

# Blind Channel Equalization Using Conditional Fuzzy C-Means

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## ABSTRACT

In this paper, the use of conditional Fuzzy C-Means (CFCM) aimed at estimation of desired states of an unknown digital communication channel is investigated for blind channel equalization. In the proposed CFCM, a collection of clustered centers is treated as a set of pre-defined desired channel states, and used to extract channel output states. By considering the combinations of the extracted channel output states, all possible sets of desired channel states are constructed. The set of desired states characterized by the maximal value of the Bayesian fitness function is subsequently selected for the next fuzzy clustering epoch. This modification of CFCM makes it possible to search for the optimal desired channel states of an unknown channel. Finally, given the desired channel states, the Bayesian equalizer is implemented to reconstruct transmitted symbols. In a series of simulations, binary signals are generated at random with Gaussian noise, and both linear and nonlinear channels are evaluated. The experimental studies demonstrate that the performance (being expressed in terms of accuracy and speed) of the proposed CFCM is superior to the performance of the existing method exploiting the "conventional" Fuzzy C-Means (FCM).

**Key words:** Conditional Fuzzy C-Means, Bayesian Likelihood Fitness, Blind Equalization, Desired Channel States, Channel Output States

## 1. INTRODUCTION

Most of digital communication channels suffer from inter-symbol-interference (ISI) due to non-ideal channel characteristics. The ISI will increase the symbol error rate at the receiver, sometimes preventing correct detection of a transmitted signal. The problem becomes more severe in the presence of additive white Gaussian noise (AWGN). Furthermore, the nonlinear character of ISI that often arises in high speed communication

channels degrades the performance of the overall digital communication system [1]. As a result, channel equalizers are required to remove the channel distortion. Most of them take advantage of the use of known training sequences to adaptively extract channel information. The problem with this approach is that it is bandwidth consuming. To alleviate this problem, blind-equalization algorithms have been proposed [2-4]. Here, instead of using training sequences, only an input signal and a knowledge of statistical properties of noise are required. The original transmitted message is recovered only from the received sequence that is corrupted by noise and ISI without any training sequence or a priori knowledge of the channel. However, because of inherent simplicity, most works for blind channel equalization deal with linear channels that are often inadequate for modeling channels which exhibit nontrivial nonlinearities [5-7]. This paucity does not mean blind

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nonlinear equalization methods exhibit less significance. Considering that nonlinear distortion exists in many communication systems such as high power amplifiers as well as high-density magnetic and optical storage channels, studying blind nonlinear system equalization methods comes with significant practical importance. Thus, in this paper, the blind equalization method, which could serve as a solution to both linear and nonlinear channels at the same time, is presented.

Early works done on blind nonlinear channel equalization have focused on channel estimation by exploiting high order statistics (HOS) [8-10]. The resulting equalizers suffer from slow convergence and when optimized could be easily trapped in local minima. The blind estimation of Volterra kernels, which characterize nonlinear channels, was presented in [11], while a maximum likelihood (ML) method implemented via expectation-maximization (EM) was introduced in [12]. Although these approaches seem to be applicable to nonlinear channels, the Volterra approach suffers from an enormous computational complexity required to construct a corresponding "inverse" Volterra filter, and the ML approach requires some prior knowledge of the nonlinear channel structure to estimate the channel parameters. The approach involving a nonlinear structure such as multilayer perceptrons being trained to minimize some cost function, has been investigated in [13]. However, in this method, the structure and the complexity of the nonlinear equalizer must be specified in advance. The support vector (SV) equalizer proposed by Santamaria et al. [14] can be viewed as a possible solution for both linear and nonlinear blind channel equalization, but it still suffers from high computational cost of its iterative reweighted quadratic programming procedure. The deterministic approach discussed in [15] based on second order statistic (SOS) has been successfully used to design blind equalizers of nonlinear channels, but its computational cost is also very high (requiring two matrix

eigen-decompositions). Another SOS-based method provided by Raz and Van Veen [16] has limited practical application as it requires that every nonlinear sub-channel be linearizable by an FIR Volterra system. Furthermore, for this method, the sampling rate for the received signal has to be higher than the baud rate, otherwise a multi-sensor array must be utilized. In addition, the signal to noise ratio (SNR) should be kept relatively high. A unique approach to blind channel equalization was offered by Lin and Yamashita [17]. In this method, they used the simplex Genetic Algorithm (GA) to estimate the optimal channel output states instead of estimating the channel parameters in a direct manner. The desired channel states of an unknown channel were constructed from these estimated channel output states, and placed at the center of their RBF equalizer. With this approach, the complex modeling of the nonlinear channel can be avoided and the method works well within a simple single input single output (SISO) communication environment. Additionally, this kind of approach can be applied to a linear channel as well, because it does not estimate the channel parameters but the channel output states directly, which is not dependent on the type of the channel structure. For the better performance in terms of speed and accuracy, this approach has been implemented with a hybrid genetic algorithm (that is genetic algorithm, GA merged with simulated annealing (SA); GASA) [18]. However, in general, the GA based algorithms may visibly suffer from their poor convergence properties. To overcome these weaknesses, FCM, one of the representative clustering algorithms which exhibits shorter processing time than the GA-based methods, has been modified, and the faster convergence speed along with the reliable estimation accuracy in search of the optimal channel output states have been achieved [19,20].

However, with low SNRs, the estimation accuracy of the modified FCM (MFCM) presented in [19] and [20] is no longer superior to GA-related

approaches even though it comes with the faster search speed. For real-time use, the search algorithm should be robust to intensive noise communication environments. This leads to consider the use of some modified version of the "conventional" FCM-based algorithm. A suitable modification comes in the form of a so-called Conditional FCM, or CFCM for brief. The CFCM was first introduced in [21], and successfully applied to channel equalization problem [22]. In CFCM, the conditioning aspect of the clustering mechanism is introduced by taking the conditioning variable defined over the corresponding patterns. More specifically, the conditioning variable describes a level of involvement of incoming input pattern in the constructed clusters. It can be helpful to reduce the influence of heavy noise-corrupted sequences in the underlying clustering procedure. Thus, in the proposed algorithm, CFCM is utilized to overcome this noise effect. It is accomplished by the use of Bayesian likelihood fitness function and the involvement of the relation between desired channel states and channel output states. The final clustered units of the CFCM with this modification represent the desired states of the unknown channel and are utilized to compute the decision probability of Bayesian equalizer for blind equalization. Its performance is compared with the one using the MFCM presented in [20]. In the experiments, both of linear and nonlinear channels with the heavy noise (SNR=0, 2.5, 5, 7.5, 10db) are evaluated.

The organization of this paper is structured as follows. Section 2 develops an optimal Bayesian solution for linear/nonlinear channel equalization; Section 3 shows the relationships between the desired channel states and the channel output states. In Section 4, the modification of CFCM is discussed. The simulation results including some comparative studies are provided in Section 5. Conclusions are given in Section 6.

## 2. OPTIMAL BAYESIAN SOLUTION FOR CHANNEL EQUALIZATION

The channel equalization system discussed here is depicted in Fig. 1. A digital information sequence  $s(k)$  is transmitted through the channel, which is composed of a linear portion described by  $H(z)$  and a nonlinear component  $N(z)$ , governed by the following expressions,

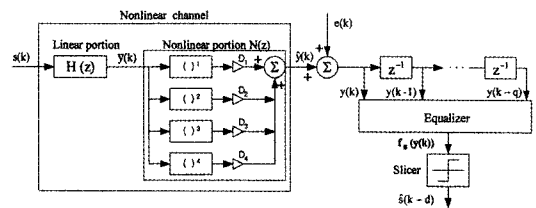


Fig. 1. An overall structure of channel equalization system.

$$\bar{y}(k) = \sum_{i=0}^p h(i)s(k-i) \quad (1)$$

$$\hat{y}(k) = D_1 \bar{y}(k) - D_2 \bar{y}(k)^2 - D_3 \bar{y}(k)^3 - D_4 \bar{y}(k)^4 \quad (2)$$

where  $p$  is the channel order and  $D_i$  stands for the coefficient of the  $i^{\text{th}}$  nonlinear term. This nonlinearity in a channel can be due to nonlinearities associated with nonlinear devices used in the transmitter and receiver. The transmitted symbol sequence,  $s(k)$ , is assumed to constitute an equiprobable and independent binary sequence taking values from a two-valued set  $\{\pm 1\}$ . The channel output,  $\hat{y}(k)$ , is assumed to be corrupted by the AWGN,  $e(k)$ . Given this, the channel observation,  $y(k)$ , can be expressed as

$$y(k) = \hat{y}(k) - e(k) \quad (3)$$

If  $q$  denotes the equalizer order (viz. a number of tap delay elements in the equalizer), then there exist  $M = 2^{p-q-1}$  different patterns of input sequences that may be received (where each component in (4) is either equal to 1 or -1).

$$s(k) = [s(k), s(k-1), \dots, s(k-p-q)] \quad (4)$$

For a specific channel order and equalizer order, these  $M$  input patterns influence the input vector

of equalizer, which is shown in (5) for a noise-free case.

$$\hat{\mathbf{y}}(\mathbf{k}) = [\hat{y}(k), \hat{y}(k-1), \dots, \hat{y}(k-q)] \quad (5)$$

The noise-free observation vector,  $\hat{\mathbf{y}}(\mathbf{k})$ , is referred to as the desired channel states, and can be partitioned into two sets,  $\mathbf{Y}_{q,d}^{+1}$  and  $\mathbf{Y}_{q,d}^{-1}$ , as shown in (6) and (7), depending on the value of  $s(k-d)$ , where  $d$  is the required time delay.

$$\mathbf{Y}_{q,d}^{-1} = \{ \hat{\mathbf{y}}(\mathbf{k}) | s(k-d) = -1 \} \quad (6)$$

$$\mathbf{Y}_{q,d}^{+1} = \{ \hat{\mathbf{y}}(\mathbf{k}) | s(k-d) = 1 \} \quad (7)$$

In case of a linear channel ( $D_1=1, D_2=0, D_3=0$  and  $D_4=0$ ),  $\hat{\mathbf{y}}(\mathbf{k})$  in (3) and (5)-(7) is just replaced with  $\bar{\mathbf{y}}(\mathbf{k})$  in (1). The task of the equalizer is to recover the transmitted symbols,  $s(k-d)$ , based on the observation vector,  $\mathbf{y}(k)$ . Because of the AWGN, the observation vector is a random process having conditional Gaussian density functions centered at each of the desired channel states,  $\hat{\mathbf{y}}(\mathbf{k})$ . The determination of the value of  $s(k-d)$  becomes a decision problem. Bayes decision theory [23] provides the optimal solution to the general decision problems. It is applied here and the optimal decision function for Bayesian equalizer can be represented as follows, see [24,25]

$$f_B(\mathbf{y}(\mathbf{k})) = \sum_{i=1}^{n_s^{+1}} \exp(-\|\mathbf{y}(\mathbf{k}) - \mathbf{y}_i^{+1}\|^2 / 2\sigma_e^2) \quad (8)$$

$$- \sum_{i=1}^{n_s^{-1}} \exp(-\|\mathbf{y}(\mathbf{k}) - \mathbf{y}_i^{-1}\|^2 / 2\sigma_e^2)$$

$$\hat{s}(k-d) = \text{sgn}(f_B(\mathbf{y}(\mathbf{k}))) = \begin{cases} +1, & f_B(\mathbf{y}(\mathbf{k})) \geq 0 \\ -1, & f_B(\mathbf{y}(\mathbf{k})) < 0 \end{cases} \quad (9)$$

where  $\mathbf{y}_i^{+1}$  and  $\mathbf{y}_i^{-1}$  are the desired channel states belonging to sets  $\mathbf{Y}_{q,d}^{+1}$  and  $\mathbf{Y}_{q,d}^{-1}$ , respectively, and their number of elements in these sets are denoted by  $n_s^{+1}$  and  $n_s^{-1}$ . Furthermore  $\sigma_e^2$  is the noise variance. The optimal equalizer solution in (8) depends on the desired channel states. In other words, the solution of blind channel equalization

crucially depends on how to find the desired channel states,  $\mathbf{y}_i^{+1}$  and  $\mathbf{y}_i^{-1}$ , only from the observation vector  $\mathbf{y}(k)$ . In this study, the modified version of CFCM is investigated in search of the optimal output states of an unknown channel, and its desired channel states are configured with the searched channel output states. The construction of desired channel states by using the relation with channel output states will be explained in the next section. The optimal Bayesian decision probability in (8) is used to derive the fitness function of proposed CFCM, and also utilized as an equalizer, along with (9), for the reconstruction of the transmitted symbols.

### 3. DESIRED CHANNEL STATE CONSTRUCTED BY CHANNEL OUTPUT STATE

In the previous section, it has been observed that the knowledge of the desired channel states is essential for the evaluation of the optimal decision function in the Bayesian equalizer. The estimation of channel states requires the knowledge of the channel. However, under most circumstances, it may not be available. Additionally, the estimation of channels for nonlinear channels is very difficult in a direct manner. Thus, in the proposed algorithm, the estimation of desired channel states is accomplished by using the scalar channel states called "channel output states". The determination of these channel output states is simple and its computational complexity is independent from the equalizer order. Once the desired channel states have been constructed by using the estimated channel output states, finding the decision function of Bayesian equalizer is straightforward.

The following example is considered to illustrate the relationship of desired channel states and channel output states. If the channel order is taken as  $p=1$  with  $H(z) = 0.5 - z^{-1}$ , the equalizer order  $q$  is equal to 1, the time delay  $d$  is also set to 1, and

the nonlinear portion is described by  $D_1 = 1, D_2 = 0.1, D_3 = 0.05, D_4 = 0.0$  (see Fig. 1), then the eight different desired channel states ( $2^{p-l} = 8$ ) may be observed at the receiver in a noise-free case. Here, the output of the equalizer should be  $\hat{s}(k-1)$ , as shown in Table 1. From this table, it can be seen that the desired channel states  $[\hat{y}(k), \hat{y}(k-1)]$  are composed of the elements of the channel output states,  $\{a_1, a_2, a_3, a_4\}$ , where for this particular channel we have

$$a_1 = 1.89375, a_2 = -0.48125, a_3 = 0.53125 \text{ and } a_4 = -1.44375.$$

The length of dataset,  $\tilde{n}$ , is determined by the channel order,  $p$ , such as  $2^{p-l} = 4$ , which is independent from the equalizer order. In general, if  $q=1$  and  $d=1$ , the desired channel states for  $\mathbf{Y}_{l,i}^+$  and  $\mathbf{Y}_{l,i}^-$  are  $(a_1, a_1)$ ,  $(a_1, a_2)$ ,  $(a_3, a_1)$ ,  $(a_3, a_2)$ , and  $(a_2, a_3)$ ,  $(a_2, a_4)$ ,  $(a_4, a_3)$ ,  $(a_4, a_4)$ , respectively. A change in the decision delay only changes some of the positive states to negative states and equal number of negative states to positive states. For example, in case of  $d=0$ , the channel states,  $(a_1, a_1)$ ,  $(a_1, a_2)$ ,  $(a_2, a_3)$ ,  $(a_2, a_4)$ , belong to  $\mathbf{Y}_{l,i}^+$ , while  $(a_3, a_1)$ ,  $(a_3, a_2)$ ,  $(a_4, a_3)$ ,  $(a_4, a_4)$  belong to  $\mathbf{Y}_{l,i}^-$ . This relation is always valid for the channel that has a one-to-one mapping between the channel inputs and outputs [17]. Thus,

the desired channel states can be derived from the channel output states if the channel order,  $p$ , is assumed to be known, and the main problem of blind equalization can be changed to focus on the determination of the optimal channel output states from the received patterns.

It is known that the Bayesian likelihood ( $BL$ ), given by (10), is always maximized with respect to the desired channel states derived from the optimal channel output states [26].

$$BL = \prod_{k=0}^{L-1} \max(f_B^{+1}(k), f_B^{-1}(k)) \quad (10)$$

where  $f_B^{+1}(k) = \sum_{i=1}^{n_i^+} \exp(-\|y(k) - y_i^+\|^2 / 2\sigma_e^2)$ ,

$f_B^{-1}(k) = \sum_{i=1}^{n_i^-} \exp(-\|y(k) - y_i^-\|^2 / 2\sigma_e^2)$  and  $L$  is the length of the received sequences. Therefore, the  $BL$  is utilized as the fitness function ( $FF$ ) of the proposed algorithm to find the optimal channel output states. Being more specific, the fitness function is taken as the logarithm of the  $BL$ , that is

$$FF = \sum_{k=0}^{L-1} \log(\max(f_B^{+1}(k), f_B^{-1}(k))) \quad (11)$$

The determination of the maximum  $FF$  is not possible without knowledge of channel structure

Table 1. The relation between desired channel states and channel output states

Nonlinear channel with $D_1 = 1, D_2 = 0.1, D_3 = 0.05, D_4 = 0.0$ , and $d=1$				
Transmitted symbols	Desired channel states		Output of equalizer	
$s(k)s(k-1)s(k-2)$	$\hat{y}(k)$	$\hat{y}(k-1)$	By channel output states, $\{a_1, a_2, a_3, a_4\}$	$\hat{s}(k-1)$
1 1 1	1.89375	1.89375	$(a_1, a_1)$	1
1 1 -1	1.89375	-0.48125	$(a_1, a_2)$	1
-1 1 1	0.53125	1.89375	$(a_3, a_1)$	1
-1 1 -1	0.53125	-0.48125	$(a_3, a_2)$	1
1 -1 1	-0.48125	0.53125	$(a_2, a_3)$	-1
1 -1 -1	-0.48125	-1.44375	$(a_2, a_4)$	-1
-1 -1 1	-1.44375	0.53125	$(a_4, a_3)$	-1
-1 -1 -1	-1.44375	-1.44375	$(a_4, a_4)$	-1

[17]. In addition, from the relation between  $FF$  and channel output states shown in Fig. 2 (where several local maxima exist), it cannot be easily solved by conventional gradient-based methods. That is one of the reasons a clustering algorithm is considered as a way to find the maximum  $FF$ . In this approach, a new CFCM based algorithm is developed and evaluated in search of the optimal channel output states which maximize  $FF$ .

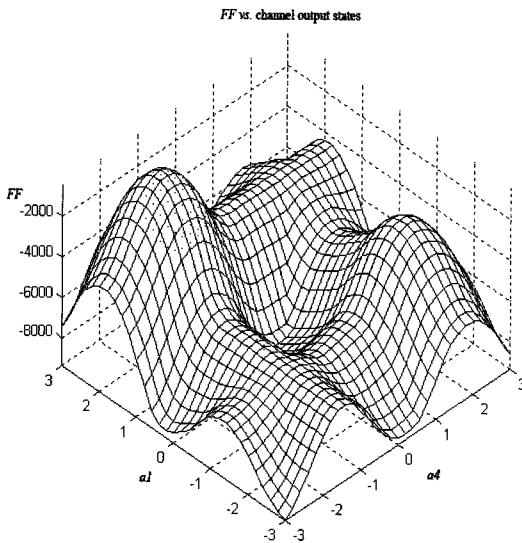


Fig. 2.  $FF$  vs. channel output states ( $a_1$  and  $a_4$ ) for the channel in Table 1 ( $a_2$  and  $a_3$  are set to their optimal values).

#### 4. CONDITIONAL FUZZY CLUSTERING FOR OPTIMAL CHANNEL STATES

Before introducing the proposed CFCM to be used to search the optimal channel states for blind channel equalization, the previously developed version of MFCM presented in [20] should be described at first. This is also justified by the fact that these two algorithms exhibit the same structure.

In comparison with the standard version of the Fuzzy C-Means (FCM) presented in [27], the MFCM comes with two additional stages. One of them concerns the construction stage of all possi-

ble data set of desired channel states with the estimated elements of channel output states. The other is the selection stage for the optimal desired channel states among them based on the Bayesian likelihood fitness function shown by (11). For the channel shown in Table 1, the four elements ( $2^{p-1} = 4$ ) of channel output states,  $\{a_1, a_2, a_3, a_4\}$ , are required to construct the optimal desired channel states. If the candidates for these elements,  $\{c_1, c_2, c_3, c_4\}$ , are randomly initialized, twelve ( $4!/2$ ) different possible data sets of desired channel states can be constructed by completing matching between  $\{c_1, c_2, c_3, c_4\}$  and  $\{a_1, a_2, a_3, a_4\}$ . To facilitate fast matching, the arrangements of  $\{c_1, c_2, c_3, c_4\}$  are saved as a certain mapping set  $C$  such that  $C(1)=1,2,3,4$ ,  $C(2)=1,2,4,3$ , ...,  $C(12)=3,2,1,4$  before the search process starts. For example, the notation  $C(2)=1,2,4,3$  means that the set of desired channel states is constructed with  $c_1$  for  $a_1$ ,  $c_2$  for  $a_2$ ,  $c_4$  for  $a_3$ , and  $c_3$  for  $a_4$  in Table 1. The desired channel states for this set are described as  $\mathbf{Y}_{i, C(2)}$  ( $\mathbf{Y}_{i, C(2)}^1$  and  $\mathbf{Y}_{i, C(2)}^{-1}$  for sets  $\mathbf{Y}_{i,1}^1$  and  $\mathbf{Y}_{i,1}^{-1}$ , respectively), and its fitness function in (11) is presented by  $FF(2)$ . As mentioned at the end of Section 3, if the set of desired channel states by a combination  $C(2)$  is optimal, it has a maximum value [26]. Thus at the next stage, a data set of desired channel states, which has a maximum Bayesian fitness value, is selected as shown below

$$[index\_j, \max\_FF] = \max(FF(1), FF(2), \dots, FF(12)) \tag{12}$$

This data set ( $\mathbf{Y}_{i, C(index\_j)}$ ), which is the set of desired channel states configured by the selected  $C(index\_j)$ , is utilized as a center set in the conventional FCM algorithm. Subsequently the partition matrix  $U$  is updated and a new center set,  $\mathbf{Y}_i$ , is sequentially derived with the use of this updated matrix  $U$ . These are expressed as

$$U_{ik}^{(m+1)} = \frac{1}{\sum_{l=1}^{n_s} \left( \frac{\|\mathbf{y}(\mathbf{k}) - \mathbf{y}_{i_{-C}(\text{index}_j)}^{(m)}\|}{\|\mathbf{y}(\mathbf{k}) - \mathbf{y}_{l_{-C}(\text{index}_j)}^{(m)}\|} \right)^2} \quad (13)$$

$$\mathbf{y}_i^{(m+1)} = \frac{\sum_{k=0}^{L-1} (U_{ik}^{(m+1)})^2 \mathbf{y}(\mathbf{k})}{\sum_{k=0}^{L-1} (U_{ik}^{(m+1)})^2} \quad (14)$$

where  $\mathbf{y}_i^{(m-1)}$  is the estimated center set at the  $(m+1)^{\text{th}}$  iteration and  $n_s$  is the total number of center vectors ( $n_s=8$  for the channel in Table 1). In the next epoch, the new four candidates for the elements of optimal output states are extracted from this new center set,  $\mathbf{y}_i^{(m-1)}$ , based on the relation presented in Table 1. The eight centers in the new center set,  $\mathbf{y}_i^{(m-1)}$ , are treated as the desired channel states constructed by the elements of channel output states,  $\{a_1, a_2, a_3, a_4\}$ , shown in Table 1, and thus each value of the new  $\{c_1, c_2, c_3, c_4\}$  is replaced with each one of the  $\{a_1, a_2, a_3, a_4\}$  in the new center set as in (15), respectively.

$$\mathbf{c}_r^{(m-1)} = \mathbf{a}_r \text{ in } \mathbf{y}_i^{(m-1)} \text{ where } r=1,2,3,4 \quad (15)$$

With this new set of candidates, the steps are repeated again until the Bayesian likelihood fitness function is not changed or the maximum number of iteration has been achieved. More details about MFCM can be found in [20].

The MFCM illustrated above showed the better performance than the existing hybrid GA algorithm in terms of speed and estimation accuracy, however, at low SNRs, the differences of accuracy for both algorithms are not significant [19,20]. As mentioned in Section 2, the received symbol,  $\mathbf{y}(\mathbf{k})$ , is a random process having conditional Gaussian density functions centered at each of the desired channel states because of the use of the AWGN. Thus, under low SNRs, the noise variance  $\sigma_e^2$  is high and the received patterns are quite scattered, which makes it difficult for the MFCM to estimate their correct centers. This weakness of MFCM to

significant level of noise can be overcome by applying the different weights to each of received patterns, which depend on their distances to the constructed clusters. To be more specific, the closer the received patterns to the clusters, the higher weight is attached and consequently more influential it becomes in the clustering process. This can be accomplished by using the clustering procedure of the CFCM. In the CFCM, the conditioning aspect of the clustering mechanism is introduced by taking into consideration the conditioning variable assuming values,  $f_1, f_2, \dots, f_k$  on the corresponding patterns [21]. Here  $f_k$  taking values in the unit interval describes a level of involvement of received symbol,  $\mathbf{y}(\mathbf{k})$ . For example, if  $f_i = 0$ , the  $i^{\text{th}}$  received pattern is regarded as meaningless in the clustering procedure and the calculations of the resulting prototypes are not affected by this element. Subsequently, the calculations of the partition matrix  $U$  do not take this into consideration. On the other hand, the pattern for which  $f_i = 1$  contributes to the clustering process to the highest extent. The membership degree is described as follows

$$U_{ik}^{(m+1)} = \frac{f_k}{\sum_{l=1}^{n_s} \left( \frac{\|\mathbf{y}(\mathbf{k}) - \mathbf{y}_{i_{-C}(\text{index}_j)}^{(m)}\|}{\|\mathbf{y}(\mathbf{k}) - \mathbf{y}_{l_{-C}(\text{index}_j)}^{(m)}\|} \right)^2} \quad (16)$$

For this application to search the optimal channel states from the noise-corrupted received patterns, the new conditional constraint  $f_k$  in (16) should contain the distance information of each of received patterns, and it has a high value if the corresponding pattern is closely located at the estimated center. It is known that the Bayesian likelihood ( $BL$ ) by (10) is always maximized with respect to the optimal desired channel states and utilized as the fitness function of proposed CFCM by (11). In addition, for the calculation of  $BL$ , if a received pattern is located near the optimal desired channel states,  $\mathbf{y}_i^{-1}$  or  $\mathbf{y}_i^1$ , this pattern produces

a higher value of  $f_B^{-1}(k)$  or  $f_B^{+1}(k)$  in (10), and the  $BL$  becomes to be larger. Therefore each component of  $BL$  for the received patterns is utilized as the conditional constraint  $f_k$  (after normalization). The computational details are described as follows

$$nf_k = \max(f_B^{+1}(k), f_B^{-1}(k)) \quad (17)$$

$$f_k = nf_k / \max(nf_k) \quad (18)$$

while its effectiveness is demonstrated in Fig. 3.

As shown in Fig. 3, the values of the conditional constraint for noisy patterns are relatively very

low (indicated by the bright color in Fig. 3(c)). On the other hand, in the proposed CFCM, the received patterns located near the optimal channel states are more weighted by the conditional constraint  $f_k$  (close to black color in Fig.3(c)) and generated a higher contribution to the clustering procedure. The resulting estimation accuracy is increased even with low SNRs as it will be shown in the next section. Thus in the proposed search algorithm, the partition matrix  $U$  is updated by (16) instead of (13) in MFCM and the conditional constraint by (17) and (18) is applied. It is summarized

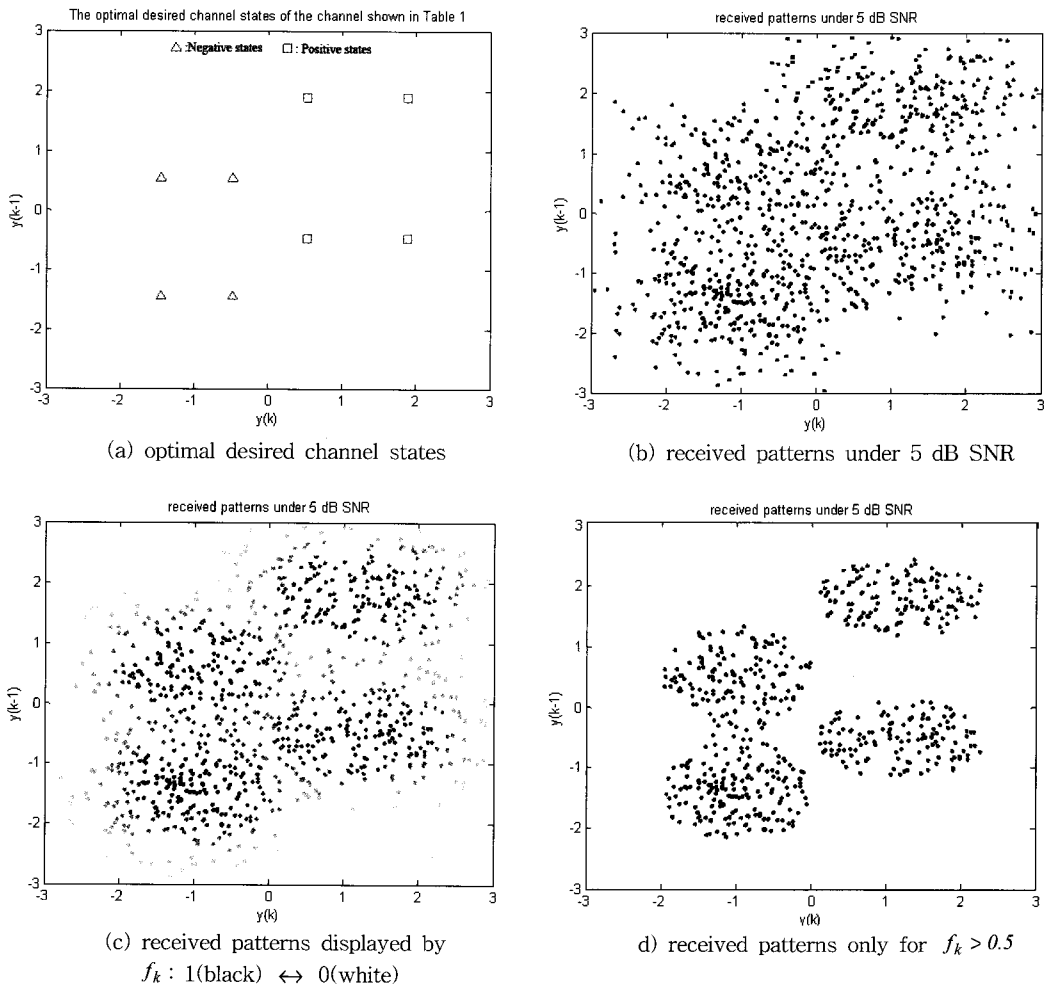


Fig. 3. The optimal desired states for the channel shown in Table 1(a), the received patterns under 5 dB SNR(b), and patterns with their conditional constraint  $f_k$  after 10 epochs of proposed CFCM (c)(d).



in the following pseudo-code.

*begin*

save arrangements of candidates,  $\{c_1, c_2, c_3, c_4\}$ ,  
to  $C$

randomly initialize the candidates,  $\{c_1, c_2, c_3, c_4\}$

while (new fitness function - old fitness  
function) < threshold value

for  $j=1$  to  $C$  size

map the arrangement of candidates,  $C[j]$ ,  
to  $\{a_1, a_2, a_3, a_4\}$

construct a set of desired channel states  
based on the structure shown in table 1

calculate its fitness function ( $FF[j]$ )

by eq. (11)

*end*

find a data set which has a maximum  $FF$   
in  $j=1..C$  size : eq. (12)

find the conditional constraint  $f_k$  for the  
selected data set:: eqs. (17) & (18)

update the membership matrix  $U$  by the data  
set utilized as a center set : eq. (16)

derive a new center set by the  $U$ : eq. (14)

extract the candidates,  $\{c_1, c_2, c_3, c_4\}$ , from the  
new center set based on the structure shown  
in table 1 and  $C(1)$ : eq. (15)

*end*

*end*

In the proposed search algorithm, all possible sets of desired channel states are constructed with the candidates by using the structure shown in Table 1 and a data set which exhibits a maximum fitness value is always selected. Therefore, the set of desired channel states produced by the proposed CFCM is always close to the optimal set, and its first half presents the desired channel states for  $\mathbf{Y}_{i,l}^{-1}$  and the rest presents for  $\mathbf{Y}_{i,l}^1$ , or reversely. In addition, as the fast searching of MFCM in [19] and [20], the proposed CFCM does not need to check all of the possible arrangements,  $C(1), C(2), \dots, C(12)$ , to find the data set which has a maximum  $FF$  after the first couple of *while*-loop. It is because the new candidates,  $\{c_1, c_2, c_3, c_4\}$ , are extracted by using the arrangement  $C(1)$  as shown in (15) at the end of *while*-loop and thus the set of desired channel states constructed by  $C(1)$  always has the maximum  $FF$  after couple of clustering epochs. Therefore, in our experiments, for the fast searching of proposed CFCM, the *for*-loop in the pseudo-code is skipped if the selected *index\_j* has not been changed during the last 5 epochs. From this moment, the set of desired channel states only by  $C(1)$  is constructed with the new candidates and utilized for further process. The flowchart of the proposed CFCM with fast searching is illustrated in Fig. 4.

Fig. 4. Flowchart of the proposed CFCM with fast searching.

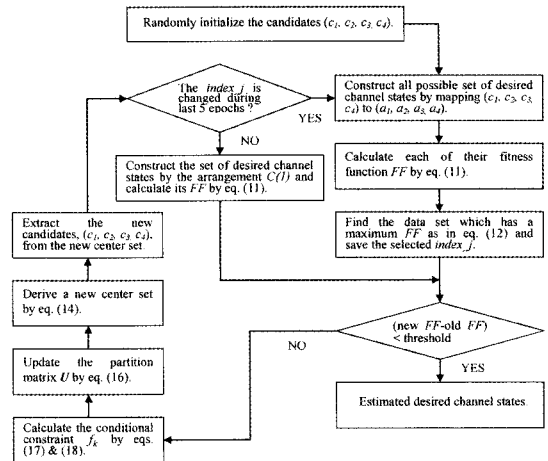


Fig. 4. Flowchart of the proposed CFCM with fast searching.

## 5. EXPERIMENTAL RESULTS AND PERFORMANCE EVALUATION

In this section, the proposed CFCM is compared and evaluated vis-a-vis the previously developed algorithm which also estimates the optimal channel states of unknown channel to solve the problem of blind equalization. As mentioned in the introduction, the MFCM presented in [19] and [20] showed better performance than the GASA [18]

and the simplex GA [17] in terms of speed and accuracy. To demonstrate the effectiveness of the method, blind equalizations realized with the use of the MFCM and the proposed CFCM are considered in the experiments. Four channels including a linear model are discussed. Channel 1 (shown in Table 1) and 2 stand for two different nonlinear models with the channel order  $p=1$ , and channel 3 concerns a linear model (here the nonlinear terms in channel 1 and 2 have been removed). These three channels were discussed in [17], [18] and [20]. The last one (channel 4) is for a nonlinear model with the channel order  $p=2$  as presented in [19] and [28]. The detailed description of the channels is presented below.

Channel 2 (nonlinear):  $H(z) = 0.5 - 1.0z^{-1}$ ,  
 $D_1 = 1, D_2 = 0, D_3 = -0.9, D_4 = 0$ , and  $d=1$

Channel 3 (linear):  $H(z) = 0.5 + 1.0z^{-1}$ ,  
 $D_1 = 1, D_2 = 0, D_3 = 0, D_4 = 0$ , and  $d=1$

Channel 4 (nonlinear with  $p=2$ ):  
 $H(z) = 0.3482 - 0.8704z^{-1} - 0.3482z^{-2}$ ,  
 $D_1 = 1, D_2 = 0.2, D_3 = 0, D_4 = 0$ , and  $d=1$

For channel 1, 2 and 3, the channel order  $p$ , the equalizer order  $q$ , and the time delay  $d$  are 1, 1, and 1, respectively. Thus, the output of the equalizer should be  $\hat{s}(k-1)$ , and the eight desired channel states for  $\mathbf{Y}_{1,1}^{-1}$  and  $\mathbf{Y}_{1,1}^{-1}$  composed of the four channel output states ( $2^{p-1} = 4, a_1, a_2, a_3, a_4$ ) as shown in Table 1 will be observed at the receiver in a noise-free case. In channel 2 for a nonlinear model,  $a_1, a_2, a_3, a_4$  are 1.5375, 0.3875, -0.3875 and -1.5375, respectively, and for channel 1, they are illustrated in Table 1. In channel 3 for a linear model, where nonlinear terms of channel,  $D_2, D_3$ , and  $D_4$ , are equal to zeros,  $a_1, a_2, a_3, a_4$  are 1.5, -0.5, 0.5 and -1.5, respectively. In channel 4, the channel order  $p$  is 2 and thus there exist the sixteen desired channel states ( $2^{p-1} = 16$ ) composed of the eight channel output states ( $2^{p-1} = 8, a_1, a_2, a_3, \dots, a_8$ ).

The desired channel states,  $(a_1, a_1), (a_1, a_2), (a_2, a_3), (a_2, a_4), (a_5, a_1), (a_5, a_2), (a_6, a_3), (a_6, a_4)$ , belong to  $\mathbf{Y}_{1,1}^{-1}$ , and  $(a_3, a_5), (a_3, a_6), (a_4, a_7), (a_4, a_8), (a_7, a_5), (a_7, a_6), (a_8, a_7), (a_8, a_8)$  belong to  $\mathbf{Y}_{1,1}^{-1}$ , where  $a_1, a_2, a_3, \dots, a_8$  are 2.0578, 1.0219, -0.1679, -0.7189, 1.0219, 0.1801, -0.7189 and -1.0758, respectively. These sixteen desired channel states for channel 4 are summarized in [19]. The coefficients of this channel are symmetric, which means the channel 4 has a linear phase characteristic. In this case, the number of observed channel output states becomes six instead of eight because  $a_2$  and  $a_5$ , and  $a_4$  and  $a_7$  always have the same values, 1.0219 and -0.7189 for this channel, respectively. However, in our simulations, each of all eight channel output states,  $a_1, a_2, a_3, \dots, a_8$  are searched for and evaluated for more general cases.

In the experiments, 10 independent simulations for each of three channels with five different noise levels (SNR=0, 2.5, 5, 7.5, and 10dB) were performed with 1,000 randomly generated transmitted symbols ( $L=1000$ ). Afterwards, the obtained results were averaged. The MFCM and the proposed CFCM have been implemented in a batch mode to facilitate comparative analysis. In addition, both algorithms are evaluated with the use of same parameters shown in Table 2, and these are fixed for all experiments. The choice of the specific parameter values is not critical to the performance of MFCM as well as the proposed CFCM. The fitness function described by (11) is utilized in both algorithms. With this regard, the normalized root mean squared errors (NRMSE) is determined in the form

$$\text{NRMSE} = \frac{1}{\|\mathbf{a}\|} \sqrt{\frac{1}{N} \sum_{i=1}^N \|\mathbf{a} - \hat{\mathbf{a}}_i\|^2} \quad (19)$$

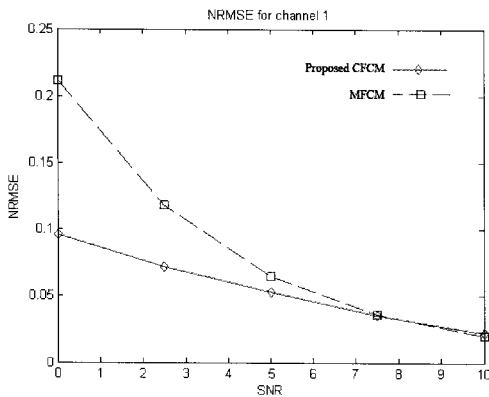
where  $\mathbf{a}$  is the data set of optimal channel output states,  $\hat{\mathbf{a}}_i$  is the data set of estimated channel output states in the  $i^{\text{th}}$  simulation, and  $N$  is the total number of independent simulations ( $N=10$ ).

Table 2. Parameters used in both MFCM and CFCM algorithms

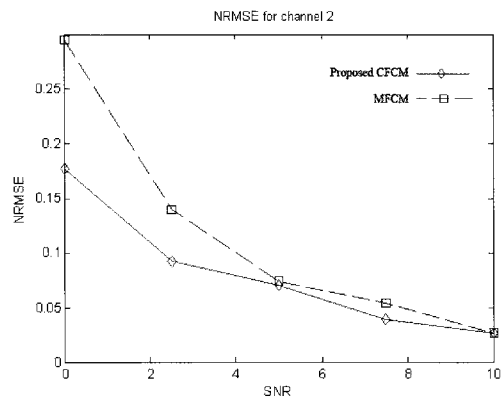
Maximum number of iteration	100
Threshold for $FF$ variation	$10^{-3}$
Exponent for the partition matrix $U$	2
Random initial channel output states	$[-0.5 \ 0.5]$

The values of NRMSEs after 10 independent simulations for each of four channels are averaged and illustrated in Fig. 5. The proposed CFCM comes with lower NRMSE for all four channels, and the performance differences are more severe in higher noise levels. As shown in Fig. 6, with a low SNR, the received patterns are widely distributed and the values of conditional constraint for each of them are quite different depending on their distances to the estimated centers. The patterns,

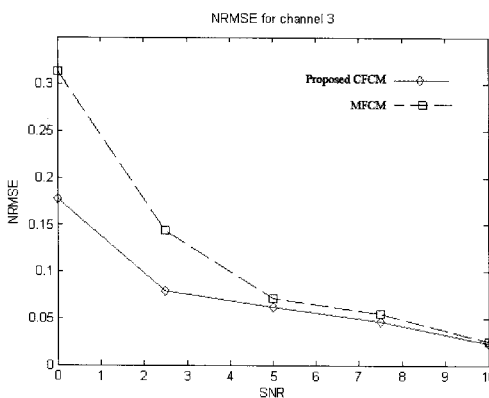
which are more distant from their centers, have lower conditional constraints and are less weighted on the clustering procedure. It helps to increase the search accuracy. However, with a high SNR, most of the received patterns are densely located near the desired channel states and the patterns even with low conditional constraints are not far from their centers as much as they are in the case of low values of the SNR. It means that the conditional constraint  $f_k$  does not much affect the clustering procedure. This is why the proposed CFCM is highly effective to find the optimal channel states when the received patterns are heavily corrupted by noise. A sample of 1,000 received symbols under 0dB SNR for channel 4 and its desired channel states constructed from the estimated channel output states by the MFCM and the proposed CFCM



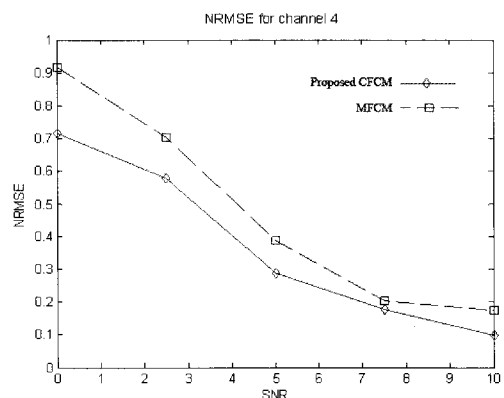
(a) for channel 1



(b) for channel 2



(c) for channel 3



(d) for channel 4

Fig. 5. NRMSE of the MFCM and the proposed CFCM.

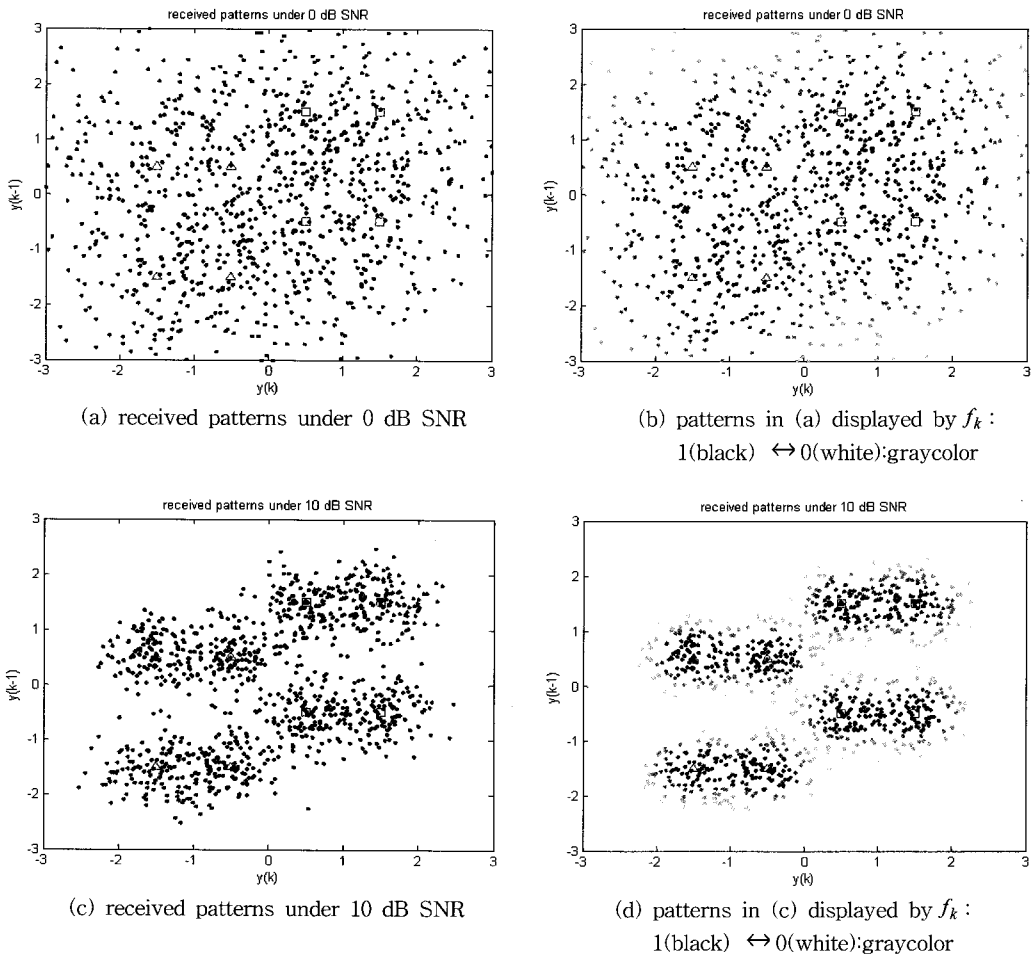


Fig. 6. Received patterns displayed by the conditional constraint  $f_k$  under high and low SNRs for channel 3 (including optimal positive( $\square$ ) and negative( $\triangle$ ) states).

are illustrated in Fig. 7.

In addition, the search times of the two algorithms are included in Table 3. Both algorithms use their fast searching procedures, which means the *for*-loop in the pseudo-code is skipped if the selected *index<sub>j</sub>* has not been changed during the last 5 epochs. The overall search time by the proposed CFCM is slightly slower because of calculation time for the conditional constraint. However, their difference is not significant where the proposed CFCM provides much better performance in terms of NRMSE. Additionally, some of search times for the CFCM (in channel 4), especially with low SNRs, are faster, and it is caused by the conditional

constraint which reduces the number of convergence epochs in heavy noise circumstances.

Finally, the bit error rates (BER) when using the Bayesian equalizer is investigated and shown in Table 4. It becomes apparent that the BER with the estimated channel output states realized by the proposed CFCM is close enough to the one with the optimal output states for all four channels. However, especially for low SNRs, the performance of the proposed CFCM does not dominate in terms of BER as much as it does in terms of NRMSE. It is resulted from the fact that the Bayesian decision function shown in (8) is affected by heavy noise (high value of noise variance  $\sigma_e^2$ )

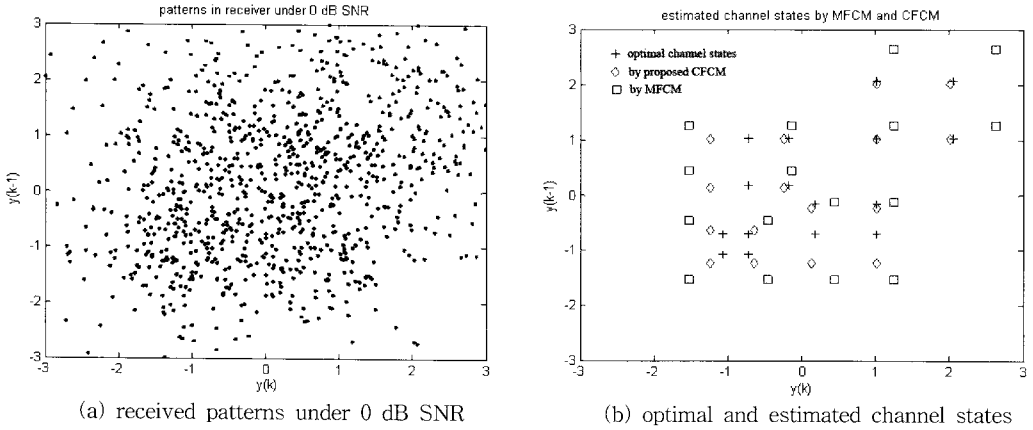


Fig. 7. A sample of received symbols under 0dB SNR for channel 4 and its sixteen desired channel states estimated by the MFCM and the proposed CFCM.

even though the desired channel states are estimated well with high accuracy by the proposed CFCM. For further improvement of BER, the deci-

sion function (or mechanism) of the equalizer should be investigated in near future.

Table 3. The averaged search time ( in sec) for MFCM and proposed CFCM (Matlab 7.0 run on Intel core 2)

Channel	SNR	MFCM	Proposed CFCM
Channel 1	0.0 dB	0.0797	0.0875
	2.5 dB	0.0734	0.0813
	5.0 dB	0.0734	0.0734
	7.5 dB	0.0688	0.0719
	10 dB	0.0594	0.0719
Channel 2	0.0 dB	0.0828	0.0859
	2.5 dB	0.0766	0.0766
	5.0 dB	0.0609	0.0734
	7.5 dB	0.0641	0.0641
	10 dB	0.0594	0.0641
Channel 3	0.0 dB	0.0859	0.0922
	2.5 dB	0.0719	0.0875
	5.0 dB	0.0688	0.0734
	7.5 dB	0.0625	0.0719
	10 dB	0.0609	0.0656
Channel 4	0.0 dB	4.8031	4.2969
	2.5 dB	9.7406	7.9047
	5.0 dB	3.7156	4.7719
	7.5 dB	4.7141	3.0375
	10 dB	3.0484	3.2531

Table 4. Averaged BER(%) (no. of errors/no. of transmitted symbols)

Channel	SNR	with optimal states	MFCM	Proposed CFCM
Channel 1	0.0 dB	17.06	17.46	17.39
	2.5 dB	12.51	12.91	12.77
	5.0 dB	8.27	8.51	8.35
	7.5 dB	4.02	4.14	4.11
	10 dB	1.43	1.44	1.44
Channel 2	0.0 dB	19.12	20.35	20.21
	2.5 dB	15.18	15.47	15.39
	5.0 dB	11.08	11.34	11.11
	7.5 dB	6.24	6.30	6.27
	10 dB	2.76	2.81	2.83
Channel 3	0.0 dB	18.77	19.51	19.38
	2.5 dB	13.13	13.31	13.13
	5.0 dB	8.99	9.09	8.92
	7.5 dB	4.53	4.53	4.46
	10 dB	1.45	1.49	1.48
Channel 4	0.0 dB	21.98	25.90	23.33
	2.5 dB	17.36	23.91	21.33
	5.0 dB	11.49	14.65	12.56
	7.5 dB	8.31	9.72	8.86
	10 dB	4.93	5.90	4.94

## 6. CONCLUSIONS

A modification of the CFCM aimed at the estimation of desired channel states of an unknown digital communication channel is provided for blind equalization, and successfully evaluated with both of linear and nonlinear channels. By taking this kind of approach, the highly demanding modeling task of an unknown channel becomes unnecessary as the construction of the desired channel states is accomplished directly on the basis of the estimated channel output states. It has been shown that the proposed CFCM offers better performance in comparison to the solution provided by the previously developed algorithm (MFCM). In particular, because of the conditional constraint, the proposed CFCM can estimate the channel output states with substantial accuracy and speed even when the received symbols are significantly corrupted by heavy noise. Therefore, the CFCM can possibly constitute a search algorithm of optimal channel states for the various problems of blind channel equalization. For future works, this algorithm could be evaluated with wider range of communication environments including higher order channels. Furthermore, as mentioned at the end of last section, the decision function of Bayesian equalizer could be investigated for the improvement of BER in low SNRs. The research for more powerful search algorithms (easier and faster to compute or implement and more robust to heavy noise) will also be continued.

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