A Synthesis on Essential Issues in the Field of Mathematics Education

Kim, Dong-Joong1) · Cho, Jeong-i2)

Acknowledging mathematics education as a research field and its relation to different domains such as mathematics, educational sciences, psychology, sociology, and history, two paradigmatic issues of theoretical research and classroom practice are focused on to synthesize the different domains in mathematics education. Six sub-categories in the field of mathematics education are proposed to have a better understanding of their role and interdependence.

Key Words : Mathematics education, synthesis, mathematics, learning, teaching, curriculum, research, policy

I. INTRODUCTION

Mathematics education has been emphasized as a research field and its relation to different domains such as mathematics, educational sciences, psychology, sociology, and history. In addition to the complex systematic nature, each of these domains has been developed with its own methodologies. Thus some researchers focused on the relationships between domains in order to have a better understanding of the role of mathematics education (see Bartolini Bussi and Bazzini, 2003). However, there is limited literature attempting to synthesize different domains embedded in mathematics education. Although the related domains are essential for the field of mathematics education to function in an optimal way, its specificity relies on the core and thus the core must be the central component. Subsequently, we will focus on two paradigmatic issues with consideration of the interaction of different domains: Theoretical research and classroom practice. Therefore, the purpose of this study is to synthesize different domains in the field of mathematics education research in order to broaden our views of mathematics.

1) Korea National University of Education (djkim@knue.ac.kr)
2) Indiana University-Purdue University at Fort Wayne (choj@ipfw.edu)
education and use combined domains to intensify the outcomes of research and practice.

The synthesis is divided into six sections: mathematics, learning, teaching and teacher education, policy, curriculum, and research. The six categories were proposed, because we believe they are necessary pieces in all mathematics education research. With their connectedness and disconnectedness, one can see strengths and weaknesses existing in any research in mathematics education. One thing to note is that the purpose of this study is not to explain one domain in detail, but to show how to synthesize different domains for research and practice in mathematics education.

In what follows, the first section describes how two different lenses to view mathematics are deeply related to contemplation on the nature of learning mathematics and its practice in mathematics classrooms. The second section addresses the exploration of two metaphors in learning mathematics. The third section illustrates issues in an approach to teaching students mathematics. In the fourth section, we will discuss a political view of mathematics education. In the fifth section, we will raise curriculum-related issues in mathematics education. In the sixth section, we will depict issues of research on mathematics education. Finally, we will summarize the six sub-categories in mathematics education on the basis of their complexity and considerable interdependence.

II. MATHEMATICS

One's view of mathematics will weigh heavily in how one thinks about and understands learning and teaching. We will address two principal views on the nature of mathematics: absolutism and relativism. In absolutism, mathematics is absolute truths and thus is viewed as a collection of both complete concepts and their operational principles. In this view, objectivity is central, rather than a view of mathematics as depending on contexts. The best known example of an absolutist view of mathematics is Platonism. Unlike a Platonic mathematics, in relativism, mathematical ideas are derived from contexts such as persons, society, and culture (Buerk, 1982). Ethnomathematics is an example of relativism because mathematical ideas in it have social and cultural context (Ascher & D'Ambrosio, 1994). In relativism, the mathematics that human beings know is a human mind-based mathematics (Hersh, 1997). Thus, mathematics is a product of the human mind and our social and cultural history. The two different views on the nature of mathematics are related to approaches to mathematical learning.

III. LEARNING

The exploration of how to conceptualize mathematical learning is older than the field of mathematics education itself. Education research on how students learn mathematics
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can be categorized with two metaphors: acquisition and participation (Sfard, 1998). While the acquisition metaphor stresses the nature of the mechanisms through which mathematical concepts are integrated into the learner's cognitive structures, the participation metaphor focuses on the process of becoming a participant in a certain community as an activity. One thing to note is that the acquisition metaphor is implicitly grounded in absolutism in which the nature of mathematics is context-independent.

In cognitive theories influenced by the work of Piaget, mathematical learning has been considered as a process of active cognitive acquisition. The acquisitionists have explained that the idea of cognitive processes is based on uniform and orderly forms of rule-following (Harré & Gillett, 1995). Most acquisitionist researchers have attempted to understand the mechanisms by examining three different aspects: semantic categorization of misconceptions or cognitive obstacles (see Fischbein, 2001; Tall & Vinner, 1981; Sierpinska, 1987), hierarchical structures of mental categories (see Brownell, 1945; Hiebert & Lefèvre, 1986; Michener, 1978; Skemp, 1987), and mechanisms of cognitive theory (see Duffin & Simpson, 2000; Gray et al., 1999; Weller et al., 2004). In the same vein, the summary of Adding it up (National Research Council, 2001) argues that mathematical proficiency, which has five components (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition), is necessary for anyone to learn mathematics successfully.

These acquisitionist tenets underestimate not only the inherently social nature of student thinking, but also the role of discourse and communication in learning and in other intellectual activities. Neither have they led to satisfactory solutions to account for inter-personal and cross-situational differences and the source of human development (Sfard, 2006). To highlight the social nature of student learning in the participation metaphor, learning should be perceived as "a process of enculturation into a community of practice" from a socio-cultural perspective, instead of "a process of active cognitive reorganization" (Cobb, 1994). In this process, not only conversation but also process of a transformation (from an inter-personal communication to an intra-personal) is of principal importance in learning mathematics as an activity (Vygotsky, 1986). From a socio-cultural perspective, relativism as a view on the nature of mathematics is a noticeable feature.

In a rapidly changing and increasingly technological society, the use of technology should not be disregarded in issues of learning mathematics. For instance, the "extent to which the technology being used highlights the mathematics that is being studied rather than obscures it" is called transparency (Heid, 1997, p.7). Rather than the issue of transparency, there are many learning issues related to technology such as changes in the nature of mathematics (Hancock, 1995) as well as in the roles in a learning context (Goos et al., 2000).

Another issue is student attitudes toward learning mathematics. Ma (1999) has pointed out that basic attitudes of a subject may be even more penetrating than its basic

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principles. Walmsley (2000) has also stated that mathematical attitude becomes a problem if it begins to determine success or failure despite the ability of a student. In addition, students' disposition toward mathematics is a major factor in determining their educational success (National Research Council, 2001). Therefore, considering the ways to improve the strand of productive disposition for mathematical proficiency is of great importance in issues of learning mathematics. Without students' productive disposition, they may not learn mathematics successfully.

IV. TEACHING AND TEACHER EDUCATION

Mathematical learning cannot be fully understood without contemplation of the contribution made by a teacher, students, mathematical content, and their interaction within environments. The approach to teaching students mathematics should be developed and characterized on the basis of solid theories of mathematical learning. However, there is a lack of theories that link teaching and learning.

Knowing a mathematical concept is different from understanding the processes necessary to teach it. Teachers' knowledge and capacity are necessary components of what teachers need to teach mathematics. Most previous research on teaching for the last few decades has focused on teachers' beliefs and knowledge for teaching mathematics (Calderhead, 1996). As for mathematics teachers' knowledge, four components can be proposed: mathematical knowledge, knowledge of how students know mathematics, knowledge for teaching mathematics, and teacher's beliefs. For teachers' capacity, they need practices which enable them to use this knowledge in their teaching.

Previous research on teachers' knowledge for teaching mathematics is based on a cognitive perspective. The following three examples are representative: pedagogical content knowledge (PCK) in which pedagogy and content are interwoven (Shulman, 1987); knowledge package and its key pieces (Ma, 1999); and subject matter knowledge for teaching as decompression of constituent elements of contents (Ball & Bass, 2000). Regardless of whether teachers' specific knowledge for teaching mathematics is referred to as PCK or by other terms, teacher's knowledge is an important component for teaching mathematics. The reason can be found in connections between teachers' knowledge and effectiveness in mathematics teaching (Borko et al., 1992; Thompson & Thompson, 1994). Both pedagogy at the expense of content and content at the expense of pedagogy are equally ineffective.

Other researchers have asserted that the beliefs of teachers are important for teaching mathematics by emphasizing the relationship between teachers' belief structures (of mathematics, learning, and teaching) and teaching (Borko et al., 1992; Pajares, 1992; Thomson, 1984). However, although these are important research findings in teacher's knowledge and beliefs, not only what works, but also how things work should be more emphasized because the same teaching method might facilitate different kinds of
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learning.

Assessing students' learning is another important issue in the work of teaching. How to assess students' learning is an important component of knowledge both of how students know mathematics and for teaching mathematics. As the NCTM Standards (NCTM, 2000) state, assessing students' learning as a complex task should support the learning of important mathematics and furnish useful information to both teachers and students. The use of technology is another important issue in teaching. To make appropriate changes in both student roles and the nature of mathematics, teachers need to consider the interactions between the students and the technology, and also the interactions between the curriculum and the technology (Guin & Trouche, 1999). Technological pedagogical content knowledge (TPCK) is probably one of the areas in need of research attention in learning mathematics through technology.

V. POLICY

Mathematics education is influenced by what is the view of mathematics, how students learn, and how teachers teach. Moreover, a political lens to view mathematics education is another important factor which decides what is taught, how mathematics curricula are selected, and what is assessed. For instance, the standards are not decided only by research because there are values about social expectations which influence the standards (Hiebert, 1999). The alignment between the standards and educational implementations is another important issue in educational policies. When the standards are implemented in school practices, there are multiple ways of applying the standards according to contextually based interpretations (Ferrini-Mundy & Martin, 2003).

To systematically reform school, changing the complex system of the school may be needed rather than a single factor. Thus, how local and national policies can implement systematic changes in actual teaching practice is another important issue in the politics of education (Schorr, Firestone, & Monfils, 2003; Spillane & Zeuli, 1999). Specifically, in order to design a systematic structure which has an impact on mathematics classrooms, the role of each local government (Smith & O'Day, 1990) as well as the federal government (Lappan & Wanko, 2003) is an important and complex issue.

One big question is how to ensure that research has an impact on practice through educational policies. For instance, the main goal of the federal No Child Left Behind (NCLB) legislation is to improve the academic achievement of every student through highly-qualified teaching. To achieve this objective, two major components of this policy are assessment systems and accountability measures. The NCLB Act requires states to develop annual assessments. Based on the assessment results, accountability requirements interpret whether districts and schools meet the adequate yearly progress (AYP) objective established by each state (U.S. Department of Education, 2002). Under the guidance of the NCLB, the federal government aims at improving schools in the
United States. One important lesson in the California mathematics reform story (Wilson, 2003) is to always consider a possible complexity that the politics of education are too complicated to be moved in the direction in which policy makers intend to.

VI. CURRICULUM

Mathematics curricula are based on learning and teaching perspectives, each school's policy, trends in mathematics, etc. Thus, philosophical and political beliefs in education underlie curricular selection. Many theories about learning that guide design of curriculum, such as behaviorism and cognitivism, have come out and faded out (Howson, Keitel, & Kilpatrick, 1981). In these processes, philosophical and political convictions are interwoven. For instance, the "back-to-basics" movement of the 1970s did not serve philosophically as an effective model for curricular revision in the 1980s because of the political attentions on the crisis in education (American Association for the Advancement of Science, 1984; National Commission on Excellence in Education, 1983).

The intended curriculum is the planned curriculum by state education department or school district, whereas the enacted curriculum is what students actually learn (Burkhardt, Fraser, & Ridgway, 1988). In the process of developing the intended curriculum, one important issue is how to incorporate rigorous mathematical ideas to school mathematics-"the tension between the needs of the child and the needs of mathematics" (Sfard, 2003). Another issue to consider is breadth and depth of mathematics curriculum-"a curriculum with many broad topics versus one with a few deep topics (Mesa & Kilpatrick, 1998). Another issue is how to select one perspective among several perspectives on mathematical content (e.g., many different perspectives in the introduction of school algebra: generalization, problem solving, modeling, functions, and language and representation (Bednarz, Kieran, & Lee, 1996).

VII. RESEARCH

Research in education is different from research in other fields by virtue of several aspects. First, educational research is characterized by a need to respond to practical issues arising from school and society (Labaree, 1998). Second, educational research produces a different knowledge. Educational research would not produce the theoretical objective knowledge but the practical common-sense knowledge (Carr, 1995). Finally, it's difficult to implement findings due to the embedded perspectives of educational phenomena in social life which results in the numerous interactions among contexts (Berliner, 2002).

Throughout the 1980s, growing familiarity with qualitative research fostered a variety of new relationships between education research and practice (Lagemann, 2000). In order
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to understand the differences between qualitative and quantitative research, it is important to recognize how these two approaches to research influence the types of research questions asked as well as the methods of investigation employed. Based on a qualitative process of inquiry, the goal is to take a holistic picture of phenomena of interest in order to gain insights in contexts. Case studies, ethnography, phenomenology, and grounded theory are representatives of qualitative methods (Strauss & Corbin, 1998; Wolcott, 1990). In contrast, quantitative research is an inquiry into systematic connections between factors in educational systems based on statistical techniques. Experiments, quasi-experiments, and surveys are general types of quantitative methods (Campbell & Stanley, 1963).

One of the validities in educational research can be the fit between research questions, data collection, and analysis techniques. In *The Sources of a Science of Education* (Dewey, 1929), Dewey insisted that theory and practice should be integral to one another and worried that research had developed at too great a separation from practice. We need to find many links between research and practice for a better validity in educational research.

VIII. SUMMARY

We tried to produce a synthesis in order to gather the ideas derived from previous research in mathematics education in terms of relevant psychological issues in learning and teaching mathematics. Most sub-categories (e.g., mathematics, learning, and teaching subcategories) have a thread of connection with each other, whereas one sub-category (e.g., policy subcategory) seems to float alone in a labyrinthine jungle. It is important to explore and witness this labyrinthine jungle (so called mathematics education) and the interdependent tendency of the six categories in a bigger picture. Through this holistic consideration, we, as mathematics education researchers, can not only see the tip of an iceberg (as what we can see in our research), but also be fully aware of its hidden part (as what we cannot see through our research methodology).

Every research has its own strength and weak point. Through the above holistic analysis of the six categories and their interdependent tendency in mathematics education, we believe that one can more clearly see connectedness as well as disconnectedness among the six categories and gain a better understanding of strength and weak point existing in one’s own research. If we, as mathematics educators, can see strength and weak point in our research, we would be capable of balancing the "unbalanceable" in the field of mathematics education. For instance, with an understanding of the connectedness between the policy domain and the learning and teaching domains, through a political lens, one can perceive the urgent need for an additional approach to confront learning and teaching issues in mathematics education. This new lens may help one obtain different insights in one's learning and teaching.
research and promote more holistic and balanced perspectives on mathematics education research.

However, it is not easy and there is no short-cut to see the whole jungle because of too many trees, labyrinths, and our misconceptions and biases. In addition to such complexities, different domains in the field of mathematics education play dynamically different roles and affect each other depending on their contexts as well as times. In spite of such difficulties, the analysis on how different domains are interrelated can help mathematics educators to dig further into the complex field of mathematics education, pursue a practical connection between research and practice, and help students who are struggling in learning mathematics.

References


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수학교육분야에서 중요한 이슈들에 대한 통합

김동중 3) · 조정일 4)

초 록

수학교육이 연구의 한 분야이며 수학, 교육과학, 심리학, 사회학, 역사학동과 같은 다른 영역들과의 연계성을 인정하면서 다른 영역들의 통합을 위해 이론적 연구와 수업실습이란 두 가지의 실용적 이슈들에 초점을 맞춘다. 서로 다른 영역들의 역할과 의존성을 더 잘 이해하기 위해서 수학교육분야에서 6가지의 중요한 영역들을 제기한다.

주요 용어 : 수학교육, 통합, 수학, 학습, 교수, 교육과정, 연구, 정치

3) 한국교원대학교 (djkim@kme.ac.kr), 교신저자
4) 인디애나-퍼듀대학 포트웨인 (choj@ipfw.edu)