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FUZZY STRONGLY (r, s)-PREOPEN AND PRECLOSED MAPPINGS

Seok Jong Lee and Jin Tae Kim

Abstract. In this paper, we introduce the notions of fuzzy strongly (r, s)-preopen and preclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense. The relationships among fuzzy (r, s)-open, fuzzy strongly (r, s)-semiopen, fuzzy (r, s)-preopen, and fuzzy strongly (r, s)-preopen mappings are discussed. The characterizations for the fuzzy strongly (r, s)-preopen and preclosed mappings are obtained.

1. Introduction and preliminaries

The concept of fuzzy set was introduced by Zadeh [16]. Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [15], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [3], and by Ramadan [14].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Çoker and his colleagues [4, 6] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [5] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. Biljana Krsteska [8] introduced the concepts of fuzzy strongly preopen and preclosed mappings on Chang's fuzzy topological spaces and Yong Chan Kim and his colleagues [7] considered this concepts on smooth topological spaces.

In this paper, we introduce the notions of fuzzy strongly (r, s)-preopen and preclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense. The relationships among fuzzy (r, s)-open, fuzzy strongly (r, s)-semiopen, fuzzy (r, s)-preopen, and fuzzy strongly (r, s)-preopen mappings are discussed. The

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characterizations for the fuzzy strongly (r,s)-preopen and preclosed mappings are obtained.

Let $I(X)$ be a family of all intuitionistic fuzzy sets in X and let $I \otimes I$ be the set of the pair (r,s) such that $r,s \in I$ and $r+s \leq 1$.

Definition 1.1 ([5])**.** Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense* (SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a mapping $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties:

(1) $\mathcal{T}_1(\underline{0}) = \mathcal{T}_1(\underline{1}) = 1$ and $\mathcal{T}_2(\underline{0}) = \mathcal{T}_2(\underline{1}) = 0$.
(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.
(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

The $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense* (SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

Definition 1.2 ([10])**.** Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then A is said to be

(1) *fuzzy strongly* (r,s)*-preopen* if $A \subseteq \mathrm{int}(\mathrm{pcl}(A,r,s),r,s)$,
(2) *fuzzy strongly* (r,s)*-preclosed* if $A \supseteq \mathrm{cl}(\mathrm{pint}(A,r,s),r,s)$.

Definition 1.3 ([10])**.** Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r,s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy strongly* (r,s)*-preinterior* is defined by

$$\mathrm{stpint}(A,r,s) = \bigcup\{B \in I(X) \mid B \subseteq A,\ B \text{ is fuzzy strongly } (r,s)\text{-preopen}\}$$

and the *fuzzy strongly* (r,s)*-preclosure* is defined by

$$\mathrm{stpcl}(A,r,s) = \bigcap\{B \in I(X) \mid A \subseteq B,\ B \text{ is fuzzy strongly } (r,s)\text{-preclosed}\}.$$

For the nonstandard definitions and notations we refer to [9, 10, 11, 12, 13].

2. Fuzzy strongly (r,s)-preopen and preclosed mappings

Now, we introduce the concepts of fuzzy strongly (r,s)-preopen and preclosed mappings, and then we investigate some of their characteristic properties.

Definition 2.1. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is called

(1) a *fuzzy strongly* (r,s)*-preopen* mapping if $f(A)$ is a fuzzy strongly (r,s)-preopen set in Y for each fuzzy (r,s)-open set A in X,
(2) a *fuzzy strongly* (r,s)*-preclosed* mapping if $f(A)$ is a fuzzy strongly (r,s)-preclosed set in Y for each fuzzy (r,s)-closed set A in X.

Remark 2.2. It is easy to see that the following implications are true:

(1) fuzzy (r,s)-open $\Rightarrow$ fuzzy strongly (r,s)-preopen.
(2) fuzzy strongly (r,s)-semiopen $\Rightarrow$ fuzzy strongly (r,s)-preopen.
(3) fuzzy strongly (r,s)-preopen $\Rightarrow$ fuzzy (r,s)-preopen.

However, the following examples show that all of the converses need not be true.

Example 2.3. Let $X = \{x, y, z\}$ and let A, B, and C be intuitionistic fuzzy sets in X defined as

$$A(x) = (0.8, 0.1),\ A(y) = (0.7, 0.2),\ A(z) = (0.6, 0.4);$$
$$B(x) = (0.3, 0.6),\ B(y) = (0.1, 0.8),\ B(z) = (0.7, 0.2);$$

and

$$C(x) = (0.8, 0.1),\ C(y) = (0.8, 0.1),\ C(z) = (0.4, 0.5).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(D) = (T_1(D), T_2(D)) = \begin{cases} (1,0) & \text{if } D = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } D = A, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(D) = (\mathcal{U}_1(D), \mathcal{U}_2(D)) = \begin{cases} (1,0) & \text{if } D = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } D = B, C, B \cap C, B \cup C, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $g : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $g(x) = x$, $g(y) = y$, and $g(z) = z$. Then g is fuzzy strongly $(\frac{1}{2}, \frac{1}{3})$-preopen, but g is neither fuzzy $(\frac{1}{2}, \frac{1}{3})$-open nor fuzzy strongly $(\frac{1}{2}, \frac{1}{3})$-semiopen.

Example 2.4. Let $X = \{x, y, z\}$ and let A and B be intuitionistic fuzzy sets in X defined as

$$A(x) = (0.3, 0.6),\ A(y) = (0.1, 0.8),\ A(z) = (0.7, 0.2);$$

and

$$B(x) = (0.8, 0.1),\ B(y) = (0.8, 0.1),\ B(z) = (0.4, 0.5).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(D) = (\mathcal{T}_1(D), \mathcal{T}_2(D)) = \begin{cases} (1,0) & \text{if } D = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } D = A, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(D) = (\mathcal{U}_1(D), \mathcal{U}_2(D)) = \begin{cases} (1,0) & \text{if } D = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } D = B, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $g : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $g(x) = x$, $g(y) = y$, and $g(z) = z$. Then g is fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen, but g is not a fuzzy strongly $(\frac{1}{2}, \frac{1}{3})$-preopen mapping.

Theorem 2.5. *Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent*:

(1) *f is a fuzzy strongly (r,s)-preopen mapping.*

(2) *For each intuitionistic fuzzy set A in X,*

$$f(\mathrm{int}(A,r,s)) \subseteq \mathrm{stpint}(f(A),r,s).$$

(3) *For each intuitionistic fuzzy set B in Y,*

$$\mathrm{int}(f^{-1}(B),r,s) \subseteq f^{-1}(\mathrm{stpint}(B,r,s)).$$

(4) *For each intuitionistic fuzzy set B in Y,*

$$f^{-1}(\mathrm{stpcl}(B,r,s)) \subseteq \mathrm{cl}(f^{-1}(B),r,s).$$

(5) *For each intuitionistic fuzzy set A in X,*

$$f(\mathrm{int}(A,r,s)) \subseteq \mathrm{int}(\mathrm{pcl}(f(A),r,s),r,s).$$

Proof. (1) $\Rightarrow$ (2) Let $A \in I(X)$. Then $\mathrm{int}(A,r,s)$ is a fuzzy (r,s)-open set in X. Since f is fuzzy strongly (r,s)-preopen, $f(\mathrm{int}(A,r,s))$ is a fuzzy strongly (r,s)-preopen set in Y. From the result $f(\mathrm{int}(A,r,s)) \subseteq f(A)$, we have

$$f(\mathrm{int}(A,r,s)) = \mathrm{stpint}(f(\mathrm{int}(A,r,s)),r,s) \subseteq \mathrm{stpint}(f(A),r,s).$$

(2) $\Rightarrow$ (3) Let $B \in I(Y)$. Then $f^{-1}(B) \in I(X)$. By (2), we obtain

$$f(\mathrm{int}(f^{-1}(B),r,s)) \subseteq \mathrm{stpint}(f(f^{-1}(B)),r,s) \subseteq \mathrm{stpint}(B,r,s).$$

Thus $\mathrm{int}(f^{-1}(B),r,s) \subseteq f^{-1}(\mathrm{stpint}(B,r,s))$.

(3) $\Rightarrow$ (1) Let A be a fuzzy (r,s)-open set in X. Then $f(A) \in I(Y)$. By (3), we have

$$\mathrm{int}(A,r,s) \subseteq \mathrm{int}(f^{-1}(f(A)),r,s) \subseteq f^{-1}(\mathrm{stpint}(f(A),r,s)).$$

Thus $f(A) = f(\mathrm{int}(A,r,s)) \subseteq \mathrm{stpint}(f(A),r,s) \subseteq f(A)$. Hence $f(A) = \mathrm{stpint}(f(A),r,s)$. Therefore f is a fuzzy strongly (r,s)-preopen mapping.

(3) $\Rightarrow$ (4) Let $B \in I(Y)$. By (3), we obtain

$$\mathrm{int}(f^{-1}(B^c),r,s) \subseteq f^{-1}(\mathrm{stpint}(B^c,r,s)).$$

Hence we have

$$\begin{aligned} f^{-1}(\mathrm{stpcl}(B,r,s)) &= (f^{-1}(\mathrm{stpint}(B^c,r,s)))^c \\ &\subseteq (\mathrm{int}(f^{-1}(B^c),r,s))^c = \mathrm{cl}(f^{-1}(B),r,s). \end{aligned}$$

(4) $\Rightarrow$ (3) Let $B \in I(Y)$. By (4), we obtain

$$f^{-1}(\mathrm{stpcl}(B^c,r,s)) \subseteq \mathrm{cl}(f^{-1}(B^c),r,s).$$

Thus we have

$$\begin{aligned} \mathrm{int}(f^{-1}(B),r,s) &= (\mathrm{cl}(f^{-1}(B^c),r,s))^c \\ &\subseteq (f^{-1}(\mathrm{stpcl}(B^c,r,s)))^c = f^{-1}(\mathrm{stpint}(B,r,s)). \end{aligned}$$

(1) $\Rightarrow$ (5) Suppose that f is fuzzy strongly (r,s)-preopen. Then $f(\mathrm{int}(A,r,s))$ is a fuzzy strongly (r,s)-preopen set in Y. Hence

$$\begin{aligned} f(\mathrm{int}(A,r,s)) &\subseteq \mathrm{int}(\mathrm{pcl}(f(\mathrm{int}(A,r,s)),r,s),r,s) \\ &\subseteq \mathrm{int}(\mathrm{pcl}(f(A),r,s),r,s). \end{aligned}$$

(5) $\Rightarrow$ (1) Let A be a fuzzy (r,s)-open set in X. Then by hypothesis, $f(A) = f(\text{int}(A,r,s)) \subseteq \text{int}(\text{pcl}(f(A),r,s),r,s)$. Thus $f(A)$ is a fuzzy strongly (r,s)-preopen set in Y. Therefore f is fuzzy strongly (r,s)-preopen. □

Theorem 2.6. *Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then the following statements are equivalent*:

(1) *f is fuzzy strongly (r,s)-preclosed.*

(2) *For each intuitionistic fuzzy set A in X,*

$$\text{stpcl}(f(A),r,s) \subseteq f(\text{cl}(A,r,s)).$$

(3) *For each intuitionistic fuzzy set A in X,*

$$\text{cl}(\text{pint}(f(A),r,s),r,s) \subseteq f(\text{cl}(A,r,s)).$$

Proof. (1) $\Rightarrow$ (2) Suppose that f is fuzzy strongly (r,s)-preclosed. Let $A \in I(X)$. Then $f(\text{cl}(A,r,s))$ is a fuzzy strongly (r,s)-preclosed set in Y. Since $f(A) \subseteq f(\text{cl}(A,r,s))$, we have

$$\text{stpcl}(f(A),r,s) \subseteq \text{stpcl}(f(\text{cl}(A,r,s)),r,s) = f(\text{cl}(A,r,s)).$$

(2) $\Rightarrow$ (1) Let A be a fuzzy (r,s)-closed set in X. According to the assumption,

$$f(A) \subseteq \text{stpcl}(f(A),r,s) \subseteq f(\text{cl}(A,r,s)) = f(A).$$

Thus $f(A) = \text{stpcl}(f(A),r,s)$. Hence $f(A)$ is fuzzy strongly (r,s)-preclosed in Y. Therefore f is a fuzzy strongly (r,s)-preclosed mapping.

(1) $\Rightarrow$ (3) Let f be fuzzy strongly (r,s)-preclosed. Then $f(\text{cl}(A,r,s))$ is fuzzy strongly (r,s)-preclosed in Y. Hence

$$\begin{aligned} \text{cl}(\text{pint}(f(A),r,s),r,s) &\subseteq \text{cl}(\text{pint}(f(\text{cl}(A,r,s)),r,s),r,s) \\ &\subseteq f(\text{cl}(A,r,s)). \end{aligned}$$

(3) $\Rightarrow$ (1) Let A be a fuzzy (r,s)-closed set in X. According to the assumption, we obtain

$$\text{cl}(\text{pint}(f(A),r,s),r,s) \subseteq f(\text{cl}(A,r,s)) = f(A).$$

Hence $f(A)$ is fuzzy strongly (r,s)-preclosed in Y. Therefore f is a fuzzy strongly (r,s)-preclosed mapping. □

Theorem 2.7. *Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a bijective mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is fuzzy strongly (r,s)-preopen if and only if it is fuzzy strongly (r,s)-preclosed.*

Proof. Straightforward. □

Theorem 2.8. *Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then the following statements are true*:

(1) *If $f(\text{int}(\text{pcl}(A,r,s),r,s)) \subseteq \text{int}(\text{pcl}(f(A),r,s),r,s)$ for each fuzzy (r,s)-open set A in X, then f is a fuzzy strongly (r,s)-preopen mapping.*

(2) *If* $f(\mathrm{cl}(\mathrm{pint}(A, r, s), r, s)) \supseteq \mathrm{cl}(\mathrm{pint}(f(A), r, s), r, s)$ *for each fuzzy* (r, s)*-closed set* A *in* X*, then* f *is a fuzzy strongly* (r, s)*-preclosed mapping.*

Proof. (1) Let A be a fuzzy (r, s)-open set in X. Then

$$A \subseteq \mathrm{int}(\mathrm{pcl}(A, r, s), r, s).$$

By hypothesis, we have

$$f(A) \subseteq f(\mathrm{int}(\mathrm{pcl}(A, r, s), r, s)) \subseteq \mathrm{int}(\mathrm{pcl}(f(A), r, s), r, s),$$

and hence $f(A)$ is a fuzzy strongly (r, s)-preopen set in Y. Thus f is a fuzzy strongly (r, s)-preopen mapping.

(2) Similar to (1). □

Theorem 2.9. *Let* $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ *be a mapping from a SoIFTS* X *to a SoIFTS* Y *and* $(r, s) \in I \otimes I$*. Then* f *is fuzzy strongly* (r, s)*-preopen if and only if for each intuitionistic fuzzy set* B *in* Y *and each fuzzy* (r, s)*-closed set* A *in* X *with* $f^{-1}(B) \subseteq A$*, there is a fuzzy strongly* (r, s)*-preclosed set* C *in* Y *such that* $B \subseteq C$ *and* $f^{-1}(C) \subseteq A$*.*

Proof. Let B be any intuitionistic fuzzy set in Y and A a fuzzy (r, s)-closed set in X with $f^{-1}(B) \subseteq A$. Then $A^c \subseteq f^{-1}(B^c)$, and hence $f(A^c) \subseteq f(f^{-1}(B^c)) \subseteq B^c$. Since f is fuzzy strongly (r, s)-preopen and A^c is fuzzy (r, s)-open, we have $f(A^c) \subseteq \mathrm{stpint}(B^c, r, s)$. Thus $A^c \subseteq f^{-1}(f(A^c)) \subseteq f^{-1}(\mathrm{stpint}(B^c, r, s))$. Hence $A \supseteq f^{-1}(\mathrm{stpcl}(B, r, s))$. Let $C = \mathrm{stpcl}(B, r, s)$. Then C is fuzzy strongly (r, s)-preclosed in Y such that $B \subseteq C$ and $f^{-1}(C) \subseteq A$.

Conversely, let A be a fuzzy (r, s)-open set in X. Then $A^c \supseteq (f^{-1}(f(A)))^c = f^{-1}(f(A)^c)$, where A^c is fuzzy (r, s)-closed. According to the assumption, there is a fuzzy strongly (r, s)-preclosed set B in Y such that $f(A)^c \subseteq B$ and $f^{-1}(B) \subseteq A^c$. From $f(A)^c \subseteq B$ follows $\mathrm{stpcl}(f(A)^c, r, s) \subseteq \mathrm{stpcl}(B, r, s) = B$, and hence $B^c \subseteq \mathrm{stpcl}(f(A)^c, r, s)^c = \mathrm{stpint}(f(A), r, s)$. Since $f^{-1}(B) \subseteq A^c$, we have $f^{-1}(B^c) \supseteq A$, so $B^c \supseteq f(f^{-1}(B^c)) \supseteq f(A)$. Thus $f(A) = \mathrm{stpint}(f(A), r, s)$. Hence $f(A)$ is a fuzzy strongly (r, s)-preopen set in Y. Therefore f is fuzzy strongly (r, s)-preopen. □

Corollary 2.10. *Let* $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ *be a mapping from a SoIFTS* X *to a SoIFTS* Y *and* $(r, s) \in I \otimes I$*. If* f *is a fuzzy strongly* (r, s)*-preopen mapping, then for each intuitionistic fuzzy set* B *in* Y*,*

$$f^{-1}(\mathrm{cl}(\mathrm{pint}(B, r, s), r, s)) \subseteq \mathrm{cl}(f^{-1}(B), r, s).$$

Proof. Let B be an intuitionistic fuzzy set in Y. Then $\mathrm{cl}(f^{-1}(B), r, s)$ is fuzzy (r, s)-closed in X containing $f^{-1}(B)$. By Theorem 2.9, there is a fuzzy strongly (r, s)-preclosed set C in Y such that $B \subseteq C$ and $f^{-1}(C) \subseteq \mathrm{cl}(f^{-1}(B), r, s)$. Hence

$$\begin{aligned} f^{-1}(\mathrm{cl}(\mathrm{pint}(B, r, s), r, s)) &\subseteq f^{-1}(\mathrm{cl}(\mathrm{pint}(C, r, s), r, s)) \\ &\subseteq f^{-1}(C) \subseteq \mathrm{cl}(f^{-1}(B), r, s). \end{aligned}$$

□

Theorem 2.11. *Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ and $g : (Y, \mathcal{U}) \to (Z, \mathcal{S})$ be mappings and $(r, s) \in I \otimes I$. If g is fuzzy strongly (r, s)-preopen and f is fuzzy (r, s)-open, then $g \circ f$ is fuzzy strongly (r, s)-preopen.*

Proof. Straightforward. □

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Seok Jong Lee
Department of Mathematics
Chungbuk National University
Cheongju 361-763, Korea
E-mail address: sjl@cbnu.ac.kr

Jin Tae Kim
Department of Mathematics
Chungbuk National University
Cheongju 361-763, Korea
E-mail address: kjtmath@hanmail.net