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Semiparametric Seasonal Cointegrating Rank Selection

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Abstract

This paper considers the issue of seasonal cointegrating rank selection by information criteria as the extension of Cheng and Phillips (2009). The method does not require the specification of lag length in vector autoregression, is convenient in empirical work, and is in a semiparametric context because it allows for a general short memory error component in the model with only lags related to error correction terms. Some limit properties of usual information criteria are given for the rank selection and small Monte Carlo simulations are conducted to evaluate the performances of the criteria.

Keywords: Seasonal cointegration, information criteria, nonparametric model selection.

1. Introduction

Various procedures have been proposed to determine cointegrating(CI) ranks in nonseasonal and seasonal models. They are mostly the likelihood ratio type of tests, considered by Johansen (1996) for nonseasonal cointegration, Johansen and Schaumburg (1999), Cubadda (2001), and Seong *et al.* (2006) for seasonal cointegration. However, these procedures are based on parametric models that require the specification of a full model such as lag length in vector autoregression and can occur the misspecification of CI rank possibly through an inappropriate specification of the lag length. Recently, the works by Phillips (2008), and Cheng and Phillips (2008, 2009) consider semiparametric models as alternatives to parametric ones in determining CI rank. They regard CI rank as an order parameter for which information criteria are particularly well suited since there are only a finite number of possible choices.

In this paper, we extend the issue of cointegrating rank selection by information criteria to seasonal models, by using Gaussian Reduced Rank(GRR) procedure of Ahn and Reinsel (1994). The GRR estimation has a special characteristic that simultaneously imposes rank conditions at all existing

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seasonal unit roots (Seong and Yi, 2008). If we use a partial regression procedure, often used by previous literature such as Johansen and Schaumburg (1999), it is impossible to construct the simultaneous estimation because it ignores the constraints of reduced ranks (*i.e.*, cointegrated structures) of the other unit roots except one unit root focused by the partial regression. Note that seasonal CI rank tests, performed independently by focusing on one unit root at a time, can result in a seriously inflated Type I error in terms of multiple hypothesis testing (Seong, 2009). Therefore, this paper combines three advantages: (i) simultaneous estimation from the GRR, (ii) semiparametric model without the specification of complete form, and (iii) convenience for practical implementation.

The paper is organized as follows. In Section 2, the semiparametric error correction model(ECM) is presented for seasonal cointegration. The asymptotic results are given in Section 3. In Section 4, small Monte Carlo simulations are conducted to evaluate performances of the proposed methods. Conclusions are drawn in Section 5.

2. The Semiparametric Seasonal ECM

We consider the following semiparametric seasonal ECM:

$$Z_t = A_1B_1U_{t-1} + A_2B_2V_{t-1} + (A_3B_4 + A_4B_3)W_{t-1} + (A_4B_4 - A_3B_3)W_{t-2} + e_t, \tag{2.1}$$

where X_t is an m-vector time series with $Z_t = (1 - L^4)X_t$ stationary,

$$U_t = (1 + L)(1 + L^2)X_t, \qquad V_t = (1 - L)(1 + L^2)X_t, \qquad W_t = (1 - L^2)X_t,$$

and A_j and B_j are $m \times r_{0j}$ and $r_{0j} \times m$ matrices, respectively, with rank equal to r_{0j} for $j = 1, \ldots, 4$, and $r_{03} = r_{04}$. The error term e_t is weakly dependent stationary time series with zero mean and continuous spectral density matrix $f_e(\lambda)$. We assume that the initial value X_0 is fixed and, for brevity, X_t is observed on a quarterly basis. Models with the other seasonal periods, e.g., monthly, can be easily implemented as in Ahn *et al.* (2004). Note that r_{01}, r_{02}, and $r_{03}(r_{04})$ denote the CI ranks at seasonal unit roots 1, -1, and $i(-i)$, respectively, (*i.e.*, frequencies 0, π, and $\pi/2(3\pi/2)$, respectively), and B_1U_t, B_2V_t, $(B_3 + B_4L)W_t$, and $(B_4 - B_3L)W_t$ are stationary processes, *i.e.*, CI relationships.

As in Phillips (2008) and Cheng and Phillips (2009), we treat model (2.1) semiparametrically with regard to e_t and identify the seasonal CI ranks r_{01}, r_{02}, and $r_{03}(r_{04})$ directly and simultaneously in model (2.1) by information criteria and the GRR estimation. Specifically, we identify the ranks as follows: model (2.1) is estimated by the GRR for all combinations of $\mathbf{r} = (r_1, r_2, r_3)$ with $r_j = 0, 1, \ldots, m$ $(j = 1, 2, 3)$ just as if e_t were a martingale difference, and the combination of $\mathbf{r} = (r_1, r_2, r_3)$ is chosen to minimize the corresponding information criteria as if model (2.1) were a correctly specified parametric framework up to the ranks parameter $\mathbf{r}$. Thus, the selection method is convenient for practical implementation in empirical work because no explicit account is taken of the weak dependence structure of e_t in the process. The criterion used to evaluate the seasonal CI ranks takes the following form:

$$\mathrm{IC}(\mathbf{r}) = \log\left|\hat{\Sigma}(\mathbf{r})\right| + C_n n^{-1}\left\{\left(2mr_1 - r_1^2\right) + \left(2mr_2 - r_2^2\right) + 2\left(2mr_3 - r_3^2\right)\right\}, \tag{2.2}$$

where $\hat{\Sigma}(\mathbf{r})$ denotes the residual covariance matrix from the GRR and coefficient $C_n = \log n$, $2\log\log n$, or 2 corresponds to the BIC (Schwarz, 1978), HQ (Hannan and Quinn, 1979), and AIC (Akaike, 1973) penalties, respectively. Note that, in Equation (2.2), the degrees of freedom terms

$2mr_1 - r_1^2$, $2mr_2 - r_2^2$, and $2(2mr_3 - r_3^2)$ account for the $2mr_j$ $(j = 1, 2, 3, 4)$ elements of the matrices A_j and B_j that have to be estimated, adjusted for the r_j^2 restrictions on B_j that ensure a unique parameterization.

For each combination $\mathbf{r} = (r_1, r_2, r_3)$ with $r_j = 0, 1, \ldots, m$ $(j = 1, 2, 3)$, we estimate the $m \times r_j$ matrices A_j and B_j by the GRR estimation, denoted by $\hat{A}_j$ and $\hat{B}_j$, and, for use in Equation (2.2), we form the corresponding residual covariance matrices

$$\hat{\Sigma}(\mathbf{r}) = n^{-1} \sum_{t=4}^{n} \hat{e}_t \hat{e}_t', \qquad \text{for } r_j = 0, 1, \ldots, m \ (j = 1, 2, 3),$$

where

$$\hat{e}_t = Z_t - \hat{A}_1 \hat{B}_1 U_{t-1} - \hat{A}_2 \hat{B}_2 V_{t-1} - \left(\hat{A}_3 \hat{B}_4 + \hat{A}_4 \hat{B}_3 \right) W_{t-1} - \left(\hat{A}_4 \hat{B}_4 - \hat{A}_3 \hat{B}_3 \right) W_{t-2}$$

and $r_j = 0$ and $r_j = m$ imply that $A_j B_j' = O_m$ and $B_j = I_m$, respectively. Model evaluation based on $\mathrm{IC}(\mathbf{r})$ then leads to the seasonal CI ranks selection criterion

$$\hat{\mathbf{r}} = \underset{0 \le r_1, r_2, r_3 \le m}{\arg\min} \ \mathrm{IC}(\mathbf{r}). \tag{2.3}$$

Similarly in the Cheng and Phillips (2009), the information criterion $\mathrm{IC}(\mathbf{r})$ is expected to be weakly consistent for selecting the CI ranks $\mathbf{r} = (r_1, r_2, r_3)$ provided that the penalty term in Equation (2.2) satisfies the weak requirements that $C_n \to \infty$ and $C_n/n \to 0$as $n \to \infty$. No minimum expansion rate for C_n such as $\log \log n$ is required and no more complex parametric model needs to be estimated. The approach is therefore quite straightforward for practical implementation.

3. Asymptotic Results

We consider the weak consistency of the information criteria for selecting the true seasonal CI ranks $\mathbf{r}_0 = (r_{01}, r_{02}, r_{03})$ under suitable regular conditions which are standard in the study of linear process condition of the innovations, and seasonal cointegration. The following theorem can be conjectured from that of Cheng and Phillips (2009).

Theorem 3.1. *Under suitable assumptions,*

(a) the criterion $IC(\mathbf{r})$ is weakly consistent for selecting the seasonal CI ranks provided $C_n \to \infty$ and $C_n/n \to 0$ as $n \to \infty$;

(b) the asymptotic distribution of the AIC criterion is given by

$$\lim_{n\to\infty} P(\hat{\mathbf{r}}_{AIC} = \mathbf{r}_0) = \xi_1 > 0,$$
$$\lim_{n\to\infty} P(\hat{\mathbf{r}}_{AIC} = \mathbf{r} | \mathbf{r} \succ \mathbf{r}_0) = \xi_2 > 0,$$
$$\lim_{n\to\infty} P(\hat{\mathbf{r}}_{AIC} = \mathbf{r} | \mathbf{r} \prec \mathbf{r}_0) = 0,$$

where $\mathbf{r}_1 \succ \mathbf{r}_2$ and $\mathbf{r}_1 \prec \mathbf{r}_2$ denote that all components of the vector $\mathbf{r}_1 - \mathbf{r}_2$ and $\mathbf{r}_2 - \mathbf{r}_1$, respectively, are positive.

Part (a) of theorem implies that all information criteria with $C_n \to \infty$ and $C_n/n \to 0$, such as BIC and HQ, are consistent for the selection of seasonal CI rank with semiparametric estimation approach, *i.e.*, without the specification of a full model. Part (b) implies that AIC is inconsistent

Table 4.1. Seasonal CI rank selection when DGP I with $e_t \sim$ AR(1)

(r_1, r_2, r_3)	BIC		HQ		AIC	
	$T = 100$	$T = 400$	$T = 100$	$T = 400$	$T = 100$	$T = 400$
$(0, 0, 0)$	0	0	0	0	0	0
$(0, 1, 0)$	0	0	0	0	0	0
$(0, 2, 0)$	0	0	0	0	0	0
$(0, 0, 1)$	0	0	0	0	0	0
$(0, 1, 1)$	0	0	0	0	0	0
$(0, 2, 1)$	0	0	0	0	0	0
$(0, 0, 2)$	0	0	0	0	0	0
$(0, 1, 2)$	0	0	0	0	0	0
$(0, 2, 2)$	0	0	0	0	0	0
$(1, 0, 0)$	0	0	0	0	0	0
$(1, 1, 0)$	0	0	0	0	0	0
$(1, 2, 0)$	0	0	0	0	0	0
$(1, 0, 1)$	0	0	0	0	0	0
$(1, 1, 1)$	8688	9342	7228	7942	5194	5334
$(1, 2, 1)$	177	59	550	358	1050	1059
$(1, 0, 2)$	0	0	0	0	0	0
$(1, 1, 2)$	605	332	1154	979	1656	1692
$(1, 2, 2)$	12	2	83	37	336	319
$(2, 0, 0)$	0	0	0	0	0	0
$(2, 1, 0)$	0	0	0	0	0	0
$(2, 2, 0)$	0	0	0	0	0	0
$(2, 0, 1)$	0	0	0	0	0	0
$(2, 1, 1)$	474	253	801	589	1145	980
$(2, 2, 1)$	14	3	64	21	243	214
$(2, 0, 2)$	0	0	0	0	0	0
$(2, 1, 2)$	30	9	108	73	293	333
$(2, 2, 2)$	0	0	12	1	83	69
Total	10000	10000	10000	10000	10000	10000

in that it asymptotically never underestimates CI ranks and, instead, asymptotically overestimates them. This outcome is analogous to the well-known overestimation tendency of AIC or the result of Cheng and Phillips (2009).

4. Monte Carlo Simulations

Monte Carlo simulations are conducted to evaluate the performances of the information criteria in identifying seasonal CI ranks.

The first data generating process(DGP I) considered is the bivariate quarterly process modified from that of Ahn and Reinsel (1994):

$$\left(1 - L^4\right) X_t = A_1 B_1 U_{t-1} + A_2 B_2 V_{t-1} + A_4 B_2 W_{t-1} - A_3 B_2 W_{t-2} + e_t,$$

where

$$A_1 = (0.6, 0.6)', \quad A_2 = (-0.4, 0.6)', \quad A_3 = (0.6, -0.6)', \quad A_4 = (0.4, -0.8)',$$
$$B_1 = (1, -0.7), \quad B_2 = (1, 0.4).$$

Note that the characteristic roots are ± 1, $\pm i$, $0.9715 \pm 0.7328i$, and $-1.3508 \pm 0.3406i$ when e_t is

Table 4.2. Seasonal CI rank selection when DGP II with $e_t \sim$ AR(1)

(r_1, r_2, r_3)	BIC		HQ		AIC	
	$T = 100$	$T = 400$	$T = 100$	$T = 400$	$T = 100$	$T = 400$
$(0, 0, 0)$	6568	8508	2736	4233	526	522
$(0, 1, 0)$	543	160	1423	1091	1035	1005
$(0, 2, 0)$	5	0	58	38	116	125
$(0, 0, 1)$	1540	798	2051	1984	1063	1070
$(0, 1, 1)$	64	9	718	464	1819	1865
$(0, 2, 1)$	0	0	29	16	197	217
$(0, 0, 2)$	127	42	274	217	209	239
$(0, 1, 2)$	4	0	76	45	357	406
$(0, 2, 2)$	0	0	2	1	30	44
$(1, 0, 0)$	824	380	935	878	378	318
$(1, 1, 0)$	36	9	425	200	770	660
$(1, 2, 0)$	1	0	13	5	92	80
$(1, 0, 1)$	133	42	592	352	754	632
$(1, 1, 1)$	1	0	152	80	1150	1096
$(1, 2, 1)$	0	0	10	0	122	109
$(1, 0, 2)$	8	1	71	46	142	129
$(1, 1, 2)$	0	0	18	7	230	240
$(1, 2, 2)$	0	0	0	0	22	24
$(2, 0, 0)$	120	48	191	188	122	131
$(2, 1, 0)$	6	2	82	46	213	257
$(2, 2, 0)$	0	0	4	1	22	36
$(2, 0, 1)$	19	1	90	81	204	225
$(2, 1, 1)$	0	0	27	16	278	391
$(2, 2, 1)$	0	0	1	0	42	32
$(2, 0, 2)$	1	0	16	8	40	47
$(2, 1, 2)$	0	0	6	3	62	94
$(2, 2, 2)$	0	0	0	0	5	6
Total	10000	10000	10000	10000	10000	10000

regarded as a martingale difference and, then, X_t is seasonally cointegrated with CI rank of one at unit roots 1, -1, and i each.

In the second data generating process(DGP II), we consider the case without cointegration at any seasonal unit root,

$$\left(1 - L^4\right) X_t = e_t.$$

In both DGPs, the error process e_t is assumed, as in Cheng and Phillips (2008), to be AR(1), MA(1), and ARMA(1,1) errors, corresponding to the models

$$e_t = \psi I_m e_{t-1} + \eta_t, \quad e_t = \eta_t + \phi I_m \eta_{t-1} \quad \text{and} \quad e_t = \psi I_m e_{t-1} + \eta_t + \phi I_m \eta_{t-1},$$

where $|\psi| < 1$, $|\phi| < 1$, and η_t are *i.i.d.* $N(0, \Sigma)$ with $\Sigma = \text{diag}\{1 + \theta, 1 - \theta\}$. The parameters are set to $\psi = \phi = 0.4$, and $\theta = 0.25$.

We generate 10,000 replications of the sample sizes with $T = 100$ and 400. We use initial values that are set to zero; however, discard the first 50 observations in order to eliminate dependence on the starting conditions. The performances of the criteria BIC, HQ, and AIC are investigated for the samples sizes and the results are summarized in Table 4.1 and Table 4.2 which show the results for DGP I and DGP II, respectively. The tables display the results for the model with AR(1) error. We omit the tables for that with the other errors because similar results are observed.

From Table 4.1, as expected from Theorem 3.1, all the information criteria are generally minimized when selected seasonal CI ranks coincide with true ranks, *i.e.*, $(r_1, r_2, r_3) = (1, 1, 1)$. Of note is that they never select the case that at least one rank among CI ranks is underestimated and the criteria have a tendency to overestimate ranks. The tendency is strengthened when only one rank is overestimated, such as $(r_1, r_2, r_3) = (1, 1, 2)$, $(1, 2, 1)$, and $(2, 1, 1)$. Nevertheless, BIC performs better than HQ and AIC which show a strong tendency to overestimate CI ranks.

As underestimation of CI ranks cannot occur in DGP II, we can analysis overestimation better than in DGP I. From Table 4.2, we observe similar results to Table 4.1 but the overestimation by AIC is noticeably strong, especially, in that AIC selects $(r_1, r_2, r_3) = (0, 1, 0)$, $(0, 0, 1)$, $(0, 1, 1)$, and $(1, 1, 1)$ comparatively often, instead of true ranks $(r_1, r_2, r_3) = (0, 0, 0)$. This tendency is noticeably attenuated when CI rank is overestimated at unit root 1.

Note that, from Table 4.1 and Table 4.2, we can conjecture the results that the information criteria will give when we use the partial regression procedure by Johansen and Schaumburg (1999) instead of the GRR. As the partial regression is performed by focusing on one unit root by regarding the ranks of the other unit roots as full ranks, the selection frequencies at the cases $(r_1, r_2, r_3) = (0, 2, 2)$, $(1, 2, 2)$, and $(2, 2, 2)$ can show the conjecture.

5. Conclusions

In this paper, we show that information criteria can consistently select seasonal CI ranks if they satisfy weak conditions on the expansion rate of the penalty coefficient, as extension of the nonseasonal model in Cheng and Phillips (2009) to seasonal. The method by the criteria offers substantial convenience to the empirical researcher because it is robust to weak dependence of error term.

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