Process of Artificial Data for Recursive Multinomial Probit Model

1. Model structure

(1) $U_{ij}^{*} = z_{ij1}\alpha_1 + z_{ij2}\alpha_2 + \nu_{ij}, \quad j = 1,2$

(2) $V_{il}^{*} = e_{il1}\theta_1 + e_{il2}\theta_2 + d_{l1}\delta_{l1} + d_{l2}\delta_{l2} + \rho_{il}, \quad l = 1,2$

2. Generate explanatory variables and error terms

(1) $z_{ij1} \sim N(0,1)$, $z_{ij2} \sim N(0,1)$, $e_{il1} \sim N(0,1)$, $e_{il2} \sim N(0,1)$

(2) $\varepsilon_i = [\nu_{i1}, \nu_{i2}, \rho_{i1}, \rho_{i2},]' \overset{iid}{\sim} N(0, \Sigma)$.

$\Sigma = \text{diag}(\sigma) A \text{diag}(\sigma)$ with $\sigma' = (\sigma_{11}, 1, \sigma_{33}, 1)$

3. Input the true values

(1) (α_1, α_2)=(-0.5, 1.5), (θ_1, θ_2)=(0.5, -0.5), (δ_{11}, δ_{21})=(-1, 1), (δ_{12}, δ_{22})=(0.5, -0.5)

(2) ($\sigma_{11}^2, \sigma_{33}^2$)=(0.5, 0.8) and $A = \begin{bmatrix} 1 & 0.1 & 0.5 & 0.5 \\ 0.1 & 1 & 0.1 & 0.1 \\ 0.5 & 0.1 & 1 & 0.1 \\ 0.5 & 0.1 & 0.1 & 1 \end{bmatrix}$

4. Generate y_i, d_1 and d_2 according to the following rule:

$$\begin{matrix} d_{l1} = 1 \text{ and } d_{l2} = 0 \text{ if } y_i = 1 \\ d_{l1} = 0 \text{ and } d_{l2} = 1 \text{ if } y_i = 2 \end{matrix}, \quad l = 1,2$$

5. Generate h_i

For prior, this study uses the following prior hyperparameter values: b=**0**, A=400I , $\boldsymbol{v}$=7, Q =$\boldsymbol{v}I$. For the estimation, this study generates 11,000 draws from Gibbs sampling, discarding the first 1,000 draws as burn-in iterations. Restricted model estimation is done through two stages: (1) the first discrete choice equation is estimated by the MNP model and (2) the second discrete choice equation is also estimated by the MNP model in which endogenous dummies from the results of first choice are assumed to be exogenous. The estimation results of the full model and restricted model are given in Table 2.

TABLE 2 A Simulated Example of a Recursive Multinomial Probit Model

		Full model		Restricted model	
		Posterior moments		Posterior moments	
Parameter	**True value**	**Mean**	**Std. dev.**	**Mean**	**Std. dev.**
First choice equation				MNP	
α_1	-0.5	-0.5089	0.0484	-0.5229	0.0451
α_2	1.5	1.5058	0.0965	1.4608	0.0995
Second choice equation				MNP	
θ_1	0.5	0.5593	0.0591	0.5558	0.0542
θ_2	-0.5	-0.5147	0.0539	-0.5374	0.0568
δ_{11}	-1	-1.1626	0.2599	-1.0139	0.2393
δ_{21}	1	1.0664	0.1053	1.2763	0.1021
δ_{12}	0.5	0.6148	0.0951	0.4137	0.0699
δ_{22}	-0.5	-0.5045	0.1255	-0.7115	0.1073
Variance-covariance					
σ_{11}^2	0.5	0.6403	0.1222	0.4676	0.0927
σ_{33}^2	0.8	1.2997	0.3263	1.0029	0.2457
Σ_{31}	0.3162	0.5522	0.1304	-	-
Σ_{41}	0.3536	0.2462	0.0994	-	-

We compare the results between the full model and restricted model using the root mean square deviation (RMSD) which is a measure of the differences between values estimated by a model and the values entered in the model. The posterior means of identified parameters in the full model are close to the true values used to generate the artificial data. RMSD between values estimated by the full model and the values actually used in the artificial data is calculated for the second choice equation; the value of RMSD is 0.0892. When the endogeneity of dummy variable is ignored, it results in an estimation bias. The estimates of parameters (δ_{21}, δ_{12}, δ_{22}) in the restricted model (second choice equation estimated by MNP model) that are parameters for the endogenous dummies in the second choice equation are not close to the true values. The RMSD between values estimated by the restricted model and the true values is 0.1490 for the second choice equation that is almost two times bigger than that of full model. This means the differences between values estimated by restricted model and the true values are significant and suggest potentially serious consequence in ignoring the endogenous dummy problem. Of note is that the absolute value of difference between the estimated parameters in the restricted model and the true values is somewhat small. This may be because the relative influence of endogenous dummies on choice is not large enough to change the results of the second choice equation. The estimation method developed in the previous section performs well and the estimated results are consistent with the true values.

5. POSSIBLE APPLICATION OF THE MODEL IN INNOVATION RESEARCH

From an empirical perspective, the recursive multinomial probit model can be applied to any situation where successive two discrete choices can be modeled jointly or in one-way causal relationships between two discrete choices can be modeled. The model can be applied to any research that uses a recursive probit model where a single choice from more than two alternatives can be considered. This section discusses the possible applicability of a recursive multinomial probit model in innovation research. For this purpose, some previous studies that use a recursive probit model in innovation research are reviewed and the examples of the studies that can be extended through the use of a recursive multinomial probit model are discussed.

Shaver (1998) examined the effect of entry mode choice (especially acquisition) on Foreign Direct Investment (FDI) survival using a recursive bivariate probit model. In their model, the first probit equation determines whether the entry mode is in the form of acquisition or not. The dummy variable that represents the entry mode choice is then entered as the endogenous explanatory variable in the second probit equation that represents whether the firm survived or not; however, the entry modes of multinational firm are various such as a Green Field Investment, joint ventures, and acquisition. Shaver's study has a limitation in that it consider only one entry mode (i.e., acquisition) without considering any other entry mode. To analyze the effect of various entry modes on FDI survival, each entry mode should be considered as an alternative in the first choice equation. This means that a single choice from more than two mutually exclusive alternatives should be considered in the estimation model and then the recursive multinomial probit model can be applied to this situation. Therefore, Shaver's study can be extended as a research examining the effect of various entry modes on FDI survival using the recursive multinomial probit model.

Bowman and Mehay (1999) examined the effect of graduate education on job success using the recursive bivariate probit model. In their model, the first probit equation determines whether and individual attends graduate school or not, the endogenous dummy that represents the attendance is then entered as an explanatory variable in the second probit equation that indicates whether the individual is promoted or not. Although Bowman and Mehay (1999) examined the effect of graduate education on job success, we can generalize their study as an examination of the effect of education on job success since the education level of the individual varies. By using a recursive probit model, we can examine the effect of education level on job success and then the first probit equation is changed into a multinomial probit equation where various education levels such as bachelor's degree, master's degrees, and PhD degrees can be considered as alternatives.

The recursive multinomial probit model can be applied to the examination of the causal relationship between various factors and innovation activity. Skuras et al. (2008) examined the impact of investment decisions on a firm's innovation activity using the recursive bivariate probit model. In their model, the first probit equation determines whether a firm has decided to invest in a fixed capital asset, and then the endogenous dummy that represents the decision to invest is entered as explanatory variable in the second probit equation represents the occurrence of innovative activity such as product innovation. Savignac (2008) examined the impact of financial constraints on innovation through use of the recursive bivariate probit model. In their model, the first probit equation is determining whether the firm faces financial constraints or not, and then the endogenous dummy that represents the existence of financial constraints is entered as an explanatory variable in the second probit equation that represents the occurrence of innovative activity. Since, these studies consider binary dependent variables (such as the decision to invest, the existence of financial constraints, or the occurrence of innovation activity) it is enough to use a recursive bivariate probit model to examine the causal relationship between binary dependent variables. However, if the alternatives in the first (or second) equation are mutually exclusive and are more than two alternatives, then the recursive multinomial probit model can be applied to address the simultaneous choices when a single decision from more than two options is considered in each choice.

6. CONCLUDING REMARKS

This study suggests a recursive multinomial probit model of a multinomial probit model with an endogenous dummy and the method of its estimation. A recursive bivariate probit model (which is a bivariate probit with endogenous dummy) is widely used to analyze one-way causal relationships between two choice behaviors; however, it has an obvious limitation in that only a binary dependent variable is considered in each equation. A recursive multinomial probit model extends a recursive probit model by making it possible to consider a single decision among two or more alternatives.

A recursive multinomial probit model is developed based on standard framework of the multinomial probit model, and a Bayesian approach with Gibbs sampling is adopted for the estimation. A simulation exercise with artificial data sets was conducted and the results show that the model performs well.

From an empirical perspective, the recursive multinomial probit model can be applied to any research that uses a recursive bivariate probit model, since the recursive multinomial probit model makes it possible to consider a single choice from more than two alternatives. In conclusion, the recursive multinomial probit model can play an important role in innovation research since it can extend the methodology of the causal relationship analysis.

References

Amemiya, T., "The Estimation of a Simultaneous Equation Generalized Probit Model.", *Econometrica*(1978), Vol. 46, No.5, pp. 1193-1205.

Angrist, J. D., "Estimation of Limited Dependent Variable Models with Dummy.", 2001.

Endogenous Regressors, "Simple Strategies for Empirical Practice.", *Journal of Business & Economic Statistics*, Vol. 19, No.1, pp. 2-16.

Blundell, R. W., & Smith, R. J., "Estimation in a Class of Simultaneous Equation Limited Dependent Variable Models.", *The Review of Economic Studies*(1989), Vol. 56, No.1, pp. 37-57.

Bowman, W. R., & Mehay, S. L., "Graduate education and employee performance: evidence from military personnel.", *Economics of Education Review*(1999), Vol. 18, No.4, pp. 453-463.

Chib, S., & Jacobi, L., "Modeling and calculating the effect of treatment at baseline from panel outcomes.", *Journal of Econometrics*(2007), Vol. 140, No.2, pp. 781-801.

Geweke, J., "Efficient simulation from the multivariate normal and student-t distributions subject to linear constraints and the evaluation of constraint probabilities.", Paper presented at the Computer Science and Statistics: Proceedings of the Twenty-Third Symposium on the Interface, Fairfax, 1991.

Greene, W. H., "Gender Economics Courses in Liberal Arts Colleges: Further Results.", *Journal of Economic Education*(1998), Vol. 29, No.4, pp. 291-300.

Heckman, J. J., "Dummy Endogenous Variables in a Simultaneous Equation System." *Econometrica*(1978), Vol. 46, No.4, pp. 931-959.

Koop, G. M., Poirier, D. J., & Tobias, J. L., "Bayesian Econometric Methods.", New York: Cambridge University Press, 2007.

Li, K., "Bayesian inference in a simultaneous equation model with limited dependent variables.", *Journal of Econometrics*(1998), Vol. 85, No.2, pp. 387-400.

McCulloch, R., & Rossi, P. E., "An exact likelihood analysis of the multinomial probit model.", *Journal of Econometrics*(1994), Vol. 64, No.1-2, pp. 207-240.

Monfardini, C., & Radice, R., "Testing Exogeneity in the Bivariate Probit Model: A Monte Carlo Study.", *Oxford Bulletin of Economics and Statistics*(2008), Vol. 70, No.2, pp. 271-282.

Munkin, M. K., & Trivedi, P. K., "Bayesian analysis of a self-selection model with multiple outcomes using simulation-based estimation: an application to the demand for healthcare.", *Journal of Econometrics*(2003), Vol. 114, No.2, pp. 197-220.

Nobile, A., "Comment: Bayesian multinomial probit models with a normalization constraint.", *Journal of Econometrics*(2000), Vol. 99, No.2, pp. 335-345.

Savignac, F., "Impact of financial constraints on innovation : what can be learned from a direct measure?", *Economics of Innovation and New Technology*(2008), Vol. 17, No.6, pp. 553-569.

Shaver, J. M., "Accounting for Endogeneity When Assessing Strategy Performance: Does Entry Mode Choice Affect FDI Survival?", *Management Science*(1998), Vol. 44, No.4, pp. 571-585.

Skuras, D., Tsegenidi, K., & Tsekouras, K., "Product innovation and the decision to invest in fixed capital assets: Evidence from an SME survey in six European Union member states.", *Research Policy*(2008), Vol. 37, No.10, pp. 1778-1789.

Wilde, J., "Identification of multiple equation probit models with endogenous dummy regressors.", *Economics Letters*(2000), Vol. 69, No.3, pp. 309-312.

Ye, X., Pendyala, R. M., & Gottardi, G., "An exploration of the relationship between mode choice and complexity of trip chaining patterns.", *Transportation Research Part B: Methodological*(2007), Vol. 41, No.1, pp. 96-113.