시변 지연을 가진 불확실 뉴럴 네트워크에 대한 지연의존 강인 수동성

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Delay-dependent Robust Passivity for Uncertain Neural Networks with Time-varying Delays

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Abstract - In this paper, the problem of passivity analysis for neural networks with time-varying delays and norm-bounded parameter uncertainties is considered. By constructing a new augmented Lyapunov functional, a new delay-dependent passivity criterion for the network is established in terms of LMIs (linear matrix inequalities) which can be easily solved by various convex optimization algorithms. Two numerical example are included to show the effectiveness of proposed criterion.

Key Words : Neural network, Passivity, Time-varying delays, Lyapunov method, Linear matrix inequality

1. Introduction

Recently, neural networks have been successfully applied in various science and systems in Engineering such as pattern recognition, image processing, signal processing, fixed-point computations, associative memory, and other scientific areas. In application of neural networks, the equilibrium points of the designed networks should be stable. Also, in the hardware implementation of neural networks, time delays often occurs due to the finite switching speed of amplifiers and the inherent communication of neurons. Since it is well known that the existence of time delays may cause poor performance, oscillation, or instability, the stability analysis of neural networks with time delays has been extensively studied. For references, see [1]-[5] and references therein.

On the other hand, in various engineering and scientific areas, stability analysis can be linked to the theory of dissipative systems which postulates that the energy dissipated inside a dynamic system is less than the energy supplied from external source [6]. In the field of nonlinear control, the concept of dissipativeness was firstly introduced by Willems [7] in the form of inequality including the storage and supply rate. Since then, passivity analysis has been one of important method for analysing the stability of nonlinear system. The main advantage of passivity analysis is that the passivity properties of a concerned system can keep the system internal stability by only using input-output characteristics and thus can lead to general conclusions on stability. In this regard, many efforts have been devoted to passivity analysis for neural networks with time-varying delays [8]-[11]. Chen et al. [9] investigated the problem of passivity for uncertain neural networks with time-varying discrete and distributed by utilizing delay-decomposition technique and free weight matrices. Recently, the problem of passivity analysis for uncertain stochastic neural networks with interval time-varying delays was studied by Fu et al. [10]. Very recently, in [11], without ignoring any useful information in the deriving process, improved passivity criteria for uncertain neural networks with time-varying delays was derived in terms of LMIs. However, there are still rooms for further improvements in passivity criteria for neural networks with time-varying delays.

In this paper, the problem of passivity analysis for uncertain neural networks with time-varying delays and norm-bounded parameter uncertainties is investigated. Unlike the methods of [9]-[11], no free weighting matrices are employed in deriving passivity criterion of the concerned networks. Instead, by taking an augmented vector $\zeta(t)$ which includes integral terms of activation functions such as $\int_{t-h(t)}^{t} f(x(s))ds$ and $\int_{t-h_U}^{t-h(t)} f(x(s))ds$. a delay-dependent sufficient condition such that the considered neural networks are passive is derived in terms of LMIs. By utilizing these integral terms in deducing passivity criterion, more past information of

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$f(x(t))$ will be utilized in Theorem 1. Furthermore, inspired by the work of [12]-[13], a triple integral forms of Lyapunov-Krasovskii's functional will be utilized. Through two numerical examples utilized in previous literature, it will be shown that our proposed criterion provide larger feasible region for guaranteeing passivity than the results in other literature.

Notations: $\mathbf{R}^n$ is the n-dimensional Euclidean space, $\mathbf{R}^{m\times n}$ denotes the set of $m\times n$ real matrices. For symmetric matrices X and Y, the notation $X>Y$ (respectively, $X\geq Y$) means that the matrix $X-Y$ is positive definite (respectively, nonnegative). $diag\{\ldots\}$ denotes the block diagonal matrix. $X_{[f(t)]}$ means that the elements of matrix $X_{[f(t)]}$ include the scalar function of $f(t)$. The subscript "T" represents the transpose.

2. Problem formulation

Consider the following uncertain neural networks with time-varying delays:

$$\begin{aligned}\dot{x}(t) &= -(A+\Delta A(t))x(t)+(W_0+\Delta W_0(t))f(x(t)) \\ &\quad +(W_1+\Delta W_1(t))f(x(t-h(t)))+u(t) \\ y(t) &= C_1f(x(t))+C_2f(x(t-h(t))),\end{aligned} \tag{1}$$

where $x(t)=[x_1(t),\ldots x_n(t)]^T\in\mathbf{R}^n$ means the neuron state vector, n denotes the number of neurons in a neural networks, $y(t)=[y_1(t),\ldots,y_n(t)]^T\in\mathbf{R}^n$ is the output vector, $f(x(t))=[f_1(x_1(t)),\ \ldots,\ f_n(x_n(t))]^T\in\mathbf{R}^n$ is the neuron activation function of neural networks, $g(y(t-h(t)))=[g_1(y_1(t-h(t))),\ \ldots,\ g_n(y_n(t-h(t)))]^T\in\mathbf{R}^n$ is the delayed neuron activation function, $A=diag\{a_i\}\in\mathbf{R}^{n\times n}$ is a positive diagonal matrix, $W_0=(w_{ij}^0)_{n\times n}\in\mathbf{R}^{n\times n}$, and $W_1=(w_{ij}^1)_{n\times n}\in\mathbf{R}^{n\times n}$ are the interconnection matrices representing the weight coefficients of the neurons, $u(t)=[u_1(t),u_2(t),\ldots,u_n(t)]^T\in\mathbf{R}^n$ is an external input vector to neurons, $C_1\in\mathbf{R}^{n\times n}$ and $C_2\in\mathbf{R}^{n\times n}$ are known constant matrices, and $\Delta A(t)$, $\Delta W_0(t)$, and $\Delta W_1(t)$ are the uncertainties of system matrices of the form

$$[\Delta A(t)\ \ \Delta W_0(t)\ \ \Delta W_1(t)]=DF(t)[E_1\quad E_2\quad E_3] \tag{2}$$

where the time-varying nonlinear function $F(t)$ satisfies

$$F^T(t)F(t)\leq I,\quad \forall t\geq 0. \tag{3}$$

The delay, $h(t)$, is a time-varying continuous function satisfying

$$0\leq h(t)\leq h_U,\ \ \dot{h}(t)\leq h_D, \tag{4}$$

where h_U is a positive scalar and h_D is any constant one.

Assume that the activation functions, $f_i(x_i(t)), i=1,\ldots,n,$ be nondecreasing, bounded and globally Lipschitz;that is,

$$k_i^-\leq f_i(\zeta)/\zeta\leq k_i^+,\ \forall\zeta\neq 0,\ i=1,\ldots,n \tag{5}$$

which is equivalent to the following inequality

$$[f_i(\zeta)-k_i^-\zeta][f_i(\zeta)-k_i^+\zeta]\leq 0,\ i=1,\ldots,n. \tag{6}$$

Let us define

$$\begin{aligned}q(t) &= -E_1x(t)+E_2f(x(t))+E_3f(x(t-h(t))), \\ p(t) &= F(t)q(t),\end{aligned} \tag{7}$$

then, system (1) can ber rewritten as

$$\begin{aligned}\dot{x}(t) &= -Ax(t)+W_0f(x(t))+W_1f(x(t-h(t)))+Dp(t)+u(t) \\ y(t) &= C_1f(x(t))+C_2f(x(t-h(t))).\end{aligned} \tag{8}$$

The objective of this paper is to investigate robust delay-dependent passivity criteria for the above system (8). Before deriving main results, the following definition and lemmas are needed.

Definition 1 [6]. The system (1) is called passive if there exists a scalar $\gamma\geq 0$ such that

$$2\int_0^{t_p}y^T(s)u(s)ds\geq -\gamma\int_0^{t_p}u^T(s)u(s)ds \tag{9}$$

for all $t_p\geq 0$ and all solution of (1) with $x(0)=0$.

Lemma 1 (Finsler's Lemma) [14]. Let $\zeta\in\mathbf{R}^n$, $\Phi=\Phi^T\in\mathbf{R}^{n\times n}$, and $B\in\mathbf{R}^{m\times n}$ such that $rank(B)<n$. The following statement is equivalent:

i) $\zeta^T\Phi\zeta<0,\ \forall B\zeta=0,\ \zeta\neq 0,$

ii) $(B^\perp)^T\Phi B^\perp<0$

where $B^\perp$ is a right orthogonal complement of B.

Lemma 2 [15]. For any constant matrix $M\in\mathbf{R}^{n\times n}$, $M=M^T>0$, a scalar $\gamma>0$ and a vector function $x:[0,\gamma]\rightarrow\mathbf{R}^n$ such that the integrations concerned are well defined, then

$$\gamma\int_0^\gamma x^T(s)Mx(s)ds\geq\left(\int_0^\gamma x(s)ds\right)^TM\left(\int_0^\gamma x(s)ds\right) \tag{10}$$

Lemma 3 [13]. From Lemma 1, for any constant matrix G, $G=G^T>0$, the following integral inequalities can be easily obtained:

$$\begin{aligned}&(h^2/2)\int_{t-h}^t\int_s^t x^T(u)Gx(u)duds \\ &\geq\left(\int_{t-h}^t\int_s^t x(u)duds\right)^TG\left(\int_{t-h}^t\int_s^t x(u)duds\right)\end{aligned} \tag{11}$$

Lemma 4 [5]. For any constant matrices Φ_1 and Φ_2 of appropriate dimensions, a symmetric matrix $Y<0$, a positive scalar h_U, and a scalar function $h(t):\mathbf{R}^{+}\rightarrow[0,h_U]$, the following inequality

$$h(t)\Phi_1+(h_U-h(t))\Phi_2+Y<0 \tag{12}$$

holds, if and only if

$$h_U\Phi_1+Y<0, \tag{13}$$
$$h_U\Phi_2+Y<0. \tag{14}$$

3. Main result

In this section, new robust passivity criterion for system (8) will be proposed. To simplify the notation of our main results, $e_i\,(i=1,\,2,\,\ldots,\,14)\in R^{14n\times n}$ are defined as block entry matrices. For example, $e_5^T=[0\,0\,0\,0\,I\;0\,0\,0\,0\,0\,0\,0\,0\,0]$. The notations for some matrices are defined as:

$$\begin{aligned}
\xi^T(t)=&[x^T(t)\;\; x^T(t-h(t))\;\; x^T(t-h_U)\;\; \dot{x}(t)\;\; \dot{x}(t-h_U)\\
&\int_{t-h(t)}^{t}x^T(s)ds\;\; \int_{t-h_U}^{t-h(t)}x^T(s)ds\;\; f^T(x(t))\;\; f^T(x(t-h(t)))\\
&f^T(x(t-h_U))\;\; \int_{t-h(t)}^{t}f^T(x(s))ds\;\; \int_{t-h_U}^{t-h(t)}f^T(x(s))ds\\
&u^T(t)\;\; p^T(t)],
\end{aligned}$$

$$\alpha(t)=\left[x^T(t)\;\; \dot{x}^T(t)\;\; f^T(x(t))\right],$$

$$\Pi=\left[-A\;\;0\;\;0\;\;-I\;\;\;0\;\;0\;\;0\;\;W_0\;W_1\;\;\;0\;\;0\;\;0\;\;I\;D\right],$$

$$\Gamma_1=\left[e_1\;\;e_3\;\;e_6+e_7\;\;e_{11}+e_{12}\right],\;\;\Gamma_2=\left[e_4\;\;e_5\;\;e_1-e_3\;\;e_8-e_{10}\right],$$

$$\Gamma_3=\left[e_1\;e_4\;\;e_8\right],\;\;\Gamma_4=\left[e_3\;e_5\;\;e_{10}\right],\;\;\Gamma_5=\left[e_1\;e_8\right],\;\;\Gamma_6=\left[e_2\;e_9\right],$$

$$\begin{aligned}
\Omega_{[h(t)]}=&(-2+h_U^{-1}h(t))[e_6G_{11}e_6^T+e_6G_{12}e_1^T+e_1G_{12}^Te_6^T-e_6G_{12}e_2^T\\
&-e_2G_{12}^Te_6^T+e_1G_{22}e_1^T\;-e_1G_{22}e_2^T-e_2G_{22}e_1^T\\
&+e_2G_{22}e_2^T+e_6G_{13}e_{11}^T+e_{11}G_{13}^Te_6^T+e_1G_{23}e_{11}^T+e_{11}G_{23}^Te_1^T\\
&-e_2G_{23}e_{11}^T-e_{11}G_{23}^Te_2^T+e_{11}G_{33}e_{11}^T]\\
&+(-1-h_U^{-1}h(t))[e_7G_{11}e_7^T+e_7G_{12}e_2^T+e_2G_{12}^Te_7^T\\
&-e_7G_{12}e_3^T-e_3G_{12}^Te_7^T+e_2G_{22}e_2^T-e_2G_{22}e_3^T-e_3G_{22}e_2^T\\
&+e_3G_{22}e_3^T+e_7G_{13}e_{12}^T+e_{12}G_{13}^Te_7^T+e_2G_{23}e_{12}^T+e_{12}G_{23}^Te_2^T\\
&-e_3G_{23}e_{12}^T-e_{12}G_{23}^Te_3^T+e_{12}G_{33}e_{12}^T],
\end{aligned}$$

$$\begin{aligned}
\Psi=&e_1G_{11}e_1^T+e_1G_{12}e_4^T+e_4G_{22}e_4^T+e_1G_{13}e_8^T+e_4G_{23}e_8^T+e_8G_{33}e_8\\
&+e_4G_{12}^Te_1^T+e_8G_{13}^Te_1^T+e_8G_{23}^Te_4^T,
\end{aligned}$$

$$\begin{aligned}
\Phi=&-2e_1K_mH_1K_pe_1^T+e_1(K_m+K_p)H_1e_8^T+e_8H_1(K_m+K_p)e_1^T\\
&-2e_8H_1e_8^T-2e_2K_mH_2K_pe_2^T+e_2(K_m+K_p)H_2e_9^T\\
&+e_9H_2(K_m+K_p)e_2^T-2e_9H_2e_9^T-2e_3K_mH_3K_pe_3^T\\
&+e_3(K_m+K_p)H_3e_{10}^T+e_{10}H_3(K_m+K_p)e_3^T-2e_{10}H_3e_{10}^T,
\end{aligned}$$

$$\begin{aligned}
\Xi=&-e_8C_1^Te_{13}^T-e_{13}C_1e_8^T-e_9C_2^Te_{13}^T-e_{13}C_2e_9^T+\varepsilon e_1E_1^TE_1e_1^T\\
&-\varepsilon e_1E_1^TE_2e_8^T-\varepsilon e_8E_2^TE_1e_1^T-\varepsilon e_1E_1^TE_3e_9^T-\varepsilon e_9E_3^TE_1e_1^T\\
&+\varepsilon e_8E_2^TE_2e_8^T+\varepsilon e_8E_2^TE_3e_9^T+\varepsilon e_9E_3^TE_2e_8^T+\varepsilon e_9E_3^TE_3e_9^T-\varepsilon e_{14}e_{14}^T,
\end{aligned}$$

$$\Theta=(h_U^4/4)e_4Me_4^T-(h_Ue_1-e_6-e_7)M(h_Ue_1-e_6-e_7)^T,$$

$$\begin{aligned}
\Omega=&\;e_8\Lambda e_4^T+e_4\Lambda e_8^T-e_1K_m\Lambda e_4^T-e_4\Lambda K_me_1^T+e_1K_p\Delta e_4^T\\
&+e_4\Delta K_pe_1^T-e_8\Delta e_4^T-e_4\Delta e_8^T,
\end{aligned}$$

$$\begin{aligned}
\Sigma=&\;\Gamma_1\boldsymbol{R}\Gamma_2^T+\Gamma_2\boldsymbol{R}\Gamma_1^T+\Gamma_3\boldsymbol{N}\Gamma_3^T-\Gamma_4\boldsymbol{N}\Gamma_4^T+\Gamma_5\boldsymbol{Q}\Gamma_5^T-(1-h_D)\Gamma_6\boldsymbol{Q}\Gamma_6^T\\
&+h_U^2\Psi+\Phi+\Xi.
\end{aligned} \tag{15}$$

Now, the following Theorem 1 will be introduced.

Theorem 1. For given scalars $h_U>0$ and h_D, diagonal matrices $K_p=diag\{\mathrm{k}_1^+,\ldots,\mathrm{k}_\mathrm{n}^+\}$, and $K_m=diag\{\mathrm{k}_1^-,\ldots,\mathrm{k}_\mathrm{n}^-\}$, the system (8) is passive for $0\le h(t)\le h_U$ and $\dot{h}(t)\le h_D$ if there exists positive diagonal matrices $H_j=diag\mathrm{h}_{\mathrm{j}1},\ldots,\mathrm{h}_{\mathrm{jn}}$ $(j=1,2,3)$, $\Lambda=diag\{\lambda_1,\ldots,\lambda_\mathrm{n}\}$, $\Delta=diag\{\delta_1,\ldots,\delta_\mathrm{n}\}$, positive definite matrices $\boldsymbol{R}=[R_{ij}]_{4\times4}\in\mathbf{R}^{4n\times4n}$, $\boldsymbol{N}=[N_{ij}]_{3\times3}\in\mathbf{R}^{3n\times3n}$, $\boldsymbol{G}=[G_{ij}]_{3\times3}\in\mathbf{R}^{3n\times3n}$, $\boldsymbol{Q}=[Q_{ij}]_{2\times2}\in\mathbf{R}^{2n\times2n}$, $M\in\mathbf{R}^{n\times n}$, and a positive scalar ε and γ satisfying the following LMIs:

$$(\Pi^{\perp})^T\left\{\Sigma+\Omega_{[h(t)=0]}\right\}(\Pi^{\perp})<0, \tag{16}$$
$$(\Pi^{\perp})^T\left\{\Sigma+\Omega_{[h(t)=h_U]}\right\}(\Pi^{\perp})<0, \tag{17}$$

where Σ, $\Omega_{[h(t)]}$, and Π are defined in (15) and $\Pi^{\perp}$ is the right orthogonal complement of Π.

Proof. With positive diagonal matrices $\Lambda=diag\{\lambda_1,\ldots,\lambda_\mathrm{n}\}$, $\Delta=diag\{\delta_1,\ldots,\delta_\mathrm{n}\}$, positive definite matrices $\boldsymbol{R}=[R_{ij}]_{4\times4}\in\mathbf{R}^{4n\times4n}$, $\boldsymbol{N}=[N_{ij}]_{3\times3}\in\mathbf{R}^{3n\times3n}$, $\boldsymbol{G}=[G_{ij}]_{3\times3}\in\mathbf{R}^{3n\times3n}$, $\boldsymbol{Q}=[Q_{ij}]_{2\times2}\in\mathbf{R}^{2n\times2n}$, $M\in\mathbf{R}^{n\times n}$, let us take the following Lyapunov-Krasovskii's functional candidate:

$$V=\sum_{i=1}^{4}V_i$$

where

$$\begin{aligned}
V_1=&\begin{bmatrix} x(t)\\ x(t-h_U)\\ \int_{t-h_U}^{t}x(s)ds\\ \int_{t-h_U}^{t}f(x(s))ds\end{bmatrix}^T\boldsymbol{R}\begin{bmatrix} x(t)\\ x(t-h_U)\\ \int_{t-h_U}^{t}x(s)ds\\ \int_{t-h_U}^{t}f(x(s))ds\end{bmatrix},\\
V_2=&\int_{t-h(t)}^{t}\begin{bmatrix} x(s)\\ f(x(s))\end{bmatrix}^T\boldsymbol{Q}\begin{bmatrix} x(s)\\ f(x(s))\end{bmatrix}ds,\\
V_3=&\int_{t-h_U}^{t}\alpha^T(s)\boldsymbol{N}\alpha(s)ds\\
&+2\sum_{i=1}^{n}\left(\lambda_i\int_0^{x_i(t)}(f_i(s)-k_i^-s)ds+\delta_i\int_0^{x_i(t)}(k_i^+s-f_i(s))ds\right),\\
V_4=&h_U\int_{t-h_U}^{t}\int_{s}^{t}\alpha^T(u)\boldsymbol{G}\alpha(u)duds,\\
V_5=&(h_U^2/2)\int_{t-h_U}^{t}\int_{s}^{t}\int_{u}^{t}\dot{x}^T(v)M\dot{x}(v)dvduds.
\end{aligned} \tag{18}$$

By calculating the time derivative of V_1, we have

$$\dot{V}_1 = \begin{bmatrix} x(t) \\ x(t-h_U) \\ \int_{t-h(t)}^{t} x(s)ds + \int_{t-h_U}^{t-h(t)} x(s)ds \\ \int_{t-h(t)}^{t} f(x(s))ds + \int_{t-h_U}^{t-h(t)} f(x(s))ds \end{bmatrix} \times \boldsymbol{R} \begin{bmatrix} \dot{x}(t) \\ \dot{x}(t-h_U) \\ x(t) - x(t-h_U) \\ f(x(t)) - f(x(t-h_U)) \end{bmatrix},$$
$$= \zeta^T(t)[\Gamma_1 \boldsymbol{R} \Gamma_2^T + \Gamma_2 \boldsymbol{R} \Gamma_1^T]\zeta(t). \tag{19}$$

An upper bound of $\dot{V}_2$ can be

$$\dot{V}_2 \le \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \boldsymbol{Q} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} - (1-h_D) \begin{bmatrix} x(t-h(t)) \\ f(x(t-h(t))) \end{bmatrix}^T \boldsymbol{Q} \begin{bmatrix} x(t-h(t)) \\ f(x(t-h(t))) \end{bmatrix}.$$
$$= \Gamma_5 \boldsymbol{Q} \Gamma_5^T - (1-h_D)\Gamma_6 \boldsymbol{Q} \Gamma_6^T \tag{20}$$

Calculating the time derivative of V_3 leads to

$$\begin{aligned} \dot{V}_3 =& \ \alpha^T(t)\boldsymbol{N}\alpha(t) - \alpha^T(t-h_U)\boldsymbol{N}\alpha(t-h_U) \\ &+ 2[f(x(t)) - K_m x(t)]^T \Lambda \dot{x}(t) \\ &+ 2[K_p x(t) - f(x(t))]^T \Delta \dot{x}(t) \\ =& \ \zeta^T(t)[\Gamma_3 \boldsymbol{N} \Gamma_3^T - \Gamma_4 \boldsymbol{N} \Gamma_4^T + \Omega]\zeta(t). \end{aligned} \tag{21}$$

Calculation of the time derivative of V_4 yields

$$\dot{V}_4 = h_U^2 \alpha^T(t)\boldsymbol{G}\alpha(t) - h_U \int_{t-h_U}^{t} \alpha^T(s)\boldsymbol{G}\alpha(s)ds. \tag{22}$$

Here, by the use of Lemma 2 and the method of [16], the term $-h_U \int_{t-h_U}^{t} \alpha^T(s)\boldsymbol{G}\alpha(s)ds$ in (22) can be estimated as

$$\begin{aligned} & -h_U \int_{t-h_U}^{t} \alpha^T(s)\boldsymbol{G}\alpha(s)ds \\ =& -h_U \int_{t-h(t)}^{t} \alpha^T(s)\boldsymbol{G}\alpha(s)ds - h_U \int_{t-h_U}^{t-h(t)} \alpha^T(s)\boldsymbol{G}\alpha(s)ds \\ =& -(h_U - h(t)) \int_{t-h(t)}^{t} \alpha^T(s)\boldsymbol{G}\alpha(s)ds - h(t) \int_{t-h(t)}^{t} \alpha^T(s)\boldsymbol{G}\alpha(s)ds \\ & -(h_U - h(t)) \int_{t-h_U}^{t-h(t)} \alpha^T(s)\boldsymbol{G}\alpha(s)ds - h(t) \int_{t-h_U}^{t-h(t)} \alpha^T(s)\boldsymbol{G}\alpha(s)ds \\ \le& \ -h_U^{-1}(h_U - h(t))h(t) \int_{t-h(t)}^{t} \alpha^T(s)\boldsymbol{G}\alpha(s)ds \\ & -h(t) \int_{t-h(t)}^{t} \alpha^T(s)\boldsymbol{G}\alpha(s)ds \\ & -(h_U - h(t)) \int_{t-h_U}^{t-h(t)} \alpha^T(s)\boldsymbol{G}\alpha(s)ds \\ & -h_U^{-1}h(t)(h_U - h(t)) \int_{t-h_U}^{t-h(t)} \alpha^T(s)\boldsymbol{G}\alpha(s)ds \\ \le& \ (-2 + h_U^{-1}h(t)) \left(\int_{t-h(t)}^{t} \alpha(s)ds \right)^T \boldsymbol{G} \left(\int_{t-h(t)}^{t} \alpha(s)ds \right) \\ & + (-1 - h_U^{-1}h(t)) \left(\int_{t-h_U}^{t-h(t)} \alpha(s)ds \right)^T \boldsymbol{G} \left(\int_{t-h_U}^{t-h(t)} \alpha(s)ds \right) \\ =& \ \zeta^T \Omega_{[h(t)]} \zeta(t). \end{aligned} \tag{23}$$

From (22)-(23), $\dot{V}_4$ has an upper bound as

$$\dot{V}_4 \le \zeta^T(t)[h_U^2 \Psi + \Omega_{[h(t)]}]\zeta(t). \tag{24}$$

By utilizing Lemma 3, an upper bound of $\dot{V}_5$ can be

$$\begin{aligned} \dot{V}_5 =& \ (h_U^4/4)\dot{x}^T(t)M\dot{x}(t) - (h_U^2/2)\int_{t-h_U}^{t}\int_{s}^{t} \dot{x}^T(u)M\dot{x}(u)duds \\ \le& \ (h_U^4/4)\dot{x}^T(t)M\dot{x}(t) \\ & -\left(\int_{t-h_U}^{t}\int_{s}^{t} \dot{x}(u)duds \right)^T M \left(\int_{t-h_U}^{t}\int_{s}^{t} \dot{x}(u)duds \right) \\ =& \ (h_U^4/4)\dot{x}^T(t)M\dot{x}(t) \\ & -\left(h_U x(t) - \int_{t-h(t)}^{t} x(s)ds - \int_{t-h_U}^{t-h(t)} x(s)ds \right)^T \\ & \times M\left(h_U x(t) - \int_{t-h(t)}^{t} x(s)ds - \int_{t-h_U}^{t-h(t)} x(s)ds \right). \\ =& \ \zeta^T(t)\Theta\zeta(t). \end{aligned} \tag{25}$$

Here, note that Eq. (6) means

$$[f_j(x_j(t)) - k_j^- x_j(t)][f_j(x_j(t)) - k_j^+ x_j(t)] \le 0 \ \ (j=1,...,n), \tag{26}$$

$$[f_j(x_j(t-h(t))) - k_j^- x_j(t-h(t))] \times [f_j(x_j(t-h(t))) - k_j^+ x_j(t-h(t))] \le 0 \ \ (j=1,...,n), \tag{27}$$

$$[f_j(x_j(t-h_U)) - k_j^- x_j(t-h_U)] \times [f_j(x_j(t-h_U)) - k_j^+ x_j(t-h_U)] \le 0 \ \ (j=1,...,n), \tag{28}$$

From the above three equations (26)-(28), for any positive diagonal matrices $H_j = diag\{\mathrm{h_{j1}},...,\mathrm{h_{jn}}\}(\mathrm{j}=1,2,3)$, we have

$$\begin{aligned} 0 \le& -2\sum_{j=1}^{n} h_{1j}[f_j(x_j(t)) - k_j^- x_j(t)][f_j(x_j(t)) - k_j^+ x_j(t)] \\ & -2\sum_{j=1}^{n} h_{2j}([f_j(x_j(t-h(t))) - k_j^- x_j(t-h(t))] \\ & \times [f_j(x_j(t-h(t))) - k_j^+ x_j(t-h(t))]) \\ & -2\sum_{j=1}^{n} h_{3j}([f_j(x_j(t-h_U)) - k_j^- x_j(t-h_U)] \\ & \times [f_j(x_j(t-h_U)) - k_j^+ x_j(t-h_U)]) \\ =& \ \zeta^T \Phi \zeta(t). \end{aligned} \tag{29}$$

From Eq. (3) and (7), the inequality

$$p^T(t)p(t) = q^T(t)F^T(t)F(t)q(t) \le q^T(t)q(t) \tag{30}$$

can be obtained. Then, there exists a positive scalar ε such that

$$\varepsilon[q^T(t)q(t) - p^T(t)p(t)] \ge 0. \tag{31}$$

From (18)-(31) and by utilizing S-procedure [17], an upper bound of $\dot{V} - 2y^T(t)u(t) - \gamma u^T(t)u(t)$ can be estimated as

$$\dot{V} - 2y^T(t)u(t) - \gamma u^T(t)u(t) \le \zeta^T(t)[\Sigma + \Omega_{[h(t)]}]\zeta(t). \tag{32}$$

By Lemma 1, $\zeta^T(t)[\Sigma + \Omega_{[h(t)]}]\zeta(t) < 0$ with $0 = \Pi\zeta(t)$ is equivalent to $(\Pi^\perp)^T\{\Sigma + \Omega_{[h(t)]}\}(\Pi^\perp) < 0$. Since $\Omega_{[h(t)]}$ is affinely dependent on $h(t)$, the inequality $(\Pi^{\perp})^T\{\Sigma+\Omega_{[h(t)]}\}(\Pi^{\perp})<0$ with $0 \le h(t) \le h_U$ holds if and only if the two inequalities (16) and (17) hold from Lemma 4. This means

$$\dot{V}-2y^T(t)u(t)-\gamma u^T(t)u(t) \le 0. \tag{33}$$

By integrating (33) with respect to t over the time interval from 0 to t_p, it can be obtained that

$$2\int_0^{t_p} y^T(s)u(s)ds \ge V(x(t_p))-V(x(0))-\gamma\int_0^{t_p} u^T(s)u(s)ds. \tag{34}$$

Since $V(x(t_p)) \ge 0$ and $V(x(0))=0$, it follows that

$$2\int_0^{t_p} y^T(s)u(s)ds \ge -\gamma\int_0^{t_p} u^T(s)u(s)ds \tag{35}$$

which means the system (8) is passive according to Definition 1. This completes our proof. ■

Remark 1. Unlike the previous method, the proposed criterion does not have any free variables which has been well utilized to reduce the conservatism of stability criteria for time-delay systems. Instead of no using free variables, the augmented vector $\zeta(t)$ has integral terms of activation functions such as $\int_{t-h(t)}^{t} f(x(s))ds$ and $\int_{t-h_U}^{t-h(t)} f(x(s))ds$. By utilizing these terms in deducing passivity criterion, more past information of $f(x(t))$ was utilized in Theorem 1. With these considerations, it will be shown that our proposed passivity criterion provide larger feasible regions by comparing maximum delay bounds with the recent results in other literature.

Remark 2. In many cases, the information on the time-derivative of time-varying delays is unknown. For this case, without V_2 of (18), one can easily obtain delay-dependent passivity criterion for system (8) which do not need the value h_D.

4. Numerical examples

In this section, it will be shown Theorem 1 can provide less conservative results by checking maximum delay bounds for guaranteeing passivity of system (8) with utilized system parameters of other literature.

Example 1. Consider the uncertain neural networks (8) with

$$A=\begin{bmatrix}2.2 & 0\\ 0 & 1.5\end{bmatrix}, W_0=\begin{bmatrix}1 & 0.6\\ 0.1 & 0.3\end{bmatrix}, W_1=\begin{bmatrix}1 & -0.1\\ 0.1 & 0.2\end{bmatrix}, K_p=diag\{1,1\}$$
$$K_m=diag\{0,0\},\ D=0.1I, E_1=0.1I,\ E_2=0.2I,\ E_3=0.3I,$$
$$C_1=I,\ C_2=0. \tag{36}$$

For the system (8) with the parameters mentioned above, maximum delay bounds for guaranteeing the system (8) passive were investigated in [9] and [10] listed as Table 1. By applying Theorem 1 to the system (8), one can obtain maximum delay bounds with various condition of h_D which is also listed as Table 1. From Table 1, one can see that our proposed passivity criterion significantly enhanced the feasible of passivity criterion.

Table 1 Delay bounds h_U with different h_D (Example 1)

h_D	0.3	0.5	0.7	0.9	Un-known
[9]	0.5624	0.5580	0.5565	0.5523	0.5420
[10]	0.5763	0.5679	0.5566	0.5273	0.5129
Theorem 1	0.9562	0.9172	0.8884	0.8523	0.8315

Example 2. Consider the nominal neural networks

$$\dot{x}(t)=-Ax(t)+W_0f(x(t))+W_1f(x(t-h(t))+u(t) \tag{37}$$

where

$$A=\begin{bmatrix}2.2 & 0\\ 0 & 1.8\end{bmatrix}, W_0=\begin{bmatrix}1.2 & 1\\ -0.2 & 0.3\end{bmatrix}, W_1=\begin{bmatrix}0.8 & 0.4\\ -0.2 & 0.1\end{bmatrix}, K_p=diag1,1$$
$$K_m=diag\{0,0\}, C_1=I,\ C_2=0.$$

By applying Theorem 1 to the above system, maximum delay bounds such that system (37) is passive can be obtained as shown in Table 2. From Table 2, it can be confirmed that Theorem 1 also provides larger delay bounds than the very recent results of [8] and [11].

Table 2 Delay bounds h_U with different h_D (Example 2)

h_D	0.5	0.9	unknown
[8]	0.7230	0.6791	0.6791
[11]	1.3752	1.3027	1.3027
Theorem 1	1.6318	1.4879	1.4626

5. Conclusion

In this paper, the new passivity criterion for uncertain neural networks with time-varying delays and norm-bounded parameter uncertainties was proposed in the form of LMI framework by utilizing more past history about activation function and the triple integral form of Lyapunov-Krasvoskii's functional. Through two numerical examples, the improvements of our proposed criterion has been successfully verified.

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