Chaotic Predictability for Time Series Forecasts of Maximum Electrical Power using the Lyapunov Exponent

Jae-Hyeon Park, Young-Il Kim, and Yeon-Gyu Choo, *Member, KIMICS*

Abstract— **Generally the neural network and the Fuzzy compensative algorithms are applied to forecast the time series for power demand with the characteristics of a non-linear dynamic system, but, relatively, they have a few prediction errors. They also make long term forecasts difficult because of sensitivity to the initial conditions.**

In this paper, we evaluate the chaotic characteristic of electrical power demand with qualitative and quantitative analysis methods and perform a forecast simulation of electrical power demand in regular sequence, attractor reconstruction and a time series forecast for multi dimension using Lyapunov Exponent (L.E.) quantitatively. We compare simulated results with previous methods and verify that the present method is more practical and effective than the previous methods. We also obtain the hourly predictability of time series for power demand using the L.E. and evaluate its accuracy.

Index Terms— **Chaos, Lyapunov Exponent, Electrical Power, Forecast**

I. INTRODUCTION

WITH the development of the industrial society and the improvement of life style, demand and supply of electric power have gradually become more recognized and emphasized. It is possible to supply stable electric power with an accurate forecast of power demand, which is closely related with energy saving and environment preservation. The prediction of the supply of electric power is related with the characteristics of production and the consumption of electric power occurring simultaneously. It requires accurate management for electric power to be supplied continuously and stably. Because of the various factors mentioned above, the short-term prediction of the supply of electric power is closely related to the management plan of the power facility, which considers consumers' demand which changes at any time[1][2]. To analyze the peculiarity of power supply, various methods have been used such as time series, regression, neural network analysis and so on. We verified that a time series of power demand has a nonlinear characteristic so its prediction is impossible using chaotic analysis[3].

In this paper, we analyze, with quantitative and qualitative analysis methods, the power demand of a residential district, Gyeongnam Jinju City, which has a chaotic characteristic. We perform a multi dimensional short-term forecast simulation using the L.E. obtained from the analysis of a time series. We compare the simulated results with previous results and evaluate its performance by their error. Finally we estimate a predictable duration and verify the relationship between the prediction error and the duration.

II. THE CHAOTIC SIGNAL ANALYSIS

A. Analysis methods of chaotic signals

Linearity means that the rule that determines what a part of a system is going to do next is not influenced by what it is doing now. Chaos is considered as a case of nonlinearity, because it is influenced by what is yet to happen and it is an unsystematic phenomenon. Due to sensitivity, all phenomena in a chaotic system depend on the initial conditions, so that any uncertainty in the initial state of the given system, no matter how small, will lead to rapidly growing errors in any effort to predict future behavior[4][5]. The frequency spectrum of a chaotic system essentially resembles one of continuous random noise. The dynamic orbits are observed as chaotic behavior in the entire phase space.

There are three analysis methods to analyze the chaotic behavior of a given system; the first method is to compare attractors described in a phase space for a time series and noise signal. The second method is the qualitative analysis method using a frequency spectrum such as an autocorrelation function, power spectrum. The last method is the quantitative analysis method using the correlation dimension, L.E.[6]

B. Attractor reconstruction and Lyapunov Exponent

L.E. is a useful tool to analyze the chaotic attractor quantitatively and is also suitable for measuring the sensitivity of a chaotic trajectory to the initial state. L.E. indicates logarithmically the growth or shrinkage rate of small perturbations in various directions in a

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Jae-Hyeon Park is with the Department of Electronics Engineering, Gyeongsang National University, Jinju, 660-701, Korea (Email: superjay@paran.com)

Yeon-Gyu Choo(corresponding author) is with the Department of Electronics Engineering, Gyeongnam National University of Science and Technology, Jinju, 660-758, Korea (Email: ygchoo@gntech.ac.kr)

phase space. If the L.E. obtained from a time series is positive, the given system has chaotic characteristic behavior, which means its attractor is sensitive to its initial conditions and becomes chaotic. On the other hand, if the L.E. is negative, the attractor gradually shrinks or becomes extinct. Generally the largest L.E. (L.L.E.) is produced from a time series of a nonlinear system after the attractor has been reconstructed by various embedding dimensions.

In order to study various dynamic systems with a time series of multi-dimension dynamic system or ascertain whether strange attractor including noise signals exist using a mathematical solution, Takens proposed the theory and mathematical basis related reconstructed attractor[7]. For example, we may obtain the value of one of the state variables of the system. We consider the sequence of numbers from a time series shown the next equation, which one of variables, calling it Z(t), and a time interval τ

$$z_0 = z(0),\ z_1 = z(\tau),\ z_2 = z(2\tau),\ \cdots \tag{1}$$

We create a time series data set shown as equation (2) after selecting a parameter of time delay T

$$\begin{aligned} &(z(0), z(T),\ z(2T)) \\ &(z(\tau), z(\tau+T),\ z(\tau+2T)) \\ &\qquad \vdots \\ &(z(kT),\ z(k\tau+T),\ z(k\tau+2T)) \end{aligned} \tag{2}$$

We can obtain various strange attractors if the points of this data set are plotted in three dimensional space with connecting line segments. However, we may simply work in higher-dimensional spaces using vectors as shown in equation (3).

$$u(t) = (z(t),\ z(t+T),\ \cdots,\ z(t+2NT)) \tag{3}$$

If N is chosen large enough, the attractor will fit in the chosen space. Here the choice of the time lag T is almost arbitrary or selected by auto-correlation function.

Wolf *et al.* give an algorithm for obtaining L.L.E. from a time series[8]. The approach is based on following the divergence of a neighboring trajectory from a selected one. Over a time interval $t_2 - t_1$, the rate of divergence of two points that evolve from a spacing D_1' to a spacing D_2 may be characterized as shown in equation (4). After n repetitions of stretching and renormalizing the spacing, the rates are weighted by the fraction of time between each renormalization and then added to yield an experimental value for L.L.E. shown as equation (5).

$$\frac{\log_e(D_2 / D_1')}{t_2 - t_1} \tag{4}$$

$$\lambda_1 = \sum_{i=1}^{n-1}\left\{\left[\frac{(t_{i+1}-t_i)}{\left[\sum_{i=1}^{n-1}(t_{i+1}-t_i)\right]}\right]\left[\frac{\log_e(D_{i+1}/D_i')}{(t_{i+1}-t_i)}\right]\right\} = \frac{\sum_{i=1}^{n-1}\log_e(D_{i+1}/D_i')}{t_n - t_1} \tag{5}$$

III. SHORT-TERM PREDICTION USING L.E.

The proposed prediction algorithm proceeds according to the following steps: the first is to reconstruct the time domain vectors of a time series in an embedding phase space. The second is to calculate an unknown point in a phase space using chaotic behavior. The third is to recover the prediction point of a phase space to the time domain and obtain the predicted value[9].

A data set of a time series is given as equation (6), where N is the length of data set.

$$x(1),\ x(2),\ x(3),\ \cdots,\ x(N) \tag{6}$$

A known embedding point in D-dimension phase space is written using the time delay T shown as equation (7).

$$Y(I),\ I \in [1\,[N-(D-1)T]] \tag{7}$$

N-$(D$-$1)T$ embedding points of that space can be obtained using equation (7). The nearest point to the $Y(N\text{-}(D\text{-}1)T)$ point can also be found due to those points. The distance between $Y(N\text{-}(D\text{-}1)T)$ and $Y(min\_dist)$ can be calculated and we call it $Diff_0$. The instant distance Diffl between $Y(N\text{-}(D\text{-}1)T+1)$ and $Y(min\_dist+1)$ also can be calculated using L.E. if $Diff_1/Diff_0$ has a little change in every step shown as equation (8).

$$Diff_1 = Diff_0 \cdot 2^{K\cdot\lambda} \tag{8}$$

Where λ is the L.E. and K is the number of steps from $Diff_0$ to $Diff_1$. A point of $Y(N\text{-}(D\text{-}1)T+1)$ can be calculated because a point of $Y(min\_dist+1)$ is known. The length of prediction is based on chaotic behavior and L.E. as shown in the following section.

IV. PREDICABILITY OF A CHAOTIC SIGNAL

When a given system, such as a time series of power demand, has a positive L.E., there is a time horizon beyond which prediction breaks down[10]. Suppose we measure the initial conditions of a given system very accurately. We consider some error $\|\delta_0\|$ between an estimated value and the true initial state. After a time t, the discrepancy grows as in equation (9).

$$\| \delta(t) \| \approx \| \delta_0 \| e^{\lambda t} \quad (9)$$

Let a be a measure of its tolerance; we consider it acceptable if a prediction is within a of the true state. The prediction becomes intolerable when $\| \delta(t) \| \geq a$. A time horizon can be described by the following equation.

$$t_{horizon} \approx \frac{1}{\lambda} \ln \frac{a}{\| \delta_0 \|} \quad (10)$$

No matter how hard we work to reduce the initial error, we cannot predict longer than a few multiples of $1/\lambda$.

V. SIMULATIONS

A. Analysis for a time series of power demand in Jinju city

A time series of electric power demand applied in this paper is based on information gathered from a residential district of Gyeongsangnamdo Jinju city. It was obtained hourly from January to December 2004. Figure 1 shows a part of an electric power time series supplied in Jinju city on 2004. The attractor of a time series described on a phase space is shown in figure 2. It is separated from the periodic signal and the noise signal distributed randomly within a phase space for checking in detail.

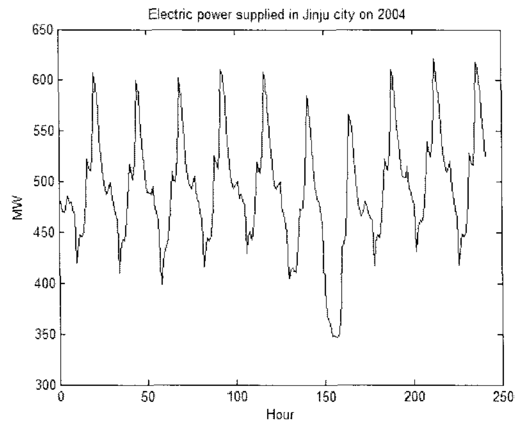


Fig. 1. Electric power supplied in Jinju city in 2004.

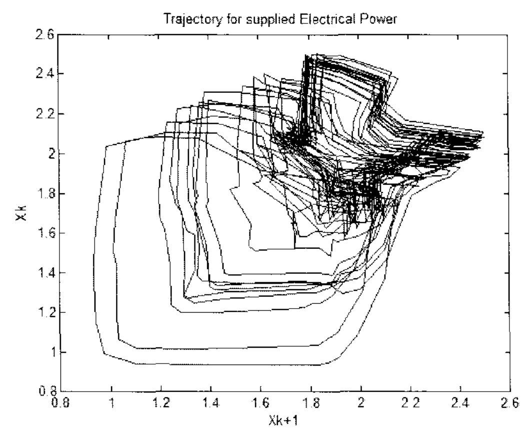


Fig. 2. Trajectory of a time series in phase space.

To analyze the chaotic characteristic of a time series, the correlation dimension and L.E. quantitative analysis were used. The correlation dimension is generally obtained from changing a specific radius of a phase space and calculating interference values for the existing trajectory in that area after a time series composed of one dimensional vectors is reconstructed by embedding the dimension. Table 1 shows correlation dimensions obtained from changing an embedding dimension (E.D.) for a time series from 1 to 6 as described in Table 1 and the correlation integral (C.I.) for each embedding dimension is shown in Figure 3.

L.L.E.'s of Table 2 are obtained with a method proposed by *Wolf et* al, in which the attractor of a time series is reconstructed by changing the embedding dimension from 2 to 6. As shown in Table 2, a time series of power demand is regarded as a nonlinear system with chaotic characteristic because the L.E.'s have positive values.

TABLE I
CORRELATION INTEGRAL BY CHANGING EMBEDDING DIMENSION.

E.D.	1	2	3	4	5	6
C.I.	0.9862	1.8437	1.9655	1.5393	1.1781	0.8771

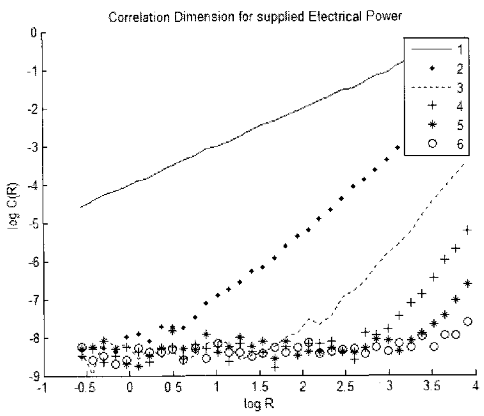


Fig. 3. Correlation Integral of a time series.

TABLE II
L.E. FOR EACH E.D. (TIME DELAY: 5)

E.D.	2	3	4	5	6
L.E.	0.3278	0.1029	0.0699	0.0311	0.0175

B. Short-term Forecast of power demand

In order to evaluate the performance of the proposed method for short-term prediction, we performed a forecast simulation for power demand with the previous Fuzzy compensative algorithm (F.C.A.) and L.L.E. method using a time series of power demand in Jinju city in 2004.

The previous prediction model using the Fuzzy compensative algorithm is proceeding by compensating the prediction error with the change rate of maximum electrical power demand due to Fuzzy algorithm and differences of the active and the predicted results.[11] A diagram of the previous model is shown as Figure 4 and the proposed model as Figure 5.

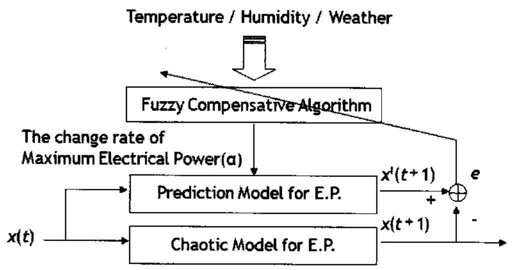


Fig. 4. Block diagram of Fuzzy compensative algorithm.

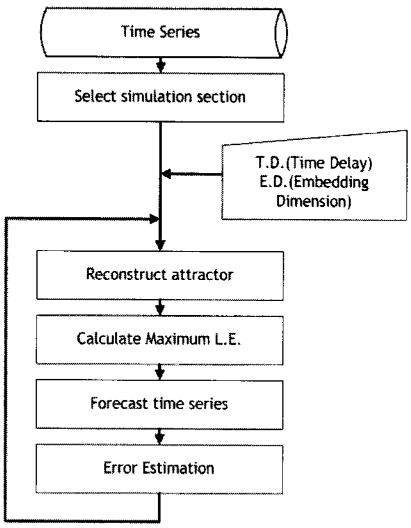


Fig. 5. Block diagram of the proposed algorithm.

The simulation to forecast power demand is applied into a section of the time series; we compute RMSE (Root Mean Square Error) and MAPE (Mean Absolute Percent Error) shown as equation (11), (12) in order to compare the actual and the predicted results.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{n}\left|x_n - \hat{x}_n\right|^2} \tag{11}$$

$$MAPE = \frac{1}{N}\sum_{i=1}^{n}\frac{\left|x_n - \hat{x}_n\right|}{x_n}\times 100\% \tag{12}$$

where x_n is nth chaotic time series point real value, $\hat{x}_n$ is nth chaotic time series point prediction value.

The results predicted by the previous algorithm and the proposed one are shown in Figure 6, 7, 8, 9. The error comparison tables are shown as Table 3, 4, 5, 6, which are the result of simulating each algorithm for some specific periods.

The result of prediction simulation shows the proposed algorithm makes a stable result for all periods but F.C.A. makes for a part of target periods.

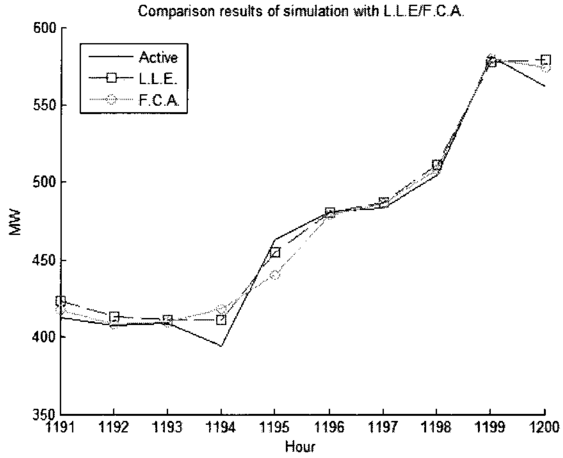


Fig. 6. Comparison result of L.L.E. and F.C.A. for random period (Period:1-1200/Year:2004)

TABLE III
ERROR COMPARISON TABLE FOR FIG. 6.

Algorithm	RMSE	MAPE
F.C.A.	11.3365	1.6501
L.L.E.	9.4396	1.6649

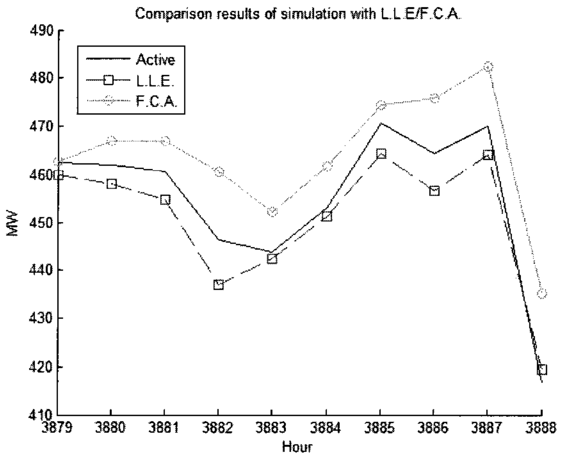


Fig. 7. Comparison result of L.L.E. and F.C.A. for random period (Period:2689-3888/Year:2004)

TABLE IV
ERROR COMPARISON TABLE FOR FIG. 7.

Algorithm	RMSE	MAPE
F.C.A.	14.7630	3.0226
L.L.E.	5.4544	1.0883

Although the simulation is performed for a part of the time series, the performance of proposed algorithm is more effective and excellent than the F.C.A. in a case of short-term forecasting of time series such as power demand.

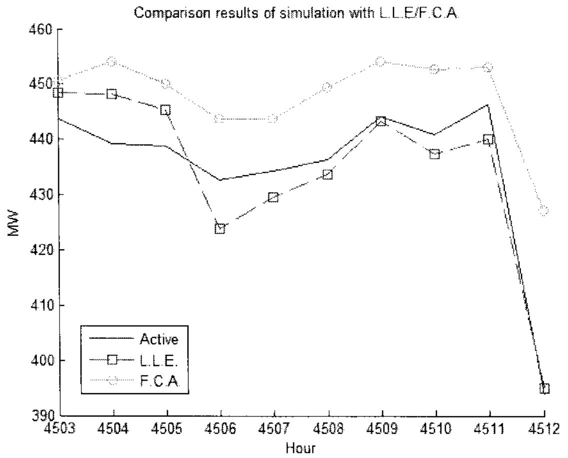


Fig. 8. Comparison result of L.L.E. and F.C.A. for random period (Period:3315-4512/Year:2004)

TABLE V
ERROR COMPARISON TABLE FOR FIG. 8.

Algorithm	RMSE	MAPE
F.C.A.	10.2588	1.9795
L.L.E.	5.4315	1.0533

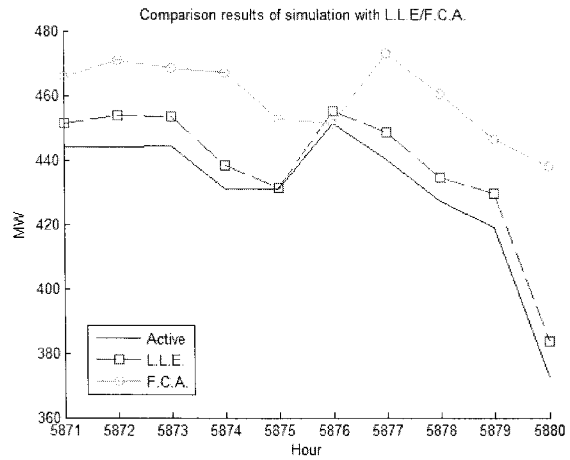


Fig. 9. Comparison result of L.L.E. and F.C.A. for random period (Period:4681-5880/Year:2004)

TABLE VI
ERROR COMPARISON TABLE FOR FIG. 9.

Algorithm	RMSE	MAPE
F.C.A.	32.9480	6.9520
L.L.E.	8.1812	1.7817

C. Predictability of Short-term Forecast

As mentioned above, the predictability of the short-term forecast is closely related with a time horizon beyond which prediction breaks down. We can estimate the sensitivity of a given system depending on the initial state by checking the error rate after the time horizon. Generally no matter how hard we work to reduce the initial error, we cannot predict longer than the time horizon. We evaluate the change of prediction error for simulated periods from the time horizon to 7 hours added. As a result of simulation for most of steps in time, the RMSE goes on increasing. It means it is impossible to predict beyond the time horizon regardless of the initial condition. Figure 10 shows the change rate of prediction error, RMSE, by time horizon and various periods.

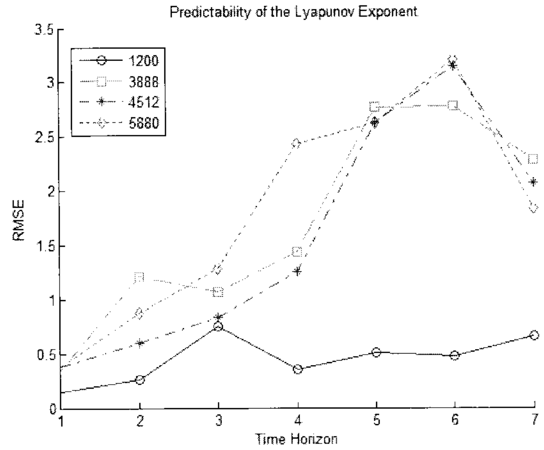


Fig. 10. Change rate of prediction error beyond the time horizon.

VI. CONCLUSIONS

In this paper, we proposed a prediction algorithm that involved time domain vectors of a time series being reconstructed in embedding phase space by various embedding dimensions, unknown points in a phase space being calculated using chaotic behavior and the prediction point of a phase space being recovered in the time domain.

L.E. is generally applied to analyze the sensitivity of the dependence on the initial state, which is commonly found in nonlinear systems exhibiting chaotic behavior. It is produced by an attractor reconstruction of the given system. The predictive data is obtained from the distances of different points in phase space and the L.E. As a result of the proposed algorithm being simulated for a restricted area, we can get the following results:

Firstly, the proposed algorithm gives an excellent performance compared to the previous Fuzzy compensative algorithm. Secondly, its algorithm is suitable for short-term forecasting of time series but makes more errors as time goes by because of the sensitivity to the initial state. The time horizon at which the prediction breaks down can be obtained using the L.E. Thirdly, the prediction error becomes smaller and more accurate for higher embedding dimensions.

To predict chaotic time series with accuracy, some intelligent algorithm such as Neural Network or Fuzzy Logic should be combined with the proposed algorithm. The simulation results of such a combined algorithm will be compared with the previous and proposed ones in subsequent papers.

REFERENCES

[1] C. E. Asburt, "Weather load model for electric demand and energy forecasting", *IEEE Trans. on Power Appr. and Sys.*, Vol. PAS-94, p.1111-1116, 1975.

[2] M. T. Hagan and S. M. Behr, "The time series approach to short-term load forecasting", paper 87. WM 044-1, *IEEE Power Engineering Society Winter Meeting*, New Orleans, LA, Feb. 1987.

[3] Gwan-Sik Yoo, Wee-Jae Shin, Yeon-Gyu Choo and Jung-Hwan Oh, "Choas Fuzzy Controller Design for Supplied Electrical Power Analyzer", *Journal of KICS*, Vol. 25 No.12, p.247-253, 2000.

[4] Heinz-Otto Peitgen and Dietmar Saupe, *Chaos and Fractals: New Frontiers of Science*, Springer-Verlag, p.745-753, 1992.

[5] Steven H. Strogatz, *Nonlinear Dynamics and Chaos*, Addison Wesley, p.320-323, 1996.

[6] In-Taek Kim, "Pattern Recognition of Chaotic Signal", *Proceeding of IEEK Spring Conference*, Vol. 3, p.107-111, 1993.

[7] F. Takens, "Detecting strange attractors in turbulence", *Dynamical systems and Turbulence*, Volume: 898 Issue: 1, p.366-381, 1981.

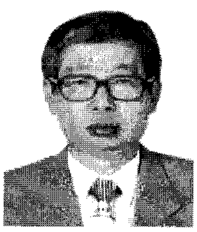
[8] Wolf Alan, Swift Jack B, Swinney Harry L. and Vastano, John A, "Determining Lyapunov exponents from a time series", *Physica D. Nonlinear Phenomena*, Vol. 16 Issue 3, p. 285-317, 1985.

[9] Jun Zhang and K.C. Lam, "Time series prediction using Lyapunov exponents in embedding phase space", *CEE Vol. 30*, pp.4-7, 2004

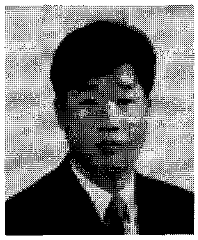
[10] G. L. Baker and J. P. Gollub, *Chaotic Dynamics*, Cambridge University Press, p.150-163, 1996.

[11] Bok-Ryeon Ha, "A Study on Supplied Forecasting of Short-Term Maximum Electrical Power using Fuzzy Compensative Algorithm", Master's thesis, Gyeongsang National University, 2004.

Jae-Hyeon Park Received B.S. and M.S. degree of Electronic Engineering in 1995, 1999 from Donga University and finished Ph.D. degree of Electric Engineering in 2004 from Gyeongsang National University. His research interests are Chaos, Artificial Intelligent and Digital Signal Processing.



Young-Il Kim Received B.S. degree of Electronic Engineering in 1979 from Kyung book National University and M.S. and Ph. D. degrees of Electronics Engineering in 1981 and 1985 respectively, from Yonsei University. He joined Gyeongsang National University in Jinju, Korea from 1987, where is currently an professor and an researcher in the Research Center of Engineering. His research interests are Digital Signal Processing, Speech Signal Processing.



Yeon-Gyu Choo Received B.S. degree of Electronic Engineering in 1988 from Pukyong National University. and M.S. and Ph.D. degrees of Electronic Engineering in 1991 and 1997 respectively, from Donga University. In 1997, he joined Gyeongnam National University of Science and Technology in Jinju, Korea, where is currently an professor and his research interests are in Intelligent Control System, Fuzzy Control System.