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Bayesian estimation in the generalized half logistic distribution under progressively type-II censoring

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Abstract

The half logistic distribution has been used intensively in reliability and survival analysis especially when the data is censored. In this paper, we provide Bayesian estimation of the shape parameter and reliability function in the generalized half logistic distribution based on progressively Type-II censored data under various loss functions. We here consider conjugate prior and noninformative prior and corresponding posterior distributions are obtained. As an illustration, we examine the validity of our estimation using real data and simulated data.

Keywords: Bayesian estimation, generalized half logistic distribution, progressively Type-II censoring, reliability.

1. Introduction

The probability density function (pdf) and cumulative distribution function (cdf) of the random variable X having the generalized half logistic distribution are given by

$$f(x;\lambda) = \lambda \left[\frac{2e^{-x}}{1+e^{-x}}\right]^{\lambda} \frac{1}{1+e^{-x}} \tag{1.1}$$

and

$$F(x;\lambda) = 1 - \left[\frac{2e^{-x}}{1+e^{-x}}\right]^{\lambda}, \tag{1.2}$$

where λ is the shape parameter. In special case, when $\lambda = 1$, this distribution is the half logistic distribution. From (1.1), the reliability function of the generalized half logistic distribution with shape parameter λ is given by

$$R(x) = \left[\frac{2e^{-x}}{1+e^{-x}}\right]^{\lambda}, \quad x > 0,\ \lambda > 0. \tag{1.3}$$

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The half logistic distribution has been used quite extensively in reliability and survival analysis particularly when the data is censored. Inferences for the half logistic distribution were discussed by several authors. Balakrishnan and Puthenpura (1986) introduced the best linear unbiased estimators of location and scale parameters of the half logistic distribution through linear functions of order statistics. Balakrishnan and Wong (1991) obtained approximate maximum likelihood estimates (AMLEs) for the location and scale parameters of the half logistic distribution with Type-II right censored sample. Recently, Kang and Park (2005) derived the AMLE of the scale parameter of the half logistic distribution based on multiply Type-II censored samples. Kang *et al.* (2009) proposed the AMLEs of the scale parameter in a half logistic distribution based on double hybrid censored samples.

The progressive censoring appears to be of great importance in planning duration experiments in reliability studies. In many life testing and industrial experiments, experiments have to be terminated early and also the number of failures must be limited for various reasons. Progressively Type-II censoring is a generalization of Type-II censoring. In this case, the first r_1 failures in a life test of n items are observed; then n_1 of the remaining $n - r_1$ unfailed items are removed from the experiment, leaving $n - r_1 - n_1$ items still present. When a further r_2 items have failed, n_2 of the still unfailed items are removed, and so on. The experiment terminates after some prearranged series of repetitions of this procedures. Also note that if $r_1 = r_2 = \cdots = r_m = 0$, so that $m = n$, progressively Type-II censoring scheme reduces to the case of no censoring, that is, complete data. Balakrishnan *et al.* (2003) suggested point and interval estimation for Gaussian distribution based on progressively Type-II censored samples. Balakrishnan *et al.* (2004) studied point and interval estimation for the extreme value distribution under progressively Type-II censored sample. Seo and Kang (2007) derived AMLEs for Rayleigh distribution based on progressively Type-II censored data. Kang *et al.* (2008) derived the AMLEs and maximum likelihood estimator of the scale parameter in a half logistic distribution based on progressively Type-II censored samples.

In Bayesian estimation, we consider three types of loss functions. The first is the squared error loss function (SELF) which is a symmetric loss function assigning equal losses to over estimation and underestimation. The Bayes estimator under the SELF is the posterior mean given by $\hat{\theta}_{BS} = E_\pi(\theta)$, where E_π denotes the posterior expectation. However, such a restriction may be impractical because an overestimate is usually much more serious than an underestimate in the estimation of reliability and failure rate functions. In this case the use of a symmetrical loss function might be inappropriate. To overcome this difficulty, Varian (1975) proposed an asymmetric loss function known as the linex loss function (LLF) and got a lot of popularity due to Zellner (1986). The Bayes estimator under the LLF is given by $\hat{\theta}_{BL} = -\frac{1}{c}\ln\left\{E_\pi\left[\exp(-c\theta)\right]\right\}$, provided that the expectation exists and is finite. Finally, in many practical situations, it appears to be more realistic to express the loss in terms of the ratio $\frac{\hat{\theta}}{\theta}$. In this case, Calabria and Pulcini (1996) proposed that a useful asymmetric loss function is the General Entropy loss function (GELF). This loss function is a generalization of the Entropy loss when shape parameter $p = 1$. The Bayes estimator under the GELF is given by $\hat{\theta}_{BE} = -\left[E_\pi(\theta^{-p})\right]^{-1/p}$, provided that the expectation $E_\pi(\theta^{-p})$ exists and is finite. Recently, Kim *et al.* (2011) suggested Bayesian estimations on the exponentiated half triangle distribution under Type-I hybrid censoring.

2. Maximum likelihood estimation

First, we derive the maximum likelihood estimator of the shape parameter and reliability function based on progressively Type-II censored sample. Let $x_{1:m:n}, \dots, x_{m:m:n}$ denote such a sample with $(r_1, \dots, r_m)$ being the progressive Type-II censoring scheme. The likelihood function based on the sample, $x_{1:m:n}, \dots, x_{m:m:n}$, is then given by

$$L(\lambda) = C\lambda^m \prod_{i=1}^{m} \left[\frac{2e^{-x_{i:m:n}}}{1+e^{-x_{i:m:n}}}\right]^{\lambda(1+r_i)} \frac{1}{1+e^{-x_{i:m:n}}}, \tag{2.1}$$

where $C = n \times (n-1-r_1) \times (n-2-r_1-r_2) \times \cdots \times (n-m+1-r_1-\cdots-r_{m-1})$. It is easy to obtain the maximum likelihood estimator (MLE) of λ to be

$$\hat{\lambda} = \frac{m}{T_1}, \tag{2.2}$$

where

$$T_1 = \sum_{i=1}^{m} (1+r_i)\left[x_{i:m:n} + \ln\left(\frac{1+e^{-x_{i:m:n}}}{2}\right)\right]. \tag{2.3}$$

By the invariance property of the MLE, we can obtain the maximum likelihood estimator of reliability function $R(t;\lambda)$ to be

$$\hat{R}(t) = \left[\frac{2e^{-t}}{1+e^{-t}}\right]^{\hat{\lambda}}. \tag{2.4}$$

3. Bayesian estimation

3.1. Estimation based on a conjugate prior

For a Bayesian inference, we consider a conjugate prior distribution and a noninformative prior for shape parameter λ, respectively. A natural conjugate prior for the shape parameter λ of the generalized half logistic distribution is an gamma prior, given by

$$\pi(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda}, \quad \lambda > 0,\ \alpha > 0,\ \beta > 0. \tag{3.1}$$

It follows, from (3.1), that the posterior distribution of λ is given by

$$\pi(\lambda|x) = \frac{(\beta+T_1)^{m+\alpha}}{\Gamma(m+\alpha)}\lambda^{m+\alpha-1}e^{-(\beta+T_1)\lambda}, \quad \alpha > 0,\ \beta > 0, \tag{3.2}$$

where T_1 is given in (2.3). That is, $\lambda|x \sim gamma(m+\alpha, \beta+T_1)$.

Substituting $\lambda = -\frac{\ln R}{T_2}$ into (3.2), we obtain the posterior probability density function of $R = R(t;\lambda)$ as

$$\pi(R|x) = \frac{1}{\Gamma(m+\alpha)}\left(\frac{\beta+T_1}{T_2}\right)^{m+\alpha}(-\ln R)^{m+\alpha-1}R^{(\beta+T_1)/T_2-1}, \quad \alpha > 0,\ \beta > 0, \tag{3.3}$$

where T_1 is given in (2.3) and

$$T_2 = t + \ln\left[\frac{1+e^{-t}}{2}\right]. \tag{3.4}$$

We compute the posterior modes of λ and R as

$$\tilde{\lambda} = \frac{m+\alpha-1}{\beta+T_1} \tag{3.5}$$

and

$$\tilde{R} = \exp\left\{-\frac{(m+\alpha-1)T_2}{\beta+T_1-T_2}\right\}, \tag{3.6}$$

respectively.

Under the squared error loss function, the Bayes estimators of λ and R can be obtained as

$$\tilde{\lambda}_S = \frac{m+\alpha}{\beta+T_1} \tag{3.7}$$

and

$$\tilde{R}_S = \left(\frac{\beta+T_1}{\beta+T_1+T_2}\right)^{m+\alpha}, \tag{3.8}$$

respectively.

Linex loss function provides the Bayes estimators of λ and R as

$$\tilde{\lambda}_L = \frac{m+\alpha}{c}\ln\left(1+\frac{c}{\beta+T_1}\right) \tag{3.9}$$

and

$$\tilde{R}_L = -\frac{1}{c}\ln\left[\sum_{j=0}^{\infty}(-1)^j\frac{c^j}{j!}\left(\frac{\beta+T_1}{\beta+T_1+jT_2}\right)^{m+\alpha}\right], \tag{3.10}$$

respectively, where c is the shape parameter of linex loss function.

Using the Entropy loss function leads the Bayes estimators of λ and R as

$$\tilde{\lambda}_E = \frac{1}{\beta+T_1}\left(\frac{\Gamma(m+\alpha)}{\Gamma(m+\alpha-p)}\right)^{1/p} \tag{3.11}$$

and

$$\tilde{R}_E = \left(1-\frac{pT_2}{\beta+T_1}\right)^{\frac{m+\alpha}{p}}, \tag{3.12}$$

respectively, where p is the shape parameter of Entropy loss function.

3.2. Estimation based on a noninformative prior

For the situation where no prior information about the shape parameter λ is available, one may use the quasi density as given by

$$\pi(\lambda) = \frac{1}{\lambda^d}, \quad \lambda > 0, \; d > 0. \tag{3.13}$$

This contains Jeffery's noninformative prior as a special case when $d = 1$. It follows, from (3.13), that the posterior distribution of λ is given by

$$\pi(\lambda|x) = \frac{\lambda^{m-d} T_1^{m-d+1}}{\Gamma(m-d+1)} e^{-T_1 \lambda}, \quad m > d. \tag{3.14}$$

That is, $\lambda|x \sim gamma(m-d+1, T_1)$.

Substituting $\lambda = -\frac{\ln R}{T_2}$ into (3.14), we obtain the posterior probability density function of $R = R(t;\lambda)$ as

$$\pi(R|x) = \frac{1}{\Gamma(m-d+1)} \left(\frac{T_1}{T_2}\right)^{m-d+1} (-\ln R)^{m-d} R^{T_1/T_2 - 1}, \quad m > d. \tag{3.15}$$

Corresponding posterior modes of λ and R are

$$\tilde{\lambda} = \frac{m-d}{T_1} \tag{3.16}$$

and

$$\tilde{R} = \exp\left\{-\frac{(m-d)T_2}{T_1 - T_2}\right\}, \tag{3.17}$$

respectively.

Under the squared error loss function, the Bayes estimators of λ and R can be obtained as

$$\tilde{\lambda}_S = \frac{m-d+1}{T_1} \tag{3.18}$$

and

$$\tilde{R}_S = \left(\frac{T_1}{T_1 + T_2}\right)^{m-d+1}, \quad m > d, \tag{3.19}$$

respectively.

Linex loss function provides the Bayes estimators of λ and R as

$$\tilde{\lambda}_L = \frac{m-d+1}{c} \ln\left(1 + \frac{c}{T_1}\right) \tag{3.20}$$

and

$$\tilde{R}_L = -\frac{1}{c} \ln\left[\sum_{j=0}^{\infty} (-1)^j \frac{c^j}{j!} \left(\frac{T_1}{T_1 + jT_2}\right)^{m-d+1}\right], \quad m > d, \tag{3.21}$$

respectively.

Using the Entropy loss function leads the Bayes estimators of λ and R as

$$\tilde{\lambda}_E = \frac{1}{\beta + T_1}\left(\frac{\Gamma(m-d+1)}{\Gamma(m-d-p+1)}\right)^{1/p} \tag{3.22}$$

and

$$\tilde{R}_E = \left(1 - \frac{pT_2}{T_1}\right)^{\frac{m-d+1}{p}}, \quad m > d, \tag{3.23}$$

respectively.

3.3. Estimation of the scale parameter σ

Considering scale parameter σ leads more general distribution with cdf

$$f(x;\lambda,\sigma) = \frac{\lambda}{\sigma}\left[\frac{2e^{-x/\sigma}}{1+e^{-x/\sigma}}\right]^{\lambda}\frac{1}{1+e^{-x/\sigma}}. \tag{3.24}$$

Basically we can consider a joint prior distibution of (λ, σ) and then perform a fully Bayesian infernce using Markov chain Monte Carlo (MCMC) algorithm. Since the shape parameter λ is the parameter of interest here, we just estimate the parameter σ and then plug it in. The nuisance paraeter σ can be estimated by maximizing its marginal likelihood,

$$L(\sigma) = \int f(\mathbf{x}|\sigma,\lambda)\pi(\lambda)d\lambda. \tag{3.25}$$

That is,

$$\hat{\sigma} = \arg\max_{\sigma>0} L(\sigma). \tag{3.26}$$

Note that the marginal likelihood of σ can be obtained by

$$\frac{\Gamma(m+\alpha)}{\beta + \sum_{i=1}^{m}(1+r_i)\log\left(\frac{1+e^{-x_{i:m:n}/\sigma}}{2e^{-x_{i:m:n}/\sigma}}\right)} \tag{3.27}$$

or

$$\frac{\Gamma(m-d+1)}{\sum_{i=1}^{m}(1+r_i)\log\left(\frac{1+e^{-x_{i:m:n}/\sigma}}{2e^{-x_{i:m:n}/\sigma}}\right)}. \tag{3.28}$$

The variance estimation of λ may require adjustment to allow proper account for the uncertainty caused by estimating σ because of

$$Var(\lambda|\mathbf{x}) = E_{\sigma|\mathbf{x}}\left[Var(\lambda|\mathbf{x},\sigma)\right] + Var_{\sigma|\mathbf{x}}\left[E(\lambda|\mathbf{x},\sigma)\right]. \tag{3.29}$$

4. Illustrative examples

In this section, we present two examples to illustrate our estimation methods discussed in the previous sections.

4.1. Real data

Consider the data given by Nelson (1982), represents failure log times to breakdown of an insulating fluid testing experiment (see Table 4.1). A progressively Type-II censored sample are generated from this data by Viverous and Balakrishnan (1994). The observations and censoring scheme are given in Table 4.2. In this example, we have $n = 16$ and $m = 8$. From (2.2) and (2.4), the MLEs $\hat{\lambda} = 0.35001$ and $\hat{R}(t = 0.5) = 0.90635$. Using the formulae presented in section 3, the Bayes estimators of λ and $R(t = 0.5)$ are calculated. These values are given in Tables 4.3 and 4.4.

Table 4.1 Failure log times to breakdown of an insulating fluid testing experiment

0.270027	1.02245	1.15057	1.42311	1.54116	1.57898	1.87180	1.99470
2.08069	2.11263	2.48989	3.45789	3.48186	3.52371	3.60305	4.28895

Table 4.2 Progressively type-II censored data

i	1	2	3	4	5	6	7	8
x	0.270027	1.02245	1.15057	1.57898	2.11263	2.48989	3.60305	4.28895
r_i	0	0	2	3	0	3	0	0

Table 4.3 Bayes estimators using the gamma prior with $\alpha = 1$ and $\beta = 1$

$c = 1.5$ and $p = 1.5$					
$\tilde{\lambda}_S$	$\tilde{\lambda}_L$	$\tilde{\lambda}_E$	$\tilde{R}_S$	$\tilde{R}_L$	$\tilde{R}_E$
0.37726	0.36587	0.32458	0.90000	0.89925	0.89859

Table 4.4 Bayes estimators using the quasi prior with $d = 0.5$

$c = 1.5$ and $p = 1.5$					
$\tilde{\lambda}_S$	$\tilde{\lambda}_L$	$\tilde{\lambda}_E$	$\tilde{R}_S$	$\tilde{R}_L$	$\tilde{R}_E$
0.37189	0.36019	0.31688	0.90137	0.90060	0.89992

4.2. Simulated data

Consider a progressively type-II censored sample of size $m = 8$ from $n = 9$ with censoring scheme $r = (0, 0, 3, 4, 0, 4, 0, 0)$ from the generalized half logistic distribution with shape parameter λ. We generate λ from the gamma distribution with $\alpha = 2.0$ and $\beta = 1.5$, then using the generated λ, generate a progressively type-II censored sample of size $m = 8$ from sample of $n = 19$ with censoring scheme $r = (0, 0, 3, 4, 0, 4, 0, 0)$ from the generalized half logistic distribution. The actual generated population values of λ and $R(t = 0.5)$ are 1.05930 and 0.74261, respectively. The observations and censoring scheme are given in Table 4.5. From (2.2) and (2.4), the MLEs $\hat{\lambda} = 1.27272$ and $\hat{R}(t = 0.5) = 0.69939$. Using the formulae presented in section 3, the Bayes estimators of λ and $R(t = 0.5)$ are calculated. These values are given in Tables 4.6 and 4.7.

Table 4.5 Progressively type-II censored sample based on simulated data

i	1	2	3	4	5	6	7	8
x	0.09962	0.20909	0.30184	0.38157	0.47794	0.74326	1.32006	1.61662
r_i	0	0	3	4	0	4	0	0

Table 4.6 Bayes estimators using the gamma prior with $\alpha = 2$ and $\beta = 1.5$

$c = 1.5$ and $p = 1.5$					
$\tilde{\lambda}_S$	$\tilde{\lambda}_L$	$\tilde{\lambda}_E$	$\tilde{R}_S$	$\tilde{R}_L$	$\tilde{R}_E$
1.2844	1.17457	1.12308	0.70155	0.69699	0.69007

Table 4.7 Bayes estimators using the quasi prior with $d = 0.5$

$c = 1.5$ and $p = 1.5$					
$\tilde{\lambda}_S$	$\tilde{\lambda}_L$	$\tilde{\lambda}_E$	$\tilde{R}_S$	$\tilde{R}_L$	$\tilde{R}_E$
1.11363	0.99871	0.91333	0.73634	0.73101	0.72337

4.3. Simulation assesment

To compare the performance of the Bayes estimators of the shape parameter λ and reliability function $R(t;\lambda)$ under three types of loss functions such as SELE, LLF, and GELF, we simulated the mean squared errors of all proposed estimators through Monte Carlo simulation method. Using the method given in section 4.1, the progressively Type-II censored data from the generalized half logistic distribution, whose shape parameter λ is from the gamma distribution with $(\alpha,\beta) = \{(1.5, 1.5)\}$, are generated for $t = 0.5$, sample size $n = 20, 30, 40$, and various censoring schemes. Using this data, the mean squared errors of the Bayes estimators of the shape parameter λ and reliability function $R(t)$ under three types of loss functions are simulated by Monte Carlo method based on 10,000 runs for sample size $n = 20, 30, 40$ and various choices of censoring under progressively Type-II censored samples. To see more clear effect of asymmetric loss function, specific values for shape parameter of loss function are considered. For simplicity in notation, we denote the scheme $(0, 0, \ldots, n-m)$ as $((m-1)\times 0, n-m)$, for example, (10×0) and $(3 \times 0, 2, 2, 0)$ denote the progressively censoring schemes $(0, 0, \ldots, 0)$ and $0, 0, 0, 2, 2, 0)$, respectively. These values are given in Tables 4.8 and 4.9.

5. Concluding remarks

In this paper we present the Bayesian and Non-Bayesian estimator of the shpae parameter λ and reliability function $R(t)$ of generalized half logistic distribution under progressively Type-II censoring. Bayes estimators under squared error loss function, linex loss function and Entropy loss function are derived. The MLE's are also obtained. The MLEs of λ and R are compared with Bayes estimates under various loss functions in terms of estimated MSE. We can see that the Bayes estimates are better than their corresponding maximum likelihood estimates for the considered cases, especially in asymmetric loss function such as linex.

Table 4.8 The relative mean squared errors for the estimators of the shape parameter λ and reliability function $R(t;\lambda)$ when prior is gamma distribution with $\alpha = 1.5$ and $\beta = 1.5$

		$\lambda = 0.754$, $c = 2$ and $p = -0.5$				
n	m	censoring scheme	$\hat{\lambda}$	$\tilde{\lambda}_S$	$\tilde{\lambda}_L$	$\tilde{\lambda}_E$
20	20	(20×0)	0.03604	0.03329	0.02690	0.03179
	10	(1 3×0 0 2 2×0 5)	0.09582	0.07456	0.04903	0.06865
	10	(5 2×0 5 6×0)	0.09703	0.07515	0.04945	0.06924
	10	(3 3×0 2 0 2 2×0 3)	0.09850	0.07625	0.05017	0.07026
	5	(0 5 0 10 0)	0.38638	0.14964	0.07172	0.13084
30	30	(30×0)	0.02249	0.02151	0.01860	0.02082
	20	(9×0 10 10×0)	0.03577	0.03301	0.02662	0.03151
	20	(5 3×0 5 15×0)	0.03607	0.03333	0.02684	0.03180
	20	(10 2×0 17×0)	0.03710	0.03426	0.02745	0.03265
	15	(5 14×0)	0.05234	0.04604	0.03455	0.04336
	15	(10 6×0 5 7×0)	0.05295	0.04672	0.03493	0.04395
40	40	(40×0)	0.01586	0.01543	0.01378	0.01503
	20	(15 5 18×0)	0.03544	0.03264	0.02649	0.03121
	20	(10 10 18×0)	0.03558	0.03290	0.02645	0.03137
	20	(5 16×0 5 5 5)	0.03565	0.03294	0.02656	0.03143
	20	(2×0 4×5 14×0)	0.03710	0.03426	0.02745	0.03265
	10	(0 15 2×0 5 0 10 3×0)	0.09609	0.07464	0.04890	0.06868
		$R = 0.809$, $c = 2$ and $p = -0.5$				
n	m	censoring scheme	$\hat{\lambda}$	$\tilde{\lambda}_S$	$\tilde{\lambda}_L$	$\tilde{\lambda}_E$
20	20	(20×0)	0.00173	0.00155	0.00162	0.00158
	10	(1 3×0 0 2 2×0 5)	0.00417	0.00317	0.00342	0.00326
	10	(5 2×0 5 6×0)	0.00420	0.00319	0.00344	0.00328
	10	(3 3×0 2 0 2 2×0 3)	0.00426	0.00324	0.00349	0.00333
	5	(0 5 0 10 0)	0.01118	0.00567	0.00644	0.00597
30	30	(30×0)	0.00110	0.00103	0.00106	0.00104
	20	(9×0 10 10×0)	0.00172	0.00154	0.00160	0.00156
	20	(5 3×0 5 15×0)	0.00173	0.00155	0.00162	0.00158
	20	(10 2×0 17×0)	0.00177	0.00159	0.00166	0.00161
	15	(5 14×0)	0.00243	0.00207	0.00219	0.00211
	15	(10 6×0 5 7×0)	0.00246	0.00210	0.00222	0.00214
40	40	(40×0)	0.00079	0.00075	0.00077	0.00076
	20	(15 5 18×0)	0.00170	0.00152	0.00159	0.00154
	20	(10 10 18×0)	0.00171	0.00153	0.00160	0.00155
	20	(5 16×0 5 5 5)	0.00171	0.00154	0.00160	0.00156
	20	(2×0 4×5 14×0)	0.00177	0.00159	0.00166	0.00161
	10	(0 15 2×0 5 0 10 3×0)	0.00416	0.00317	0.00342	0.00326

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Table 4.9 The relative mean squared errors for the estimators of the shape parameter λ and reliability function $R(t;\lambda)$ when prior is quasi density with $d = 1.5$

n	m	censoring scheme	$\hat{\lambda}$	$\tilde{\lambda}_S$	$\tilde{\lambda}_L$	$\tilde{\lambda}_E$
		$\lambda = 2.189$, $c = 2$ and $p = -0.5$				
20	20	(20×0)	0.30378	0.28002	0.20977	0.27080
	10	(1 3×0 0 2 2×0 5)	0.80760	0.68851	0.37639	0.64210
	10	(5 2×0 5 6×0)	0.81783	0.69813	0.38008	0.65143
	10	(3 3×0 2 0 2 2×0 3)	0.83020	0.70836	0.38212	0.66067
	5	(0 5 0 10 0)	3.25661	2.47006	0.68486	2.16357
30	30	(30×0)	0.18958	0.17932	0.14652	0.17531
	20	(9×0 10 10×0)	0.30150	0.27773	0.20747	0.26851
	20	(5 3×0 5 15×0)	0.30397	0.27968	0.20762	0.27021
	20	(10 2×0 17×0)	0.31267	0.28690	0.20826	0.27673
	15	(5 14×0)	0.44113	0.39535	0.26514	0.37754
	15	(10 6×0 5 7×0)	0.44629	0.39885	0.26356	0.38026
40	40	(40×0)	0.13365	0.12786	0.10918	0.12558
	20	(15 5 18×0)	0.29871	0.27633	0.21084	0.26778
	20	(10 10 18×0)	0.29984	0.27563	0.20454	0.26621
	20	(5 16×0 5 5 5)	0.30047	0.27672	0.20700	0.26751
	20	(2×0 4×5 14×0)	0.31267	0.28690	0.20826	0.27673
	10	(0 15 2×0 5 0 10 3×0)	0.80985	0.68934	0.37263	0.64228
		$R = 0.541$, $c = 2$ and $p = -0.5$				
n	m	censoring scheme	$\hat{\lambda}$	$\tilde{\lambda}_S$	$\tilde{\lambda}_L$	$\tilde{\lambda}_E$
20	20	(20×0)	0.00587	0.00542	0.00554	0.00553
	10	(1 3×0 0 2 2×0 5)	0.01233	0.01049	0.01091	0.01094
	10	(5 2×0 5 6×0)	0.01238	0.01056	0.01096	0.01101
	10	(3 3×0 2 0 2 2×0 3)	0.01258	0.01071	0.01113	0.01117
	5	(0 5 0 10 0)	0.02570	0.01967	0.02061	0.02117
30	30	(30×0)	0.00385	0.00364	0.00370	0.00370
	20	(9×0 10 10×0)	0.00578	0.00533	0.00545	0.00544
	20	(5 3×0 5 15×0)	0.00584	0.00537	0.00550	0.00549
	20	(10 2×0 17×0)	0.00594	0.00543	0.00557	0.00555
	15	(5 14×0)	0.00783	0.00701	0.00722	0.00721
	15	(10 6×0 5 7×0)	0.00794	0.00706	0.00729	0.00728
40	40	(40×0)	0.00281	0.00269	0.00272	0.00272
	20	(15 5 18×0)	0.00575	0.00535	0.00545	0.00545
	20	(10 10 18×0)	0.00576	0.00529	0.00542	0.00540
	20	(5 16×0 5 5 5)	0.00579	0.00534	0.00546	0.00545
	20	(2×0 4×5 14×0)	0.00594	0.00543	0.00557	0.00555
	10	(0 15 2×0 5 0 10 3×0)	0.01228	0.01041	0.01083	0.01087

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