Comparison of Three Modeling Methods for Identifying Unknown Magnetization of Ferromagnetic Thin Plate

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Abstract – This study presents three different magnetization models for identifying unknown magnetization of the ferromagnetic thin plate of a ship. First, the forward problem should be solved to accurately predict outboard magnetic fields due to the magnetization distribution estimated at a certain time. To achieve this, three different modeling methods for representing remanent magnetization (i.e., magnetic charge method, magnetic dipole array method, and magnetic moment method) were utilized. Material sensitivity formulas containing the first-order gradient information of an objective function were then adopted for an efficient search of an optimum magnetization distribution on the hull. The validity of the proposed methods was tested with a scale model ship, and field signals predicted from the three different models were thoroughly investigated with reference to the experimental data.

Keywords: Magnetic fields, Inverse problems, Magnetization, Material sensitivity

1. Introduction

Most surface ships are constructed using ferromagnetic material. When a ship is placed in the earth's magnetic field, the local anomaly of the underwater magnetic fields is created by induced and remanent magnetization on the hull. Field calculation due to induced magnetization is easy to compute. On the other hand, field calculation due to remanent magnetization depends on the magnetic history of the hull, such as mechanical or thermal stress; accordingly, this falls under an inverse problem because there is lack of information about such history. It is usually difficult to predict the field disturbance around the ship due to remanent magnetization [1–6]. To date, a few attempts based on the Tikhonov's regularization have been made [1–3]. However, these methods require a very careful selection of the regularization parameter and the sensor position, and also consume considerable computation time, especially for three-dimensional (3D) inverse problems.

To address the aforementioned defects, this study presents a material sensitivity analysis in conjunction with three different modeling methods for unknown magnetization of the ferromagnetic hull. For dealing with unknown magnetization, an inverse problem consisting of the forward and backward problems should be solved with an iterative calculation algorithm. The forward problem is needed for accurately predicting outboard magnetic fields due to the magnetization distribution obtained at each iterative calculation stage. On the other hand, the backward problem corresponds to a kind of optimization problem to search for an optimum magnetization distribution on the hull in the end.

The three modeling methods for remanent magnetization, namely magnetic charge method (MCM), magnetic dipole array method (MDM), and magnetic moment method (MMM), were utilized for the forward problem. MCM [4], MDM [6], and MMM represent remanent magnetization by means of an equivalent magnetic charge, magnetic dipole array, and magnetic moment, respectively. To search quickly for an optimum magnetization distribution on the hull, material sensitivity formulas containing the first-order gradient information of an objective function with respect to each equivalent magnetization source were introduced [4–6]. The feature of the proposed optimization technique is that it can provide a stable and accurate solution even with only a few outboard sensors, instead of the many onboard sensors used in [5].

Finally, the validity of the proposed methods was tested with a scale model ship, and field signals predicted from the three equivalent magnetization models were thoroughly investigated with reference to the experimental data.

2. Modeling of the Forward Problem

When a ferromagnetic hull is placed in the earth's magnetic field $\mathbf{H}_o$, a local perturbation of the field $\mathbf{H}_{red}$ expressed in terms of the magnetic reduced potential ϕ_m is created around a ship.

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$$\mathbf{H} = \mathbf{H}_o + \mathbf{H}_{red} = \mathbf{H}_o - \nabla \phi_m \qquad (1)$$

The field $\mathbf{H}_{red}$ is caused by the hull magnetization $\mathbf{M}$ being equal to the sum of the induced magnetization $\mathbf{M}_{ind}$ and the remanent one $\mathbf{M}_{rem}$.

$$\mathbf{M} = \mathbf{M}_{ind} + \mathbf{M}_{rem} \qquad (2)$$

If the thickness of the hull is relatively small and the permeability is high, the direction of the magnetization can be considered parallel to the hull surface and its magnitude can be assumed to be constant on a divided surface element [1, 2].

This study focuses only on the field anomaly due to remanent magnetization. This is because the field perturbation of induced magnetization can be calculated easily. The constitutive relation between **B** and **H** written as follows:

$$\mathbf{B} = \mu_o (\mathbf{H} + \mathbf{M}_{rem}). \qquad (3)$$

2.1 Magnetic charge modeling

In MCM, the differential field equation in the form of Poisson's equation is obtained by utilizing the solenoidal property of **B** in (3) combined with (1) and (2), except $\mathbf{M}_{ind}$, as shown below:

$$\nabla^2 \phi_m = \nabla \cdot \mathbf{M}_{rem} = \sigma_{rem} \qquad (4)$$

where σ_{rem} is the magnetic charge equivalent to the remanent magnetization distributed on the hull. For the numerical implementation of MCM, the hull is divided into sheet elements, as shown in Fig. 1(a), and the magnetic line charge is determined at each edge of the sheet elements, as shown in Fig. 1(b).

A local field perturbation caused by the equivalent magnetic charge distribution is then expressed by the following:

$$\mathbf{H}_{red} = \frac{t}{4\pi} \int_S \sigma_{rem} \frac{\mathbf{r}}{r^3} dl \qquad (5)$$

where t is the thickness of the hull, l is the edge length of each element, S is the whole hull surface, and **r** means the displacement vector from the magnetic charge to the observation point [3].

2.2 Modeling of equivalent magnetic dipole array and magnetic moment

MDM and MMM differ by the way they model the hull. In both methods, the unknowns are the magnetization with 3D components in 3D coordinate systems. In the case of
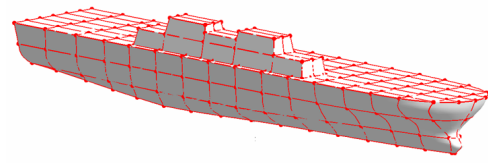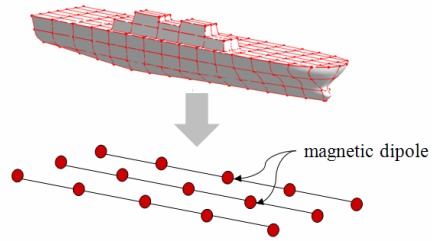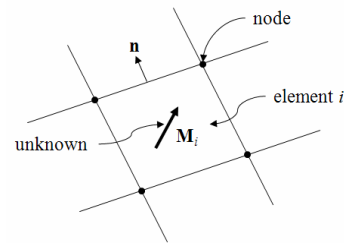

(a) Sheet elements
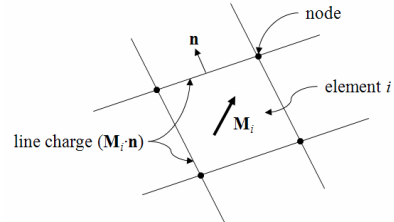
(b) Equivalent magnetic line charge

Fig. 1. Modeling of a ship hull and MCM

(a) Magnetic dipole array

(b) Equivalent magnetic moment

Fig. 2. Modeling of MDM and MMM

MDM, the equivalent magnetic dipole array, as shown in Fig. 2(a) where each circle denotes a magnetic dipole, is introduced instead of representing the hull itself. This leads to the minimization of the number of system unknowns in the forward problem being considered. The underwater field anomaly due to remanent magnetization is then calculated from the values of the equivalent dipole moment obtained. Meanwhile, like MCM, MMM requires sheet elements forming the hull surface; however, the equivalent magnetic moment vectors are sought at each center of the elements, as shown in $\mathbf{H} = \mathbf{H}_o + \mathbf{H}_{red} = \mathbf{H}_o - \nabla \phi_m$ Fig. 2(b).

If the values of the dipole moment in the array or the magnetic moment distribution on the hull have been decided, the field perturbation due to remanent magnetization is calculated by the below equation:

$$\mathbf{H}_{red} = \frac{1}{4\pi} \int_V \frac{3(\mathbf{M}_{rem} \cdot \mathbf{r})\mathbf{r} - r^2 \mathbf{M}_{rem}}{r^5} dV$$
$$= -\frac{1}{4\pi} \int_S \mathbf{M}_{rem} \cdot \mathbf{n} \frac{r}{r^3} dS \qquad (6)$$

where V is the volume enclosed by the hull and $\mathbf{n}$ is the unit vector normal to the hull surface of V [2].

3. Methodology for Solving the Backward Problem

To deal with the backward problem efficiently, two material sensitivity formulas based on the concept of the dual system are introduced. It will be shown that the two sensitivity formulas can be dealt with using the same program architecture with only minor modifications to the optimization code.

3.1 Two material sensitivity formulas

The derivation of a material sensitivity formula always starts from the variational formulation of Maxwell's equations like (4) referred to as the primary system. To obtain the analytical sensitivity formula, relatively complicated mathematical expansions are needed, but they follow a fairly routine procedure, as presented in [5–8], where the augmented Lagrangian method, material derivative concept, and adjoint variable method are exploited. An analytical formula facilitates the calculation of the first-order gradient information of an objective function with respect to system parameters, such as the magnetic charge and the magnetic dipole moment. It can also save a lot of computing time in finding an optimum solution, particularly as the number of system parameters increases. To utilize these advantages, analytical material sensitivity formulas for magnetostatic inverse problems have been developed; accordingly, mathematical procedures have been explained in previous articles [4–6].

Therefore, in this study, the final mathematical expressions of two material sensitivity formulas for an objective function F with respect to the magnetic charge and the magnetic dipole moment are briefly compared with each other. Depending on which kind of system parameter p is selected, the final sensitivity expressions will have slightly different forms, as shown in (7) and (8).

$$\frac{dF}{dp} = \int_S (-\frac{\partial \sigma_{rem}}{\partial p}) \lambda \ dS \qquad (7)$$

$$\frac{dF}{d\mathbf{p}} = \int_V \frac{\partial \mathbf{M}_{rem}}{\partial \mathbf{p}} \cdot (\nabla \times \lambda) \, dV \qquad (8)$$

where λ is the solution of the adjoint system, which is the counterpart of the primary system. In Eqs. (7) and (8), the italic and bold characters refer to the scalar and vector quantities, respectively. It can be seen that the integrands consisting of the sensitivity coefficients with respect to system parameters have very similar forms. The difference is that the surface integral is needed for the magnetic charge, whereas the volume integral is required for the magnetic dipole moment.

3.2 Numerical implementation

Using the analytical formulas (7) and (8), the first-order gradient information of an objective function with respect to system parameters can be easily calculated from the field solutions, which are obtained in the dual system consisting of the primary and adjoint systems. A program architecture applicable to the two material sensitivity formulas is presented in Fig. 3.
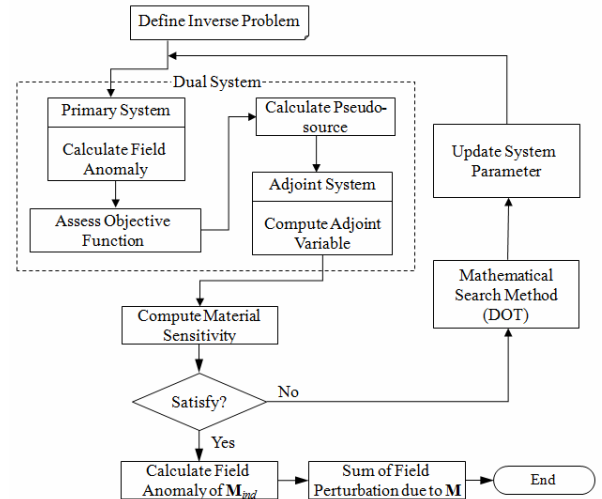

Fig. 3. Program architecture for two-material sensitivity formula

To simplify the numerical implementation, the system parameter, σ_{rem} or $\mathbf{M}_{rem}$, is forced to be a linear function of the system parameter $\mathbf{p}$ [7]. A general-purpose optimization control program, DOT, based on the Broydon-Fletcher-Goldfarb-Shanno (BFGS) algorithm in [9] is adopted to accelerate the convergence of the objective function. Under the proposed program architecture, the iterative calculation process for obtaining optimum dipole moment values involves the following steps:

Step 1: Definition of an objective function with target field data.

Step 2: Calculation of the field perturbation due to the remanent magnetization of the hull with (6) at selected observation points.

Step 3: Assessment of the objective function and then calculation of the adjoint source at the observation points.

Step 4: Computation of the adjoint variable λ and the material sensitivity value with (8).

Step 5: Updating of the magnetic dipole moment values

$\mathbf{M}_{rem}$ ($\mathbf{M}_{rem,j}^{k+1} = \mathbf{M}_{rem,j}^{k} + \alpha \Delta p$) at the kth iteration, where j and α are the jth sheet element and the relaxation factor, respectively.

Step 6: Checking of the convergence and repeating step 2 if the result is unsatisfactory.

Step 7: Calculation of the field anomaly of $\mathbf{M}_{ind}$ and then summing up of the total field perturbation due to $\mathbf{M}$ of (1).

4. Results

4.1 Experimental setup

For a comparison of the three different methods for modeling unknown magnetization of the hull, a scale model ship made out of steel plate with upper deck thickness of 0.3 mm, lower deck thickness of 0.6 mm, and relative permeability of 160 is considered, as shown in Fig. 4. The dimensions of the mock-up are 2.4 m long, 0.5 m wide, and 0.25 m high. The magnetic signature generated by the hull under the earth's magnetic field was measured with four tri-axial magnetic sensors located under the keel line of the ship, as shown in Fig. 5. Instead of using many sensors, the experiment was executed as the ship slowly moves along the x axis heading for the north magnetic pole against the sensor at a standstill. The x and z components of the earth's magnetic field at an experimental station are 307 mG and 377 mG, respectively.

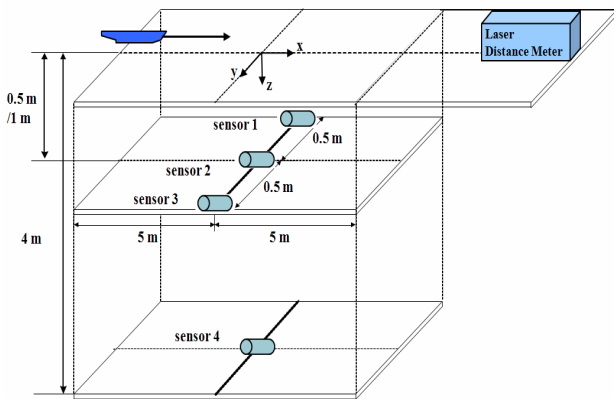**Fig. 4.** A photograph of a scale model ship

Fig. 5. Experimental setup

The field components due to remanent magnetization were taken by subtracting the induction field due to induced magnetization from the measured data. For this process, the induction field was computed using a commercial EM software package, MagNet 6 [10].

4.2 Comparison of three modeling methods

To predict the field anomaly due to the magnetization distributed over the ferromagnetic hull, the hull surface shown in Fig. 1(a) was divided into 322 sheet elements, where the optimum distributions of equivalent magnetic charges and the magnetization moments are determined in the cases of MCM and MMM, respectively. Meanwhile, instead of the hull modeling, the magnetic dipole array consisting of 15 dipoles, as shown in Fig. 2(a), was used for MDM. Each dipole is assumed to have a volume of a cubic meter, where an optimum dipole moment value is obtained.

One objective function for the three methods was defined by (10) at a reference depth of 1 m under the keel line depicted in Fig. 5.

$$F = \sum_{i=1}^{n} \sum_{j=1}^{3} (B_{ij}^{k} - B_{oij})^2 \qquad (10)$$

where n is the total observation points of 101, B_{ij}^{k} is the jth component of the magnetic flux density predicted at the ith observation point for the kth iterative calculation stage, and B_{oij} is the target fields measured at the observation points.

According to the unified program architecture presented in Fig. 3, the total field anomaly due to the hull's magnetization, $\mathbf{M}_{ind}$ and $\mathbf{M}_{rem}$, was computed along three different depths of 0.5 m, 1 m, and 4 m. After solving the inverse problems, the optimum distributions of the equivalent magnetic charges and equivalent magnetization moment vectors over the hull are illustrated in Fig. 6, where the direction and magnitude of the magnetization vector is expressed in terms of the arrows and color contours.

Fig. 7 shows a comparison of the predicted and measured fields at the depths of 1 m and 4 m apart from the keel line. The predicted fields in Fig. 7(b) were calculated by exploiting the optimum solutions obtained from solving the inverse problems based on the three different modeling methods. From the results, it can be concluded that two methods – MMM and MDM – show a good agreement with the measured data, while MCM yields a slightly different field solutions in the z component of B, as shown in Fig. 7(b). Meanwhile, the accuracy of the methods is also examined near the bottom of the ship. Fig. 8 presents the comparison of the predicted fields to the experimental data at the depth of 0.5 m. It can be noticed that MDM

(a) Equivalent magnetic charge

(b) Magnetization moment
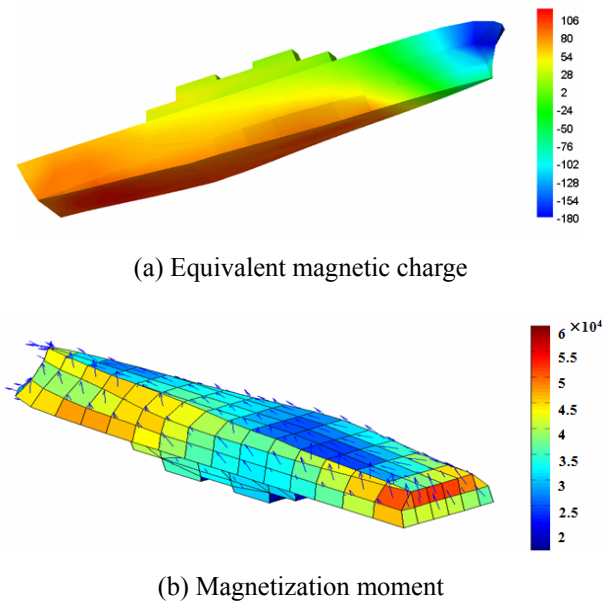
Fig. 6. Optimum distributions of equivalent magnetic charge and magnetization moment obtained after solving the inverse problems
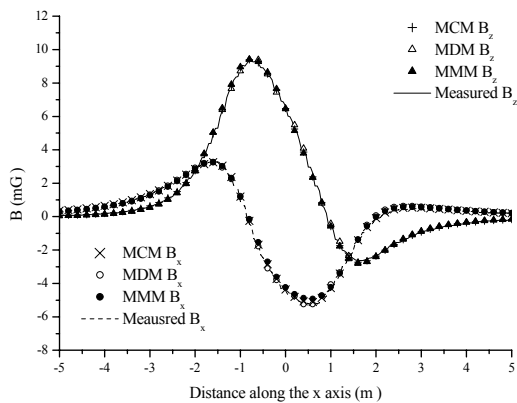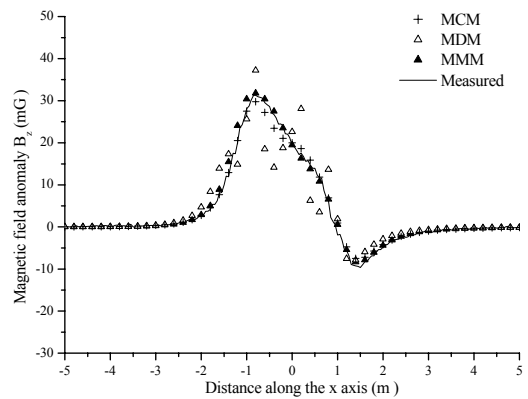

(a) 1 m in depth
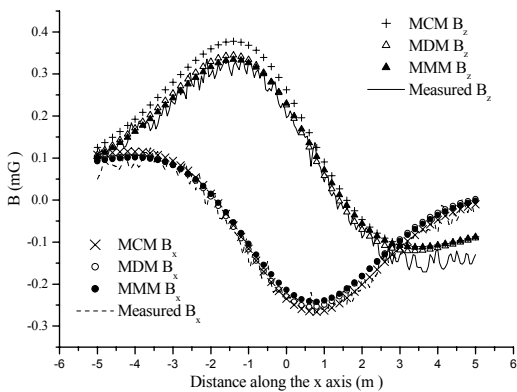
(b) 4 m in depth

Fig. 7. Comparison of the field anomaly predicted by two different methods: MDM and MCM with the measured field at different depths apart from the keel line

includes relatively large errors in the field solutions compared to MMM and MCM. Particularly in the case of MDM, it is observed that an undesirable numerical oscillation occurs in the predicted fields. This oscillation is inferred to be caused by the small number of the equivalent magnetic dipoles used for the field prediction.

The features of the three methods for predicting the underwater field anomaly around the ship due to the ferromagnetic hull are compared in Table 1. It is concluded that MMM produces a stable and reliable field solution along the three different depths compared to the others. In addition, when the proposed methodology is applied to solving the inverse problem, the computing time in finding the optimal solutions do not strongly depend on the number of system parameters.

Fig. 8. Comparison of the field anomaly predicted by two different methods: MDM and MCM with the measured field at 0.5 m under the keel line

Table 1. Comparison between the three different methods for predicting field anomaly due to ferromagnetic hull

	MCM	MDM	MMM
Unknown	Magnetic charge	Magnetic dipole	Magnetic dipole
Number of unknowns	623	45	966
Ship modeling technique	Relatively hard (3D)	Easy (2D)	Relatively hard (3D)
Total iterations	28	16	18
Computation time required for finding an optimum solution	30 seconds	28 seconds	37 seconds
Field error in depth			
0.5 m	12.40%	53.33 %	8.47 %
1 m	0.18%	0.73 %	0.51 %
4 m	20.64%	10.24 %	6.46 %

Normalized field error is computed as $|Pred\text{-}Meas|/Meas$ around the maximum field values, where *Pred* stands for the predicted field and *Meas* for the measured one.

5. Conclusion

This paper presents the three different modeling methods for identifying unknown magnetization of the ferromagnetic thin plate of a ship, namely MCM, MDM, and MMM. The most efficient methodology based on the material sensitivity formula has been successfully applied to the inverse problem. For the comparison between the equivalent magnetization models, the field signals predicted were thoroughly investigated with reference to the experimental data. MMM gives preferable features in terms of the accuracy of field solutions and the computing time required for obtaining an optimum magnetization distribution on the hull.

Acknowledgments

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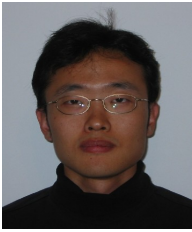
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