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A New Augmented Lyapunov Functional Approach to Robust Stability Criteria for Uncertain Fuzzy Neural Networks with Time-varying Delays

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Abstract - This paper proposes new delay-dependent robust stability criteria for neural networks with time-varying delays. By construction of a suitable Lyapunov-Krasovskii's (L-K) functional and use of Finsler's lemma, new stability criteria for the networks are established in terms of linear matrix inequalities (LMIs) which can be easily solved by various effective optimization algorithms. Two numerical examples are given to illustrate the effectiveness of the proposed methods.

Key Words : Neural networks, Takagi-Sugeno systems, Time-varying delay, Stability, Lyapunov method, LMI

1. Introduction

Neural networks (NNs) have been received considerable attentions due to their extensive applications such as combinatorial optimization, signal processing, pattern recognition, associate memory, knowledge acquisition, and so on [1]-[14]. In the implementation of neural networks, there exists time-delay due to the finite speed of information processing. It is well known that time-delay often causes undesirable oscillation, poor performance, and instability of the networks. Since the stability issue is a prerequisite to the applications of NNs, various approach to stability criteria for NNs with time-delay have been investigated in the literature [5]-[8] and references therein.

The stability criteria for systems including time delays can be classified into two categories according to their dependence on the size of the time-delay: delay-dependent ones and delay-independent ones. Since the delay-dependent stability criteria include the information on the size of delay, they are generally less conservative than the delay-independent ones particularly when the size of time-delay is small. Therefore, a great number of results on delay-dependent stability conditions for time-delay systems have been reported in the literature

[9]-[11].

On the other hand, the well-known Takagi-Sugeno (T-S) fuzzy model [12] is recognized as an efficient tool in approximating a complex nonlinear system. The T-S fuzzy modeling is a multi-model approach in which some linear models are blended into an overall single model through nonlinear membership functions to represent the nonlinear dynamics. Based on the T-S fuzzy model, the stability conditions for fuzzy systems with time-delay are addressed in [13]-[16]. In [13], by a simple analytic method, a sufficient condition for asymptotic stability of uncertain fuzzy systems with interval time-varying delay was expressed in the form of LMIs. By considering ignored terms in estimating upper bounds of the derivative of L-K functional, a further improved delay-dependent stability conditions for T-S fuzzy systems with time-varying delays were presented in [14]. By using of the theory of topological degree and applying the properties of nonsingular M-matrix, sufficient conditions for the existence, uniqueness and global asymptotic stability of equilibrium point have been established for fuzzy cellular NNs with time-varying delays [15]. In [16], the problems of robust stability for T-S fuzzy Hopfield NNs with time-varying delay based on a delay decomposition approach were considered.

In this paper, we propose new delay-dependent stability criteria for uncertain fuzzy NNs with time-varying delays. By constructing a suitable L-K functional and utilizing Finsler's lemma with no free-weighting matrices technique, new stability criteria are derived in terms of LMIs which will be driven in Theorem 1. And based on the results of Theorem 1, a further improved stability criterion, Theorem 2, is proposed by adding

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triple-integral term with free-weighting matrices. Two numerical examples are included to show the effectiveness of the proposed method.

Notation: \mathbf{R}^n is the n -dimensional Euclidean space, $\mathbf{R}^{m \times n}$ denotes the set of m by n real matrix. $\mathbf{C}_{n,h} = \mathbf{C}([-h, \mathbf{R}^n])$ denotes the Banach space of continuous functions mapping the interval $[-h, 0]$ into \mathbf{R}^n , with the topology of uniform convergence. For symmetric matrices X and Y , the notation $X > Y$ (respectively, $X \geq Y$) means that the matrix $X - Y$ is positive definite (respectively, nonnegative). I and 0 denote the identity matrix and zero matrix with appropriate dimensions. $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. $diag\{\dots\}$ denotes the block diagonal matrix. \star represents the elements below the main diagonal of a symmetric matrix. For a given matrix $X \in \mathbf{R}^{m \times n}$, such that $rank(X) = r$, we define $X^\perp \in \mathbf{R}^{n \times (n-r)}$ as the right orthogonal complement of X ; i.e., $XX^\perp = 0$.

2. Problem Statements

Consider the following uncertain NNs with time-varying delays

$$\dot{y}(t) = -(A + \Delta A(t))y(t) + (W_0 + \Delta W_0(t))g(y(t)) + (W_1 + \Delta W_1(t))g(y(t-h(t))) + b, \quad (1)$$

where $y(t) = [y_1(t), \dots, y_n(t)]^T \in \mathbf{R}^n$ is the neuron state vector, n denotes the number of neurons in a neural network, $g(\cdot) = [g_1(\cdot), \dots, g_n(\cdot)]^T \in \mathbf{R}^n$ denotes the neuron activation function vector with initial condition $g(0) = 0$, $A = diag\{a_i\} \in \mathbf{R}^{n \times n}$ is a diagonal matrix, $W_0 = (w_{ij}^0) \in \mathbf{R}^{n \times n}$ and $W_1 = (w_{ij}^1) \in \mathbf{R}^{n \times n}$ are the interconnection matrices representing the weight coefficients of the neurons, $b = [b_1, \dots, b_n]^T \in \mathbf{R}^n$ means the external bias at time t , $\Delta A(t)$, $\Delta W_0(t)$ and $\Delta W_1(t)$ are the parametric uncertainties of system matrices, and $h(t)$ are the time-varying delays satisfying

$$0 \leq h(t) \leq h_U, \quad \dot{h}(t) \leq h_D, \quad (2)$$

where h_U is a positive scalar and h_D is a any constant one.

In this paper, it is assumed that the activation functions satisfy the following condition:

Assumption 1. The neurons activation functions $g_i(\cdot)$ are assumed to be non-decreasing, bounded and globally Lipschitz; that is

$$k_i^- \leq \frac{g_i(\xi_i) - g_i(\xi_j)}{\xi_i - \xi_j} \leq k_i^+, \quad \xi_i, \xi_j \in \mathbf{R}, \quad \xi_i \neq \xi_j, \quad i = 1, \dots, n, \quad (3)$$

where k_i^- and k_i^+ are constant values.

It should be noted that by using the Brouwer's fixed-point theorem, it can be easily proved that there exists at least one equilibrium point for system (1) [6]. For simplicity, in stability analysis of the system (1), the equilibrium point $y^* = [y_1^*, \dots, y_n^*]^T$ is shifted to the origin by utilizing the transformation $x(\cdot) = y(\cdot) - y^*$, which leads the system (1) to the following form

$$\dot{x}(t) = -(A + \Delta A(t))x(t) + (W_0 + \Delta W_0(t))f(x(t)) + (W_1 + \Delta W_1(t))f(x(t-h(t))), \quad (4)$$

where $x(t) = [x_1(t), \dots, x_n(t)]^T \in \mathbf{R}^n$ is the state vector of the transformed system.

Also, $f(\cdot) = [f_1(\cdot), \dots, f_n(\cdot)]^T$ and $f_j(\cdot) = g_j(\cdot + y_j^*) - g_j(y_j^*)$ with $f_j(0) = 0$, and $f_j(\cdot)$ satisfies Assumption 1.

In this paper, a model of uncertain T-S fuzzy NNs with time-varying delays are considered as

Plant Rule l :

IF $\theta_1(t)$ is Θ_{1l} , ... , and $\theta_s(t)$ is Θ_{ls} , $l = 1, 2, \dots, r$,

THEN

$$\dot{x}(t) = -(A_l + \Delta A_l(t))x(t) + (W_{0l} + \Delta W_{0l}(t))f(x(t)) + (W_{1l} + \Delta W_{1l}(t))f(x(t-h(t))),$$

$$x(t) = \phi(t), \quad -h_U \leq t \leq 0, \quad (5)$$

where $\theta_1(t), \dots, \theta_s(t)$ are the premise variables, and $\Theta_{1s}, \dots, \Theta_{rs}$ are fuzzy sets, r is the number of IF-THEN rules, and $\phi(t) \in \mathbf{C}_{n, h_U}$ denotes the initial conditions. The parameter uncertainties $\Delta A_l(t)$, $\Delta W_{0l}(t)$ and $\Delta W_{1l}(t)$ are defined as follows

$$[\Delta A_l(t) \quad \Delta W_{0l}(t) \quad \Delta W_{1l}(t)] = DF(t)[E_{al} \quad E_{0l} \quad E_{1l}], \quad (6)$$

with $F^T(t)F(t) \leq I, \forall t \geq 0$.

Using center-average defuzzifier, product interference and singleton fuzzifier, the defuzzified output of system (5) can be inferred as follow

$$\dot{x}(t) = \sum_{l=1}^r \mu_l(\theta(t)) [-(A_l + \Delta A_l(t))x(t) + (W_{0l} + \Delta W_{0l}(t))f(x(t)) + (W_{1l} + \Delta W_{1l}(t))f(x(t-h(t)))], \quad (7)$$

where

$$\omega_l(\theta(t)) = \prod_{k=1}^s \Theta_{lk}(\theta_k(t)), \quad \mu_l(\theta(t)) = \frac{\omega_l(\theta(t))}{\sum_{l=1}^r \omega_l(\theta(t))}, \quad (8)$$

and $\Theta_{lk}(\theta_k(t))$ is the grade of membership of $\theta_k(t)$ in Θ_{lk} .

It is assumed that

$$\omega_l(\theta(t)) \geq 0, \quad \sum_{l=1}^r \omega_l(\theta(t)) > 0, \quad \forall t. \quad (9)$$

Then, we have the following condition

$$\mu_l(\theta(t)) \geq 0, \sum_{l=1}^r \mu_l(\theta(t)) = 1. \tag{10}$$

Let us define

$$\begin{aligned} \hat{A} &= \sum_{l=1}^r \mu_l(\theta(t)) A_l, \quad \hat{W}_0 = \sum_{l=1}^r \mu_l(\theta(t)) W_{0l}, \quad \hat{W}_1 = \sum_{l=1}^r \mu_l(\theta(t)) W_{1l}, \\ \hat{E}_a &= \sum_{l=1}^r \mu_l(\theta(t)) E_{al}, \quad \hat{E}_0 = \sum_{l=1}^r \mu_l(\theta(t)) E_{0l}, \quad \hat{E}_1 = \sum_{l=1}^r \mu_l(\theta(t)) E_{1l}. \end{aligned} \tag{11}$$

Then, system (7) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= -\hat{A}x(t) + \hat{W}_0 f(x(t)) + \hat{W}_1 f(x(t-h(t))) + Dp(t), \tag{12} \\ p(t) &= F(t)q(t), \\ q(t) &= -\hat{E}_a x(t) + \hat{E}_0 f(x(t)) + \hat{E}_1 f(x(t-h(t))). \end{aligned}$$

In order to investigate the delay-dependent stability analysis for uncertain T-S fuzzy NNs with time-varying delays (12), we introduce the following facts and lemmas.

Fact 1. (Schur Complement) [17] Given constant matrices $\Sigma_1, \Sigma_2, \Sigma_3$ with $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2$, then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if:

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \star & -\Sigma_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \star & \Sigma_1 \end{bmatrix} < 0.$$

Fact 2. [18] For any real vectors a, b and any matrix $Q > 0$ with appropriate dimensions, we have

$$\pm 2a^T b \leq a^T Q a + b^T Q^{-1} b.$$

Lemma 1. (Finsler's lemma) [19] Let $\zeta \in \mathbf{R}^n$, $\Phi = \Phi^T \in \mathbf{R}^{n \times n}$, and $B \in \mathbf{R}^{m \times n}$ such that $\text{rank}(B) < n$. The following statements are equivalent:

- (i) $\zeta^T \Phi \zeta < 0, \forall B\zeta = 0, \zeta \neq 0$,
- (ii) $\exists L \in \mathbf{R}^{m \times m} : \Phi + LB + B^T L^T < 0$
- (iii) $(B^\perp)^T \Phi B^\perp < 0$

Lemma 2. For any constant matrix $M > 0$, the following inequality holds:

$$(\beta - \alpha) \int_\beta^\alpha \dot{x}^T(s) M \dot{x}(s) ds \tag{13}$$

$$\leq - \left(\int_\beta^\alpha \dot{x}(s) ds \right)^T M \left(\int_\beta^\alpha \dot{x}(s) ds \right),$$

$$\begin{aligned} & - \frac{(\alpha - \beta)^2}{2} \int_\beta^\alpha \int_s^\alpha \dot{x}^T(u) M \dot{x}(u) du \\ & \leq - \left(\int_\beta^\alpha \int_s^\alpha \dot{x}(u) du \right)^T M \left(\int_\beta^\alpha \int_s^\alpha \dot{x}(u) du \right). \end{aligned} \tag{14}$$

Proof. By Jensen's inequality [10], Eq. (13) is obtained. Moreover, the following inequality holds

$$(\alpha - s) \int_s^\alpha \dot{x}^T(u) M \dot{x}(u) du \geq \left(\int_\beta^\alpha \dot{x}(u) du \right)^T M \left(\int_\beta^\alpha \dot{x}(u) du \right). \tag{15}$$

By using Fact 1, Eq. (15) is equivalent to the following

$$\left[\int_\beta^\alpha \dot{x}^T(u) M \dot{x}(u) du \quad \int_s^\alpha \dot{x}^T(u) du \right] \begin{matrix} \star \\ (\alpha - s) M^{-1} \end{matrix} \geq 0. \tag{16}$$

Integration of (16) from β to α yields

$$\left[\int_\beta^\alpha \int_s^\alpha \dot{x}^T(u) M \dot{x}(u) du ds \quad \int_\beta^\alpha \int_s^\alpha \dot{x}(u) du \right] \begin{matrix} \star \\ \int_\beta^\alpha (\alpha - s) M^{-1} ds \end{matrix} \geq 0. \tag{17}$$

Therefore, the inequality (17) is equivalent to the inequality (14) according to Fact 1. This complete the proof. ■

Lemma 3. For any scalar $h(t) \geq 0$ and any constant matrix $Q \in \mathbf{R}^{n \times n}, Q = Q^T$, the following inequality holds:

$$\begin{aligned} & - \int_{t-h(t)}^t \int_s^t \dot{x}^T(s) Q \dot{x}(s) ds \\ & \leq \frac{h^2(t)}{2} \zeta^T(t) \mathbf{X} Q^{-1} \mathbf{X}^T \zeta(t) + 2\zeta^T(t) \mathbf{X} \int_{t-h(t)}^t (x(t) - x(s)) ds, \end{aligned} \tag{18}$$

where

$$\begin{aligned} \zeta^T(t) &= [x^T(t), x^T(t-h(t)), x^T(t-h_U), \dot{x}^T(t), \dot{x}^T(t-h_U), \\ & \frac{1}{h(t)} \int_{t-h(t)}^t x^T(s) ds, \frac{1}{h_U - h(t)} \int_{t-h_U}^{t-h(t)} x^T(s) ds, \\ & f^T(x(t)), f^T(x(t-h(t))), f^T(x(t-h_U)), p^T(t)], \end{aligned}$$

and \mathbf{X} is free weighting matrix with appropriate dimensions.

Proof. From Fact 2, the following inequality holds

$$\begin{aligned} & - 2 \int_{t-h(t)}^t \int_s^t (\zeta^T(t) \mathbf{X}) \dot{x}^T(u) du ds \\ & \leq \int_{t-h(t)}^t \int_s^t [\zeta^T(t) \mathbf{X} Q^{-1} \mathbf{X}^T \zeta(t) + \dot{x}^T(u) Q \dot{x}(u)] du ds. \end{aligned} \tag{19}$$

From (19), we obtain

$$\begin{aligned} & - \int_{t-h(t)}^t \int_s^t \dot{x}^T(u) Q \dot{x}(u) (\zeta^T(t) \mathbf{X}) \dot{x}^T(u) du ds \\ & \leq \int_{t-h(t)}^t \int_s^t [\zeta^T(t) \mathbf{X} Q^{-1} \mathbf{X}^T \zeta(t)] du ds \\ & \quad + 2 \int_{t-h(t)}^t \int_s^t (\zeta^T(t) \mathbf{X}) \dot{x}^T(u) du ds \\ & = \frac{h^2(t)}{2} \zeta^T(t) \mathbf{X} Q^{-1} \mathbf{X}^T \zeta(t) + 2\zeta^T(t) \mathbf{X} \int_{t-h(t)}^t (x(t) - x(s)) ds. \end{aligned} \tag{20}$$

Therefore, the inequality (19) is equivalent to the inequality (18). This complete the proof. ■

3. Main Results

In this section, we propose stability criteria for system (12). Before introducing our main result, the notations are defined for simplicity:

$$\begin{aligned}
 \hat{\Gamma} &= [\hat{A}, 0_n, 0_n, -I, 0_n, 0_n, 0_n, \hat{W}_0, \hat{W}_1, 0_n, D], \\
 \Gamma_l &= [A_l, 0_n, 0_n, -I, 0_n, 0_n, 0_n, W_{0l}, W_{1l}, 0_n, D], (l=1, \dots, r), \\
 \hat{\Psi} &= [-\hat{E}_a, 0_n, 0_n, 0_n, 0_n, 0_n, 0_n, \hat{E}_0, \hat{E}_1, 0_n, 0_n], \\
 \Psi_l &= [-E_{al}, 0_n, 0_n, 0_n, 0_n, 0_n, 0_n, E_{0l}, E_{1l}, 0_n, 0_n], (l=1, \dots, r), \\
 \Sigma_1 &= [\hat{\Phi}_{i,j}], (i, j=1, \dots, 11), \\
 \Phi_{1,1} &= R_{13} + R_{13}^T + N_{11} + G_{11} + h_U Q_2 - 2Q_3 - 2K_m H_1 K_p, \\
 \Phi_{1,3} &= -R_{13} + R_{23}^T, \quad \Phi_{1,4} = R_{11} + N_{12} - K_m \Lambda + K_p \Delta, \quad \Phi_{1,5} = R_{12}, \\
 \Phi_{1,6} &= 2Q_3, \quad \Phi_{1,7} = h_U R_{33}, \quad \Phi_{1,8} = N_{13} + H_1 (K_m + K_p), \\
 \Phi_{2,2} &= -(1-h_D)G_{11} - 2Q_3 - 2K_m H_2 K_p, \\
 \Phi_{2,7} &= -2Q_3, \quad \Phi_{2,9} = -(1-h_D)G_{12} + H_2 (K_m + K_p), \\
 \Phi_{3,3} &= -R_{23} - R_{23}^T - N_{11} - 2K_m H_3 K_p, \quad \Phi_{3,4} = R_{12}^T, \\
 \Phi_{3,5} &= -N_{12} + R_{22}, \quad \Phi_{3,7} = -h_U R_{33}, \quad \Phi_{3,10} = N_{13} + H_3 (K_m + K_p), \\
 \Phi_{4,4} &= N_{22} + (h_U^2/2)Q_3, \quad \Phi_{4,7} = h_U R_{13}, \quad \Phi_{4,8} = N_{23} + \Lambda^T - \Delta^T, \\
 \Phi_{5,5} &= -N_{22}, \quad \Phi_{5,7} = h_U R_{23}, \quad \Phi_{5,10} = N_{23}, \quad \Phi_{6,6} = -2Q_3, \\
 \Phi_{7,7} &= -h_U Q_2 - 2Q_3, \quad \Phi_{8,8} = N_{33} - 2H_1, \\
 \Phi_{9,9} &= -(1-h_D)G_{22} - 2H_2, \quad \Phi_{10,10} = N_{33} - 2H_3, \quad \Phi_{11,11} = -\epsilon I_n, \\
 \Phi_{i,j} &= 0_n, \text{ elsewhere}, \\
 \Sigma_2 &= [\hat{\Phi}_{i,j}], (i, j=1, \dots, 11), \\
 \hat{\Phi}_{1,6} &= R_{33}, \quad \hat{\Phi}_{1,7} = -R_{33}, \quad \hat{\Phi}_{3,6} = -R_{33}, \quad \hat{\Phi}_{3,7} = R_{33}, \\
 \hat{\Phi}_{4,6} &= R_{13}, \quad \hat{\Phi}_{4,7} = -R_{13}, \quad \hat{\Phi}_{5,6} = R_{23}, \quad \hat{\Phi}_{5,7} = -R_{23}, \\
 \hat{\Phi}_{6,6} &= -Q_2, \quad \hat{\Phi}_{7,7} = Q_2, \quad \hat{\Phi}_{i,j} = 0_n, \text{ elsewhere}, \\
 \Sigma_{[h(t)]} &= \Sigma_1 + h(t)\Sigma_2, \\
 \Xi_1 &= \begin{bmatrix} -2Q_1 & 2Q_1 & 0_n & 0_n & 0_n \times 7n \\ \star & -3Q_1 & Q_1 & 0_n & 0_n \times 7n \\ \star & \star & -Q_1 & 0_n & 0_n \times 7n \\ \star & \star & \star & h_U^2 Q_1 & 0_n \times 7n \\ \star & \star & \star & \star & 0_{7n} \end{bmatrix}, \\
 \Xi_2 &= \frac{1}{h_U} \times \begin{bmatrix} Q_1 - Q_1 & 0_n & 0_n \times 8n \\ \star & 0_n & Q_1 & 0_n \times 8n \\ \star & \star & -Q_1 & 0_n \times 8n \\ \star & \star & \star & 0_{8n} \end{bmatrix}, \\
 \Xi_{[h(t)]} &= \Xi_1 + h(t)\Xi_2. \tag{21}
 \end{aligned}$$

Now, we have the following theorem.

Theorem 1. For given scalars $h_U > 0$ and h_D , diagonal matrices $K_p = \text{diag}\{k_1^+, \dots, k_n^+\}$ and $K_m = \text{diag}\{k_1^+, \dots, k_n^+\}$, the system (12) is asymptotically stable for $0 \leq h(t) \leq h_U$ and $\dot{h}(t) \leq h_D$, if there exist positive diagonal matrices

$\Lambda = \text{diag}\{\lambda_1, \dots, \lambda_n\}$, $\Delta = \text{diag}\{\delta_1, \dots, \delta_n\}$, $H_l = \text{diag}\{h_{11}, \dots, h_{1n}\}$, $H_2 = \text{diag}\{h_{21}, \dots, h_{2n}\}$, $H_3 = \text{diag}\{h_{31}, \dots, h_{3n}\}$, positive definite matrices $\mathbf{R} = [R_{ij}]_{3 \times 3} \in \mathbf{R}^{3n \times 3n}$, $\mathbf{N} = [N_{ij}]_{3 \times 3} \in \mathbf{R}^{3n \times 3n}$, $\mathbf{G} = [G_{ij}]_{2 \times 2} \in \mathbf{R}^{2n \times 2n}$, $Q_l (i=1, 2, 3) \in \mathbf{R}^{n \times n}$, and positive scalar ϵ satisfying the following LMIs for $l=1, \dots, r$:

$$\begin{bmatrix} \Gamma_l^\perp & 0_{11n \times n} \\ 0_{n \times 10n} & I_n \end{bmatrix} \begin{bmatrix} \Sigma_{[0]} + \Xi_{[0]} & \epsilon \Psi_l^T \\ \star & -I_n \end{bmatrix} \begin{bmatrix} \Gamma_l^\perp & 0_{11n \times n} \\ 0_{n \times 10n} & I_n \end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} \Gamma_l^\perp & 0_{11n \times n} \\ 0_{n \times 10n} & I_n \end{bmatrix} \begin{bmatrix} \Sigma_{[h(t)]} + \Xi_{[h(t)]} & \epsilon \Psi_l^T \\ \star & -I_n \end{bmatrix} \begin{bmatrix} \Gamma_l^\perp & 0_{11n \times n} \\ 0_{n \times 10n} & I_n \end{bmatrix} < 0, \tag{23}$$

where $\Gamma_l, \Sigma_{[h(t)], \Xi_{[h(t)], \Psi_l}$ are defined in (21), and Γ_l^\perp is the right orthogonal complement of Γ_l .

Proof. Let us consider the following L-K functional candidate as

$$V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6, \tag{24}$$

where

$$\begin{aligned}
 V_1 &= \begin{bmatrix} x(t) \\ x(t-h_U) \\ \int_{t-h_U}^t x(s) ds \end{bmatrix}^T \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ \star & R_{22} & R_{23} \\ \star & \star & R_{33} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h_U) \\ \int_{t-h_U}^t x(s) ds \end{bmatrix}, \\
 V_2 &= \int_{t-h_U}^t \begin{bmatrix} x(s) \\ \dot{x}(s) \\ f(x(s)) \end{bmatrix}^T \begin{bmatrix} N_{11} & N_{12} & N_{13} \\ \star & N_{22} & N_{23} \\ \star & \star & N_{33} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \\ f(x(s)) \end{bmatrix} ds \\
 &\quad + 2 \sum_{i=1}^n \left(\lambda_i \int_0^{x_i(t)} (f_i(s) - k_i^- s) ds + \delta_i \int_0^{x_i(t)} (k_i^+ s - f_i(s)) ds \right), \\
 V_3 &= \int_{t-h(t)}^t \begin{bmatrix} x(s) \\ f(x(s)) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ \star & G_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ f(x(s)) \end{bmatrix} ds, \\
 V_4 &= h_U \int_{t-h_U}^t \int_s^t \dot{x}^T(u) Q_1 \dot{x}(u) du ds, \\
 V_5 &= h_U \int_{t-h_U}^t \int_s^t x^T(u) Q_2 x(u) du ds, \\
 V_6 &= \int_{t-h_U}^t \int_s^t \int_u^t \dot{x}^T(v) Q_3 \dot{x}(v) dv du ds.
 \end{aligned}$$

The time-derivative of V_1 is obtained as

$$\begin{aligned}
 \dot{V}_1 &= 2 \begin{bmatrix} x(t) \\ x(t-h_U) \\ \Omega_1 \end{bmatrix}^T \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ \star & R_{22} & R_{23} \\ \star & \star & R_{33} \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \dot{x}(t-h_U) \\ x(t) - x(t-h_U) \end{bmatrix} \\
 &= 2 \begin{bmatrix} x(t) \\ x(t-h_U) \\ \frac{1}{h(t)} \int_{t-h(t)}^t x(s) ds \\ \frac{1}{h_U - h(t)} \int_{t-h_U}^{t-h(t)} x(s) ds \end{bmatrix}^T \Omega_2 \begin{bmatrix} \dot{x}(t) \\ \dot{x}(t-h_U) \\ x(t) - x(t-h_U) \end{bmatrix}, \tag{25}
 \end{aligned}$$

where

$$\Omega_1 = h(t) \left(\frac{1}{h(t)} \int_{t-h(t)}^t x(s) ds \right) + (h_U - h(t)) \left(\frac{1}{h_U - h(t)} \int_{t-h_U}^{t-h(t)} x(s) ds \right),$$

$$\leq (-1 - h_U^{-1}h(t)) \times [x(t-h(t)) - x(t-h_U)]^T Q_1 [x(t-h(t)) - x(t-h_U)]. \quad (30)$$

$$\Omega_2 = \begin{bmatrix} I0 & 0 \\ 0 I & 0 \\ 00 & h(t)I \\ 00 & (h_U - h(t))I \end{bmatrix} \times \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ \star & R_{22} & R_{23} \\ \star & \star & R_{33} \end{bmatrix}.$$

Also, the time-derivative of V_2 is

$$\dot{V}_2 = \begin{bmatrix} x(t) \\ \dot{x}(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} N_{11} & N_{12} & N_{13} \\ \star & N_{22} & N_{23} \\ \star & \star & N_{33} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \\ f(x(t)) \end{bmatrix} - \begin{bmatrix} x(t-h_U) \\ \dot{x}(t-h_U) \\ f(x(t-h_U)) \end{bmatrix}^T \begin{bmatrix} N_{11} & N_{12} & N_{13} \\ \star & N_{22} & N_{23} \\ \star & \star & N_{33} \end{bmatrix} \begin{bmatrix} x(t-h_U) \\ \dot{x}(t-h_U) \\ f(x(t-h_U)) \end{bmatrix} + 2[f(x(t)) - K_m x(t)]^T \Delta \dot{x}(t) + 2[K_p x(t) - f(x(t))] \Delta \dot{x}(t). \quad (26)$$

We can obtain an upper bound of the time-derivative of V_3 as follows

$$V_3 = \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ \star & G_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} - (1-h_D) \begin{bmatrix} x(t-h(t)) \\ f(x(t-h(t))) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ \star & G_{22} \end{bmatrix} \begin{bmatrix} x(t-h(t)) \\ f(x(t-h(t))) \end{bmatrix}. \quad (27)$$

Calculating the time-derivative of V_4 leads to

$$\dot{V}_4 = h_U^2 \dot{x}^T(t) Q_1 \dot{x}(t) - h_U \int_{t-h(t)}^t \dot{x}^T(s) Q_1 \dot{x}(s) ds - h_U \int_{t-h_U}^{t-h(t)} \dot{x}^T(s) Q_1 \dot{x}(s) ds. \quad (28)$$

By Lemma 2 and the method of [20], the first integral term in (28) can be estimated as

$$\begin{aligned} & -h_U \int_{t-h(t)}^t \dot{x}^T(s) Q_1 \dot{x}(s) ds \\ &= -(h_U - h(t)) \int_{t-h(t)}^t \dot{x}^T(s) Q_1 \dot{x}(s) ds \\ & \quad - h(t) \int_{t-h(t)}^t \dot{x}^T(s) Q_1 \dot{x}(s) ds \\ & \leq -h_U^{-1} (h_U - h(t)) h(t) \int_{t-h(t)}^t \dot{x}^T(s) Q_1 \dot{x}(s) ds \\ & \quad - h(t) \int_{t-h(t)}^t \dot{x}^T(s) Q_1 \dot{x}(s) ds \\ & \leq (-2 + h_U^{-1}h(t)) [x(t) - x(t-h(t))]^T Q_1 [x(t) - x(t-h(t))]. \end{aligned} \quad (29)$$

With the similar method introduced above, an upper bound of other integral term in (28) can be obtained as

$$\begin{aligned} & -h_U \int_{t-h_U}^{t-h(t)} \dot{x}^T(s) Q_1 \dot{x}(s) ds \\ & \leq -(h_U - h(t)) \int_{t-h_U}^{t-h(t)} \dot{x}^T(s) Q_1 \dot{x}(s) ds \\ & \quad - h_U^{-1} h(t) (h_U - h(t)) \int_{t-h_U}^{t-h(t)} \dot{x}^T(s) Q_1 \dot{x}(s) ds \end{aligned}$$

Then, from (28)–(30), an upper bound of \dot{V}_4 can be

$$\begin{aligned} \dot{V}_4 & \leq h_U^2 \dot{x}^T(t) Q_1 \dot{x}(t) \\ & \quad + (-2 + h_U^{-1}h(t)) [x(t) - x(t-h(t))]^T Q_1 [x(t) - x(t-h(t))] \\ & \quad + (-1 - h_U^{-1}h(t)) \times [x(t-h(t)) - x(t-h_U)]^T Q_1 [x(t-h(t)) - x(t-h_U)]. \end{aligned} \quad (31)$$

By use of Lemma 2, an upper bound of \dot{V}_5 can be found as

$$\begin{aligned} \dot{V}_5 &= h_U^2 x^T(t) Q_2 x(t) - h_U \int_{t-h(t)}^t x^T(s) Q_2 x(s) ds \\ & \quad - h_U \int_{t-h_U}^{t-h(t)} x^T(s) Q_2 x(s) ds \\ & \leq h_U^2 x^T(t) Q_2 x(t) \\ & \quad - h_U h^{-1}(t) \left(\int_{t-h(t)}^t x(s) ds \right)^T Q_2 \left(\int_{t-h(t)}^t x(s) ds \right) \\ & \quad - h_U (h_U - h(t))^{-1} \left(\int_{t-h_U}^{t-h(t)} x(s) ds \right)^T Q_2 \left(\int_{t-h_U}^{t-h(t)} x(s) ds \right) \\ &= h_U^2 x^T(t) Q_2 x(t) \\ & \quad - \left(\frac{1}{h(t)} \int_{t-h(t)}^t x(s) ds \right)^T (h_U h(t) Q_2) \left(\frac{1}{h(t)} \int_{t-h(t)}^t x(s) ds \right) \\ & \quad - \left(\frac{1}{h_U - h(t)} \int_{t-h_U}^{t-h(t)} x(s) ds \right)^T (h_U (h_U - h(t)) Q_2) \times \\ & \quad \left(\frac{1}{h_U - h(t)} \int_{t-h_U}^{t-h(t)} x(s) ds \right). \end{aligned} \quad (32)$$

Calculating \dot{V}_6 gives that

$$\begin{aligned} \dot{V}_6 &= \frac{h_U^2}{2} \dot{x}^T(t) Q_3 \dot{x}(t) - \int_{t-h_U}^t \int_s^t \dot{x}^T(u) Q_3 \dot{x}(u) du ds \\ &= \frac{h_U^2}{2} \dot{x}^T(t) Q_3 \dot{x}(t) - \int_{t-h(t)}^t \int_s^t \dot{x}^T(u) Q_3 \dot{x}(u) du ds \\ & \quad - \int_{t-h_U}^{t-h(t)} \int_s^t \dot{x}^T(u) Q_3 \dot{x}(u) du ds \\ & \leq \frac{h_U^2}{2} \dot{x}^T(t) Q_3 \dot{x}(t) - \int_{t-h(t)}^t \int_s^t \dot{x}^T(u) Q_3 \dot{x}(u) du ds \\ & \quad - \int_{t-h_U}^{t-h(t)} \int_s^t \dot{x}^T(u) Q_3 \dot{x}(u) du ds \\ & \leq \frac{h_U^2}{2} \dot{x}^T(t) Q_3 \dot{x}(t) \\ & \quad - \frac{2}{h^2(t)} \left(h(t)x(t) - \int_{t-h(t)}^t x(s) ds \right)^T Q_3 \times \\ & \quad \left(h(t)x(t) - \int_{t-h(t)}^t x(s) ds \right) \\ & \quad - \frac{2}{(h_U - h(t))^2} \left((h_U - h(t))x(t-h(t)) - \int_{t-h_U}^{t-h(t)} x(s) ds \right)^T Q_3 \times \\ & \quad \left((h_U - h(t))x(t-h(t)) - \int_{t-h_U}^{t-h(t)} x(s) ds \right) \end{aligned}$$

$$\begin{aligned}
 &= \frac{h_U^2}{2} \dot{x}^T(t) Q_3 \dot{x}(t) \\
 &\quad - 2 \left(x(t) - \frac{1}{h(t)} \int_{t-h(t)}^t x(s) ds \right)^T Q_3 \times \\
 &\quad \left(x(t) - \frac{1}{h(t)} \int_{t-h(t)}^t x(s) ds \right) \\
 &\quad - 2 \left(x(t-h(t)) - \frac{1}{h_U-h(t)} \int_{t-h_U}^{t-h(t)} x(s) ds \right)^T Q_3 \times \\
 &\quad \left(x(t-h(t)) - \frac{1}{h_U-h(t)} \int_{t-h_U}^{t-h(t)} x(s) ds \right).
 \end{aligned} \tag{33}$$

where Lemma 2 was utilized in (33). It should be noted that Eq.(3) means

$$\begin{aligned}
 &[f_j(x_j(t)) - k_j^- x_j(t)][f_j(x_j(t)) - k_j^+ x_j(t)] \leq 0, \\
 &[f_j(x_j(t-h(t))) - k_j^- x_j(t-h(t))][f_j(x_j(t-h(t))) - k_j^+ x_j(t-h(t))] \leq 0, \\
 &[f_j(x_j(t-h_U)) - k_j^- x_j(t-h_U)][f_j(x_j(t-h_U)) - k_j^+ x_j(t-h_U)] \leq 0,
 \end{aligned} \tag{34}$$

where $j = 1, \dots, n$.

From the inequalities (34), for any positive diagonal matrices $H_1 = \text{diag}\{h_{11}, \dots, h_{1n}\}$, $H_2 = \text{diag}\{h_{21}, \dots, h_{2n}\}$ and $H_3 = \text{diag}\{h_{31}, \dots, h_{3n}\}$, the following inequality holds

$$\begin{aligned}
 0 \leq & -2x^T(t) K_m H_1 K_p x(t) + 2x^T(t) (K_m + K_p) H_1 f(x(t)) \\
 & - 2f^T(x(t)) H_1 f(x(t)) - 2x^T(t-h(t)) K_m H_2 K_p x(t-h(t)) \\
 & + 2x^T(t-h(t)) (K_m + K_p) H_2 f(x(t-h(t))) \\
 & - 2f^T(x(t-h(t))) H_2 f(x(t-h(t))) \\
 & - 2x^T(t-h_U) K_m H_3 K_p x(t-h_U) \\
 & + 2x^T(t-h_U) (K_m + K_p) H_3 f(x(t-h_U)) \\
 & - 2f^T(x(t-h_U)) H_3 f(x(t-h_U)).
 \end{aligned} \tag{35}$$

From (12) with $F^T(t)F(t) \leq I$, we have $p^T(t)p(t) \leq q^T(t)q(t)$. Then, there exists a positive scalar ϵ satisfying the following inequality

$$\epsilon [\zeta^T(t) \Psi^T \Psi \zeta(t) - p^T(t)p(t)] \geq 0. \tag{36}$$

From (25)-(36) and S-procedure [17], the time-derivative of V has a new upper bound as

$$\dot{V} \leq \zeta^T(t) (\Sigma_{[h(t)]} + \Xi_{[h(t)]} + e^{\hat{\Psi}^T \hat{\Psi}}) \zeta(t), \tag{37}$$

where $\Sigma_{[h(t)]}$, $\Xi_{[h(t)]}$, and $\hat{\Psi}$ are defined in Eq. (21).

Also, the system (12) with the augmented vector $\zeta(t)$ can be rewritten as

$$\hat{I} \zeta(t) = 0_{n \times 1}, \tag{38}$$

where \hat{I} is defined in Eq. (21) and $\zeta(t)$ is in Eq. (18).

Therefore, an asymptotic stability condition for system (12) is

$$\zeta^T(t) (\Sigma_{[h(t)]} + \Xi_{[h(t)]} + e^{\hat{\Psi}^T \hat{\Psi}}) \zeta(t) < 0 \text{ with } \hat{I} \zeta(t) = 0_{n \times 1}. \tag{39}$$

By use of (i) and (ii) of Lemma 1, for any matrix L with appropriate dimension, inequality (39) is equivalent to

$$\Sigma_{[h(t)]} + \Xi_{[h(t)]} + e^{\hat{\Psi}^T \hat{\Psi}} L^T + \hat{I}^T L^T < 0. \tag{40}$$

By Fact 1, inequality (40) is equivalent to

$$\begin{bmatrix} \Sigma_{[h(t)]} + \Xi_{[h(t)]} + L^T + \hat{I}^T L^T & e^{\hat{\Psi}^T \hat{\Psi}} \\ \star & -I_n \end{bmatrix} < 0, \tag{41}$$

which can be represented as

$$\sum_{l=1}^r \mu(\theta(t)) \begin{bmatrix} \Sigma_{[h(t)]} + \Xi_{[h(t)]} + L^T + \hat{I}^T L^T & e^{\hat{\Psi}^T \hat{\Psi}} \\ \star & -I_n \end{bmatrix} < 0. \tag{42}$$

Note that

$$(I_l^\perp)^T (L I_l + I_l^T L^T) I_l^\perp = 0. \tag{43}$$

At each l th rule, by multiplying on the left side and right side of the inequality (42) by $\begin{bmatrix} (I_l^\perp)^T 0_{10n \times n} \\ 0_{n \times 11n} & I_n \end{bmatrix}$ and

$\begin{bmatrix} I_l^\perp & 0_{11n \times n} \\ 0_{n \times 10n} & I_n \end{bmatrix}$, we have

$$\begin{bmatrix} I_l^\perp & 0_{11n \times n} \\ 0_{n \times 10n} & I_n \end{bmatrix} \begin{bmatrix} \Sigma_{[h(t)]} + \Xi_{[h(t)]} & e^{\hat{\Psi}^T \hat{\Psi}} \\ \star & -I_n \end{bmatrix} \begin{bmatrix} I_l^\perp & 0_{11n \times n} \\ 0_{n \times 10n} & I_n \end{bmatrix} < 0, \tag{44}$$

$(l = 1, \dots, r)$

where (43) was utilized in (44).

Therefore, by convex-hull properties, if the LMIs (22) and (23) satisfy, then the inequality (44) holds. This means that system (12) is asymptotically stable. This completes our proof. ■

Theorem 1 is derived by the L-K functional without free-weighting matrices. By employing free-weighting matrices, a further improved delay-dependent stability criterion can be obtained, which will be introduced in Theorem 2.

For the sake's of simplicity, we define the following notations:

$$\begin{aligned}
 \mathbf{X}^T &= [X_1^T, X_2^T, 0_{n \times 9n}], \quad \mathbf{Y}^T = [0_n, Y_1^T, Y_2^T, 0_{n \times 8n}], \\
 \mathbf{Z}^T &= [Z_1^T, 0_{n \times 4n}, Z_2^T, 0_{n \times 5n}], \quad \mathbf{W}^T = [0_n, W_1^T, 0_{n \times 4n}, W_2^T, 0_{n \times 4n}], \\
 \Pi_1 &= [0_{n \times 3n}, I_n, 0_{n \times 7n}], \quad \Pi_2 = \begin{bmatrix} I_n - I_n & 0_n & 0_n & 0_n & 0_n & 0_n & 0_n & 0_{n \times 4n} \\ 0_n & I_n & -I_n & 0_n & 0_n & 0_n & 0_n & 0_{n \times 4n} \\ I_n & 0_n & 0_n & 0_n & 0_n & -I_n & 0_n & 0_{n \times 4n} \\ 0_n & I_n & 0_n & 0_n & 0_n & 0_n & -I_n & 0_{n \times 4n} \end{bmatrix},
 \end{aligned}$$

$$\begin{aligned}
 \hat{\Sigma}_{[h(t)]} &= \Sigma_{[h(t)]} + \Pi_1^T (h_U Q_1 + (h_U^2/2) Q_4) \Pi_1 \\
 &\quad + [\mathbf{X}, \mathbf{Y}, h(t)\mathbf{Z}, (h_U - h(t))\mathbf{W}] \Pi_2 \\
 &\quad + \Pi_2^T [\mathbf{X}, \mathbf{Y}, h(t)\mathbf{Z}, (h_U - h(t))\mathbf{W}]^T,
 \end{aligned}$$

$$\Theta_{[h(t)]} = h(t)\mathbf{X}Q_1^{-1}\mathbf{X}^T + (h_U - h(t))\mathbf{Y}Q_1^{-1}\mathbf{Y}^T + (h^2(t)/2)\mathbf{Z}Q_4^{-1}\mathbf{Z}^T + ((h_U - h(t))^2/2)\mathbf{W}Q_4^{-1}\mathbf{W}^T. \quad (45)$$

Here, $\Sigma_{[h(t)]}$ is defined in (21).

Theorem 2. For given scalars $h_U > 0$ and h_D , diagonal matrices $K_p = \text{diag}\{k_1^+, \dots, k_n^+\}$ and $K_m = \text{diag}\{k_1^+, \dots, k_n^+\}$, the system (12) is asymptotically stable for $0 \leq h(t) \leq h_U$ and $\dot{h}(t) \leq h_D$, if there exist positive diagonal matrices $A = \text{diag}\{\lambda_1, \dots, \lambda_n\}$, $\Delta = \text{diag}\{\delta_1, \dots, \delta_n\}$, $H_1 = \text{diag}\{h_{11}, \dots, h_{1n}\}$, $H_2 = \text{diag}\{h_{21}, \dots, h_{2n}\}$, $H_3 = \text{diag}\{h_{31}, \dots, h_{3n}\}$, positive definite matrices $\mathbf{R} = [R_{ij}]_{3 \times 3} \in \mathbf{R}^{3n \times 3n}$, $\mathbf{N} = [N_{ij}]_{3 \times 3} \in \mathbf{R}^{3n \times 3n}$, $\mathbf{G} = [G_{ij}]_{2 \times 2} \in \mathbf{R}^{2n \times 2n}$, $Q_i (i=1,2,3,4) \in \mathbf{R}^{n \times n}$, positive scalar ϵ , and any matrices $X_i \in \mathbf{R}^{n \times n}$, $Y_i \in \mathbf{R}^{n \times n}$, $Z_i \in \mathbf{R}^{n \times n}$, $W_i \in \mathbf{R}^{n \times n}$ ($i=1,2$) satisfying the following LMIs for $l=1, \dots, r$:

$$\begin{bmatrix} \Gamma_l^\perp & 0_{11n \times n} \\ 0_{n \times 10n} & I_n \\ 0_{3n \times 11n} & I_{3n} \end{bmatrix}^T \times \begin{bmatrix} \widehat{\Sigma}_{[0]} \epsilon \Psi_1^T & [0_{11n \times n}] & [h_U \mathbf{W}] & [h_U \mathbf{Y}] \\ \star & -\epsilon \mathbf{I} & [0_n] & [0_n] \\ \star & \star & -2Q_4 & 0_n \\ \star & \star & \star & -2Q_4 \\ \star & \star & \star & -h_U Q_1 \end{bmatrix} \begin{bmatrix} \Gamma_l^\perp & 0_{11n \times n} \\ 0_{n \times 10n} & I_n \\ 0_{3n \times 11n} & I_{3n} \end{bmatrix} < 0, \quad (46)$$

$$\begin{bmatrix} \Gamma_l^\perp & 0_{11n \times n} \\ 0_{n \times 10n} & I_n \\ 0_{3n \times 11n} & I_{3n} \end{bmatrix}^T \times \begin{bmatrix} \widehat{\Sigma}_{[h(t)]} \epsilon \Psi_1^T & [0_{11n \times n}] & [h_U \mathbf{W}] & [h_U \mathbf{Y}] \\ \star & -\epsilon \mathbf{I} & [0_n] & [0_n] \\ \star & \star & -2Q_4 & 0_n \\ \star & \star & \star & -2Q_4 \\ \star & \star & \star & -h_U Q_1 \end{bmatrix} \begin{bmatrix} \Gamma_l^\perp & 0_{11n \times n} \\ 0_{n \times 10n} & I_n \\ 0_{3n \times 11n} & I_{3n} \end{bmatrix} < 0, \quad (47)$$

where $\Gamma_l, \Psi_l, \mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{W}$, and $\widehat{\Sigma}_{[h(t)]}$ are defined in (21) and Γ_l^\perp is the right orthogonal complement of Γ_l .

Proof. Let us choose the following L-K functional candidate with free-weighting matrices as

$$\widehat{V} = V_1 + V_2 + V_3 + (1/h_U)V_4 + V_5 + V_6 + V_7, \quad (48)$$

where $V_i (i=1,2,3,4,5,6)$ is the same L-K functional used in Theorem 1, and

$$V_7 = \int_{t-h_U}^t \int_s^t \int_u^t \dot{x}^T(v) Q_4 \dot{x}(v) dv du ds.$$

Using Lemma 3, we have upper bound of the time-derivative of $(1/h_U)V_4$ as

$$\begin{aligned} (1/h_U)\dot{V}_4 &= h_U \dot{x}^T(t) Q_4 \dot{x}(t) - \int_{t-h(t)}^t \dot{x}^T(s) Q_4 \dot{x}(s) ds \\ &\quad - \int_{t-h_U}^{t-h(t)} \dot{x}^T(s) Q_4 \dot{x}(s) ds \\ &\leq h_U \dot{x}^T(t) Q_4 \dot{x}(t) \\ &\quad + h(t) \zeta^T(t) \mathbf{X} Q_1^{-1} \mathbf{X}^T \zeta(t) + 2\zeta^T(t) \mathbf{X} [x(t) - x(t-h(t))] \\ &\quad + (h_U - h(t)) \zeta^T(t) \mathbf{Y} Q_1^{-1} \mathbf{Y}^T \zeta(t) \\ &\quad + 2\zeta^T(t) \mathbf{Y} [x(t-h(t)) - x(t-h_U)]. \end{aligned} \quad (49)$$

Similarly, an upper bound of the time-derivative of V_7 can be calculated as

$$\begin{aligned} \dot{V}_7 &= \frac{h_U^2}{2} \dot{x}^T(t) Q_4 \dot{x}(t) - \int_{t-h(t)}^t \int_s^t \dot{x}^T(u) Q_4 \dot{x}(u) du ds \\ &\quad - \int_{t-h_U}^{t-h(t)} \int_s^t \dot{x}^T(u) Q_4 \dot{x}(u) du ds \\ &\leq \frac{h_U^2}{2} \dot{x}^T(t) Q_4 \dot{x}(t) \\ &\quad + \frac{h^2(t)}{2} \zeta^T(t) \mathbf{Z} Q_4^{-1} \mathbf{Z}^T \zeta(t) + \frac{(h_U - h(t))^2}{2} \zeta^T(t) \mathbf{W} Q_4^{-1} \mathbf{W}^T \zeta(t) \\ &\quad + 2\zeta^T(t) \mathbf{Z} h(t) [x(t) - \frac{1}{h(t)} \int_{t-h(t)}^t x(s) ds] \\ &\quad + 2\zeta^T(t) \mathbf{W} (h_U - h(t)) [x(t-h(t)) - \frac{1}{h_U - h(t)} \int_{t-h_U}^{t-h(t)} x(s) ds]. \end{aligned} \quad (50)$$

From (36), (49) and (50), the time-derivative of \widehat{V} has a new upper bound as

$$\dot{\widehat{V}} \leq \zeta^T(t) (\widehat{\Sigma}_{[h(t)]} + \epsilon \widehat{\Psi}^T \widehat{\Psi} + \Theta_{[h(t)]}) \zeta(t), \quad (51)$$

where $\widehat{\Sigma}_{[h(t)]}, \Theta_{[h(t)]}$ are defined in (45).

By use of (i) and (ii) of Lemma 1, for any matrix \mathbf{L} with appropriate dimension, inequality (51) is equivalent to

$$\widehat{\Sigma}_{[h(t)]} + \epsilon \widehat{\Psi}^T \widehat{\Psi} + \Theta_{[h(t)]} + \mathbf{L} \widehat{\Gamma} + \widehat{\Gamma}^T \mathbf{L}^T < 0. \quad (52)$$

From Fact 1, inequality (52) is equivalent to

$$\begin{bmatrix} \widehat{\Sigma}_{[h(t)]} + \Theta_{[h(t)]} + \mathbf{L} \widehat{\Gamma} + \widehat{\Gamma}^T \mathbf{L}^T & \epsilon \widehat{\Psi}^T \\ \star & -I_n \end{bmatrix} < 0, \quad (53)$$

which can be represented as

$$\sum_{l=1}^r \mu(\theta(t)) \begin{bmatrix} \widehat{\Sigma}_{[h(t)]} + \Theta_{[h(t)]} + \mathbf{L} \widehat{\Gamma} + \widehat{\Gamma}^T \mathbf{L}^T & \epsilon \widehat{\Psi}^T \\ \star & -I_n \end{bmatrix} < 0. \quad (54)$$

At each l th rule, by multiplying on the left side and right side of the inequality (54) by $\begin{bmatrix} (I_l^\perp)^T 0_{10n \times n} \\ 0_{n \times 11n} I_n \end{bmatrix}$ and $\begin{bmatrix} I_l^\perp & 0_{11n \times n} \\ 0_{n \times 10n} & I_n \end{bmatrix}$, we have

$$\begin{bmatrix} (I_l^\perp)^T (\Sigma_{[h(t)]} + \Xi_{[h(t)]}) I_l^\perp & \epsilon (I_l^\perp)^T \Psi_l^T \\ \star & -I_n \end{bmatrix} < 0. \quad (55)$$

From Fact 1, inequality (55) is equivalent to

$$\begin{bmatrix} (I_l^\perp)^T \left(\hat{\Sigma}_{[h(t)]} + h(t) \mathbf{X} Q_1^{-1} \mathbf{X}^T \right. \\ \left. + (h_U - h(t)) \mathbf{Y} Q_1^{-1} \mathbf{Y}^T \right) I_l^\perp & \epsilon (I_l^\perp)^T \Psi_l^T \\ \star & -\epsilon I \\ \star & \star \\ \star & \star \\ \left[h(t) (I_l^\perp)^T \mathbf{Z} \right] \left[(h_U - h(t)) (I_l^\perp)^T \mathbf{W} \right] \\ \left[\begin{matrix} 0_n \\ -2Q_4 \\ \star \end{matrix} \right] \left[\begin{matrix} 0_n \\ 0 \\ -2Q_4 \end{matrix} \right] \end{bmatrix} < 0 \quad (l=1, \dots, r). \quad (56)$$

By the properties of convex-hull, inequality (56) holds if the following two inequalities are satisfied

$$\begin{bmatrix} (I_l^\perp)^T (\hat{\Sigma}_{[0]} + h_U \mathbf{Y} Q_1^{-1} \mathbf{Y}^T) I_l^\perp & \epsilon (I_l^\perp)^T \Psi_l^T \\ \star & -\epsilon I \\ \star & \star \\ \star & \star \\ \left[\begin{matrix} 0_{11n \times n} \\ 0_n \\ -2Q_4 \\ \star \end{matrix} \right] \left[\begin{matrix} h_U (I_l^\perp)^T \mathbf{W} \\ 0_n \\ 0 \\ -2Q_4 \end{matrix} \right] \end{bmatrix} < 0 \quad (l=1, \dots, r), \quad (57)$$

$$\begin{bmatrix} (I_l^\perp)^T (\hat{\Sigma}_{[h(t)]} + h_U \mathbf{X} Q_1^{-1} \mathbf{X}^T) I_l^\perp & \epsilon (I_l^\perp)^T \Psi_l^T \\ \star & -\epsilon I \\ \star & \star \\ \star & \star \\ \left[\begin{matrix} h_U (I_l^\perp)^T \mathbf{Z} \\ 0_n \\ -2Q_4 \\ \star \end{matrix} \right] \left[\begin{matrix} 0_{11n \times n} \\ 0_n \\ 0 \\ -2Q_4 \end{matrix} \right] \end{bmatrix} < 0 \quad (l=1, \dots, r). \quad (58)$$

Using Fact 1, the above inequalities (57) and (58) can be handled by the LMIs (46) and (47), which guarantee the stability of the system (12) by the Lyapunov stability method. This completes our proof. ■

Remark 1. When the information of the time-derivative of time-delay, $\dot{h}(t)$, is unknown, then, by setting $G_{11} = G_{12} = G_{22} = 0$ in both Theorem 1 and 2, we can easily obtain delay-dependent robust stability criteria for uncertain T-S NNs with time-varying delays, which do not need the information of $\dot{h}(t)$.

Remark 2. Since a delay-partitioning idea was firstly proposed in [21], it is well known that delay-partitioning

approach can enhance the feasible region of delay-dependent stability criteria due to the fact that this method can obtain more tighter upper bounds obtained by calculation of the time-derivative of Lyapunov-Krasovskii functional. In this regard, very recently, Balasubramaniam, and Chandran [16] proposed new delay-dependent stability criteria for fuzzy neural networks with time-varying delays by utilization of delay-partitioning techniques. However, the method [16] is still conservative because the proposed criteria were only single LMI regardless of divided subinterval. Furthermore, when the number of delay-partitioning number increases, the matrix formulation becomes more complex and the computational burden and time-consuming grow bigger. Noticing this fact mentioned above, the proposed Theorem 1 and 2 do not utilize the delay-partitioning technique. Instead, the utilized augmented vector has new integral terms such as $(1/h(t)) \int_{t-h(t)}^t x(s) ds$ and $(1/(h_U-h(t))) \int_{t-h_U}^{t-h(t)} x(s) ds$ in

order to utilize more information about $h(t)$ in the derived stability condition. To do this, in Theorem 1, some novel derivation are proposed at Eq. (25), (32) and (33). In Theorem 2, further improved stability criterion was derived by employing free weighting matrices at Eq. (49) and (50). It will be shown Theorem 1 and 2 significantly improve the feasible region of stability criterion comparing with the previous works [16] which utilized delay-partitioning technique.

Remark 3. In the field of delay-dependent stability analysis, one of major concerns is to get maximum delay bounds with fewer decision variables [22]-[24]. By use of Finsler lemma, one can eliminate free variables which were used in zero equalities in the works [24]-[25]. From Lemma 2, one can check that the $(B^\perp)^T \Phi B^\perp < 0$ is equivalent to the existence of \mathbf{L} such that $\Phi + \mathbf{L}B + B^T \mathbf{L}^T < 0$ holds. Insertion of such an additional matrix \mathbf{L} does not play a role to reduce the conservatism of $(B^\perp)^T \Phi B^\perp < 0$. It only increases the number of decision variables. Therefore, as presented at Eq. (54) and (55), the variable \mathbf{L} at Eq. (54) was eliminated by multiplying both side of (54) by $\begin{bmatrix} (I_l^\perp)^T 0_{10n \times n} \\ 0_{n \times 11n} I_n \end{bmatrix}$ and $\begin{bmatrix} I_l^\perp & 0_{11n \times n} \\ 0_{n \times 10n} & I_n \end{bmatrix}$.

4. Numerical Examples

In this section, we provide two numerical examples to show the effectiveness of the proposed stability criteria.

Example 1. Consider the following system

Rule 1: IF $x_1(t)$ is Θ_{11} , THEN

$$\dot{x}(t) = -(A_1 + \Delta A_1(t))x(t) + (W_{01} + \Delta W_{01}(t))f(x(t)) + (W_{11} + \Delta W_{11}(t))f(x(t-h(t))),$$

Rule 2: IF $x_1(t)$ is Θ_{21} , THEN

$$\dot{x}(t) = -(A_2 + \Delta A_2(t))x(t) + (W_{02} + \Delta W_{02}(t))f(x(t)) + (W_{12} + \Delta W_{12}(t))f(x(t-h(t))), \quad (59)$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} 2.2 & 0 \\ 0 & 1.8 \end{bmatrix}, W_{01} = \begin{bmatrix} 0 & 1.2 \\ 0.2 & 0 \end{bmatrix}, W_{11} = \begin{bmatrix} 2.8 & 1 \\ 1 & 1.8 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 2.5 & 0 \\ 0 & 2.7 \end{bmatrix}, W_{02} = \begin{bmatrix} 1 & 0.5 \\ 1 & 0.9 \end{bmatrix}, W_{12} = \begin{bmatrix} 2 & 0.4 \\ 0 & 1.5 \end{bmatrix}, \\ D &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_{il} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, E_{ol} = E_{il} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, l=1,2, \\ f(x) &= \frac{3}{20}(|x+1|-|x-1|), F(t) = \begin{bmatrix} \sin(100t) & 0 \\ 0 & \sin(100t) \end{bmatrix}, \\ K_p &= \text{diag}\{0.3, 0.3\}, K_m = \text{diag}\{0, 0\}, \\ \mu_1(x_1(t)) &= \sin^2(x_1), \mu_2(x_1(t)) = \cos^2(x_1). \end{aligned}$$

The results of the upper bound of time-delay with different values of h_D provided by Theorem 1 and 2 are listed as Table 1. The example shows that Theorem 1 and 2 obtain the less conservative results step by step. In Theorem 2, for further improved results, we add the triple-integral term with free-weighting matrices. To

표 1 h_D 의 다른 값들을 고려한 지연 한계값 h_U .

Table 1 Delay bounds h_U with different values of h_D .

h_D	0.8	0.9	unknown
Theorem 1	1.1485	1.1338	1.1301
Theorem 2	1.3414	1.3258	1.3182

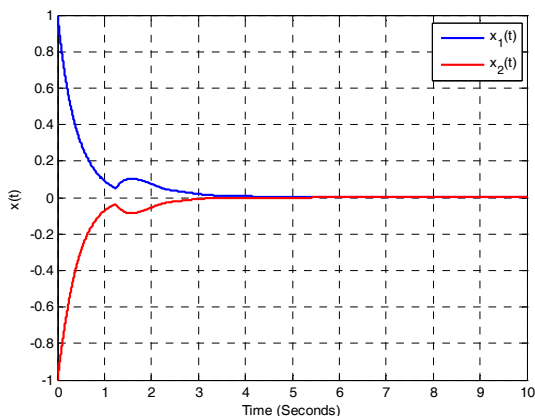


그림 1 시간지연 $h(t) = 1.3182|\sin(t)|$ 를 고려한 시스템의 상태 응답.

Fig. 1 State responses of the system with time-delay $h(t) = 1.3182|\sin(t)|$.

confirm the feasibility of our obtained results in Theorem 2 when h_D is unknown, Fig.1 shows the simulation results for the state response of system (59) when $h(t) = 1.3182|\sin(t)|$ and $x^T(0) = [1 \ -1]^T$. From Fig. 2, one can see the states of system (59) approach to zero as time increases.

Example 2. Consider the following system

Rule 1: IF $x_1(t)$ is Θ_{11} , THEN

$$\dot{x}(t) = -A_1x(t) + W_{01}f(x(t)) + W_{11}f(x(t-h(t))),$$

Rule 2: IF $x_1(t)$ is Θ_{21} , THEN

$$\dot{x}(t) = -A_2x(t) + W_{02}f(x(t)) + W_{12}f(x(t-h(t))), \quad (60)$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, W_{01} = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}, W_{11} = \begin{bmatrix} 1.8 & 1 \\ 2 & 1.8 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, W_{02} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, W_{12} = \begin{bmatrix} 2.6 & 0 \\ 0 & 2.5 \end{bmatrix}, \\ f(x) &= \frac{1}{10}(|x+1|-|x-1|), K_p = \text{diag}\{0.2, 0.2\}, K_m = \text{diag}\{0, 0\}, \\ \mu_1(x_1(t)) &= \sin^2(x_1), \mu_2(x_1(t)) = \cos^2(x_1). \end{aligned}$$

When the value of the time-derivative of time-delay is unknown, in [16], the obtained the upper bound of time-delay was 1.8450 ($N=15$). By applying Theorem 1 and 2 with Remark 1, one can obtain maximum delay bounds as listed in Table 2. From Table 2, one can confirm that the proposed Theorem 1 and 2 provides larger delay than the results of [16] in spite of no utilizing of delay-partitioning technique.

To check the effectiveness of the obtained results, the obtained LMI variables in Theorem 2 when $0 \leq h(t) \leq 2.9555$ and h_D is unknown are listed as follows

$$\begin{aligned} R_{11} &= 10^{-3} \times \begin{bmatrix} 257.5750 & 40.1372 \\ 40.1372 & 113.9509 \end{bmatrix}, \\ R_{12} &= 10^{-3} \times \begin{bmatrix} -32.4148 & -1.6964 \\ 2.7613 & -15.3564 \end{bmatrix}, \\ R_{13} &= 10^{-6} \times \begin{bmatrix} -120.8282 & 88.6497 \\ 88.7549 & -62.8308 \end{bmatrix}, \\ R_{22} &= 10^{-3} \times \begin{bmatrix} 35.0725 & -2.7844 \\ -2.7844 & 16.5553 \end{bmatrix}, \\ R_{23} &= 10^{-6} \times \begin{bmatrix} 379.5723 & -269.4190 \\ -267.7798 & 192.3107 \end{bmatrix}, \\ R_{33} &= 10^{-3} \times \begin{bmatrix} 1.5638 & -1.0979 \\ -1.0979 & 789.8479 \times 10^{-3} \end{bmatrix}, \\ N_{11} &= 10^{-3} \times \begin{bmatrix} 303.6355 & -59.2537 \\ -59.2537 & 155.6969 \end{bmatrix}, \\ N_{12} &= 10^{-3} \times \begin{bmatrix} 19.5203 & -2.6812 \\ -735.4298 \times 10^{-3} & 6.6207 \end{bmatrix}, \\ N_{13} &= 10^{-3} \times \begin{bmatrix} 2.7188 & -32.3469 \\ 70.1444 & -49.1782 \end{bmatrix}, \end{aligned}$$

$$N_{22} = 10^{-3} \times \begin{bmatrix} 6.0611 & -715.2201 \times 10^{-3} \\ -715.2201 \times 10^{-3} & 3.4485 \end{bmatrix},$$

$$N_{23} = 10^{-3} \times \begin{bmatrix} 10.4683 & -8.0475 \\ -7.3724 & 5.7918 \end{bmatrix},$$

$$N_{33} = 10^{-3} \times \begin{bmatrix} 382.5831 & -6.2382 \\ -6.2382 & 228.5301 \end{bmatrix},$$

$$Q_1 = 10^{-3} \times \begin{bmatrix} 53.7157 & 10.4774 \\ 10.4774 & 27.0569 \end{bmatrix},$$

$$Q_2 = 10^{-3} \times \begin{bmatrix} 1.4126 & -1.0097 \\ -1.0097 & 721.7090 \times 10^{-3} \end{bmatrix},$$

$$Q_3 = 10^{-6} \times \begin{bmatrix} 71.2880 & -50.9449 \\ -50.9449 & 36.4100 \end{bmatrix},$$

$$Q_4 = 10^{-6} \times \begin{bmatrix} 128.2599 & -91.6587 \\ -91.6587 & 65.5082 \end{bmatrix},$$

$$X_1 = 10^{-6} \times \begin{bmatrix} -215.9119 & 154.0798 \\ 173.1332 & -123.7650 \end{bmatrix},$$

$$X_2 = 10^{-3} \times \begin{bmatrix} 34.1866 & 6.3701 \\ 5.4227 & 16.5650 \end{bmatrix},$$

$$Y_1 = 10^{-3} \times \begin{bmatrix} -34.4206 & -6.1932 \\ -5.2696 & -16.6810 \end{bmatrix},$$

$$Y_2 = 10^{-3} \times \begin{bmatrix} 2.8388 & -2.0498 \\ -1.9869 & 1.4324 \end{bmatrix},$$

$$Z_1 = 10^{-6} \times \begin{bmatrix} -86.9822 & 62.1604 \\ 62.1638 & -44.4283 \end{bmatrix},$$

$$Z_2 = 10^{-6} \times \begin{bmatrix} 85.9105 & -61.3945 \\ -61.3932 & 43.8776 \end{bmatrix},$$

$$W_1 = 10^{-6} \times \begin{bmatrix} -86.7428 & 61.9892 \\ 61.9891 & -44.3034 \end{bmatrix},$$

$$W_2 = 10^{-6} \times \begin{bmatrix} 85.3427 & -60.9887 \\ -60.9882 & 43.5881 \end{bmatrix},$$

$$\Gamma = 10^{-3} \times \begin{bmatrix} 286.1302 & 0 \\ 0 & 263.0564 \end{bmatrix},$$

$$\Delta = 10^{-3} \times \begin{bmatrix} 385.4016 & 0 \\ 0 & 285.5186 \end{bmatrix},$$

$$H_1 = 10^{-3} \times \begin{bmatrix} 1.2122 & 0 \\ 0 & 322.8737 \end{bmatrix}, H_2 = \begin{bmatrix} 2.0952 & 0 \\ 0 & 1.0588 \end{bmatrix},$$

$$H_3 = 10^{-3} \times \begin{bmatrix} 493.3945 & 0 \\ 0 & 131.9094 \end{bmatrix}.$$

With the solutions presented above, the eigenvalues of (46) and (47) can be obtained as follows:

- I) Eigenvalues in the left half side of LMI (46) when $l=1$ and $h(t)=0$: -2.9649, -2.0784, -1.3829, -719.5350 $\times 10^{-3}$, -510.5963 $\times 10^{-3}$, -415.5768 $\times 10^{-3}$, -347.9996 $\times 10^{-3}$, -347.9996 $\times 10^{-3}$, -332.2627 $\times 10^{-3}$, -230.7363 $\times 10^{-3}$, -128.3570 $\times 10^{-3}$, -95.7480 $\times 10^{-3}$, -53.5449 $\times 10^{-3}$, -34.9795 $\times 10^{-3}$, -7.4300 $\times 10^{-3}$, -3.9802 $\times 10^{-3}$, -1.7249 $\times 10^{-3}$, -769.2002 $\times 10^{-6}$, -387.5283 $\times 10^{-6}$, -290.9591 $\times 10^{-6}$, -211.8022 $\times 10^{-6}$, -109.8547 $\times 10^{-9}$, -11.3097 $\times 10^{-9}$, -7.8198 $\times 10^{-9}$, -4.0381 $\times 10^{-9}$,

표 2 h_D 의 다른 값들을 고려한 지연 한계값 h_U .

Table 2 Delay bounds h_U with different values of h_D .

h_D	unknown
[16](N=2)	1.1104
[16](N=3)	1.3044
[16](N=4)	1.4480
[16](N=5)	1.5610
[16](N=6)	1.6530
[16](N=10)	1.8110
[16](N=15)	1.8450
Theorem 1	2.5356
Theorem 2	2.9555

(N: Delay-partitioning number in [16])

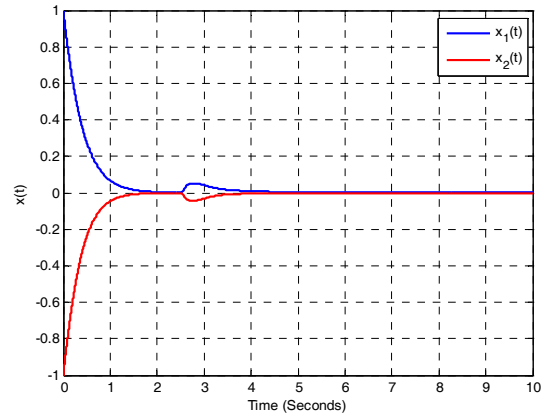


그림 2 시간지연 $h(t) = 1.9555 + |\sin(t)|$ 를 고려한 시스템의 상태 응답.

Fig. 2 State responses of the system with time-delay $h(t) = 1.9555 + |\sin(t)|$.

II) Eigenvalues in the left half side of LMI (46) when $l=1$ and $h(t)=h_U$: -2.9649, -2.0784, -1.3829,

- 719.5149 $\times 10^{-3}$, -510.5969 $\times 10^{-3}$, -415.5828 $\times 10^{-3}$, -347.9996 $\times 10^{-3}$, -347.9996 $\times 10^{-3}$, -330.0160 $\times 10^{-3}$, -232.9460 $\times 10^{-3}$, -126.7276 $\times 10^{-3}$, -96.8694 $\times 10^{-3}$, -53.6605 $\times 10^{-3}$, -35.3668 $\times 10^{-3}$, -6.8544 $\times 10^{-3}$, -3.9567 $\times 10^{-3}$, -1.7239 $\times 10^{-3}$, -779.5009 $\times 10^{-6}$, -387.5283 $\times 10^{-6}$, -356.4943 $\times 10^{-6}$, -189.5120 $\times 10^{-6}$, -109.5628 $\times 10^{-9}$, -7.8198 $\times 10^{-9}$, -7.5098 $\times 10^{-9}$, -6.3338 $\times 10^{-9}$, -152.3757 $\times 10^{-12}$,

III) Eigenvalues in the left half side of LMI (47) when $l=1$ and $h(t)=0$: -2.8298, -1.5652, -1.3829, -1.0491,

- 510.6296 $\times 10^{-3}$, -417.9014 $\times 10^{-3}$, -347.9996 $\times 10^{-3}$, -347.9996 $\times 10^{-3}$, -333.0022 $\times 10^{-3}$, -237.3941 $\times 10^{-3}$, -126.4340 $\times 10^{-3}$, -94.3961 $\times 10^{-3}$, -22.4741 $\times 10^{-3}$, -15.5304 $\times 10^{-3}$, -7.2391 $\times 10^{-3}$, -5.4993 $\times 10^{-3}$, -1.1738 $\times 10^{-3}$,

-565.9613×10^{-6} , -387.5283×10^{-6} , -214.1760×10^{-6} ,
 -16.4015×10^{-6} , -106.3722×10^{-9} , -7.8651×10^{-9} , -7.8198×10^{-9} ,
 -5.2265×10^{-9} , -3.8436×10^{-9} ,

IV) Eigenvalues in the left half side of LMI (47) when $l=1$ and $h(t)=h_U$: -2.8299 , -1.5654 , -1.3829 , -1.0491 ,
 -510.6306×10^{-3} , -417.8997×10^{-3} , -347.9996×10^{-3} ,
 -347.9996×10^{-3} , -330.5698×10^{-3} , -239.7325×10^{-3} ,
 -124.8240×10^{-3} , -95.5990×10^{-3} , -22.3386×10^{-3} ,
 -15.7009×10^{-3} , -7.0464×10^{-3} , -5.5123×10^{-3} , -1.2080×10^{-3} ,
 -387.5283×10^{-6} , -370.1519×10^{-6} , -344.6671×10^{-6} ,
 -11.0078×10^{-6} , -108.7359×10^{-9} , -7.8198×10^{-9} , -7.3497×10^{-9} ,
 -4.2064×10^{-9} , $-386.5412 \times 10^{-12}$,

From the results mentioned above, all the eigenvalues of LMs (46) and (47) are negative, which means the system (60) is asymptotically stable when $0 \leq h(t) \leq 2.9555$ and h_D is unknown.

The simulation result for the state responses of the system (60) is shown in Figure 2. We assume that the state-delay $h(t) = 1.9555 + |\sin(t)|$ and $x^T(0) = [1 \ -1]^T$. To solve the above system (60) employ Fourth-order Runge-Kutta method with sampling time 0.001[sec]. Figure 2 show that the system (60) responses converge to zero for chosen initial values of the state.

5. Conclusions

In this paper, the delay-dependent stability criteria for the uncertain fuzzy NNs with time-varying delays have been proposed. To do this, the suitable L-K functional is used to investigate the feasible region of stability criteria. Two numerical examples have confirmed the effectiveness and usefulness of the presented criteria.

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