

# Design of a bilinear robust controller for a hydrostatic driver Hydrostatic 구동기에 관한 Bilinear 강인 제어기 설계

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**Key Words** : 바이리니어(bilinear), 하이드로우스태틱 구동기(hydrostatic driver), 관측기(observer), 비선형 추정 및 보상(nonlinear estimation and compensation)

**Abstract** : 이 논문은 비선형 시스템에 대해 bilinear 강인제어기를 설계하는 새로운 방법을 제시한다. 이 설계 방법은 골칫거리인 비선형 영향을 나타내는 무거운 질량을 가지고 진동하는 시스템을 제어하기 위한 새로운 대안이고 진전된 방법이다. 이 설계 과정에, hydrostatic 구동기로 구동되는 킬(용골)이 주어진다. 첫 단계로, 킬은 물리적으로 여러 한정된 질량으로 모델화된다. 물리적 모델에 근거한 수학적 모델은 유도하는 방법은 해밀턴 원리를 적용한 유한요소법을 사용하였다. 즉, 수학적 모델은 여러 서브시스템으로 구성된다. 이것은 주어진 물리적인 시스템에 대해 기준이 되는 시스템이다. 회전하는 구동기에 대한 물리적 모델에 근거하여, 과도 거동은 구동기의 베어링에서 측정되는 운동 현상으로부터 유도된다. 물리적인 시스템은 bilinear 시스템으로 구성하였다. 이 시스템에 근거하여, 요트 킬의 거동을 제어하도록 bilinear 관측기를 설계한다. 구동기의 속도, 토크, 밸브에서의 유량 등이 관측기를 구성하는데 필요한 데이터들이다. 시뮬레이션 결과에 의하면 비선형성에 대한 추정과 보상을 통하여 무거운 질량을 갖는 회전축에 대한 위치와 힘을 제어하는 설계에 유용한 접근법임이 증명되었다.

## 1. Introduction

From the viewpoint of the increasing complexity and high requirements of the structures of modern control systems, the nonlinear behaviors of a system are strongly considered. Especially, when the interest comes to dynamic behavior by high signal amplitude or various operating points, the simplified model is used as approximation of system behavior. There are some methods of bilinearization by model adaption<sup>1)</sup> which is used the different operating points, and by parameter identification.<sup>2)</sup> One of the interesting and important application's areas is hydrostatic rotating transition drivers. These systems show highly nonlinear grade, but these must be often precisely controlled even in the range of the total velocity

and load, where the linear design strategies are limited. The main points of the interest are regarded the considered system as the bilinearized system by an alternative way of system description and control structure for the nonlinear estimation and compensation. The controller can be divided into two different types : one is with decentralized structure and the other is centralized one. By decentralized controller, each part of a driver is considered independently of the others, and implemented with a simple linear controller. The nonlinear coupling effects between the joints are neglected. This concept is referred to as *independent joint control* and it was one of the first methods used in the area of joint structure control. As drivers have to do more demanding jobs such as higher working-velocities and higher precision of positioning under the huge weight, different concepts to improve the methods of control design have been introduced to consider

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the nonlinear coupling effects. By one group of these improved controllers the decentralized structure is kept and the nonlinear coupling effects are compensated— either with the method of computed torque by feed forwarding the coupling torques obtained from the desired trajectories, or with the method of joint torque control by the feedback of coupling torque which is measured at each joint. Unlike the above mentioned methods, another group of these improved controllers use a centralized structure, in which is considered as a whole as multi-variables system, nonlinear effects are decoupled and compensated using the method of exact linearization and decoupling by state feedback which is linear control theory. The centralized control approach called the method of exact linearization is on the one hand methodically more precise than one of the decentralized concepts in the method of computed torque, since the actual but not the desired position and velocity information is used; on the other hand it is more involving due to the on-line state feedback than the latter, in which the required feed forwarding torques by robot or displacement by rotating shaft can be computed off-line. In both cases, however, a complete knowledge of the dynamic model of driver is assumed. Incompletely-known model parameters, varied operating conditions like payload variations on rotating shaft or motor, the varied frictional forces and so on lead to the need for an additional robust control design. This problem disappears, if another decentralized control concept named the method of torque control is used, since parameter inaccuracies, payload variations and varied frictional effects together with the nonlinear coupling forces are recorded by the measurement of torque or displacement. However, this method suffers from the drawback that it needs additional measuring devices as commercial machineries have. In this paper, an analytical system is going to be transformed into a bilinear system (BLS) and an approach of bilinearization of a system for a hydraulic rotating shaft and a control design modified with the Relay

with hysteresis and  $\text{sgn}$ , the so-called nonlinearity estimation and compensation is introduced, where these valve displacements can be estimated with decentralized bilinear state observers and nonlinearities can thus be compensated by the feedback of these estimated values. The results will be compared with the methods of analytic linear system (ALS). The idea of nonlinearity estimation and compensation in ALS was introduced in Muller.<sup>3)</sup> Based on a fictitious model of the time behavior of the nonlinearities, a state observer of an extended linear dynamic system is designed, which results in an estimate of the nonlinear effects. These nonlinearities are then constructed by the estimated signals by applying disturbance rejection control techniques. Using this method to design robust decentralized position controllers for both rigid and elastic multi-body system has been systematically investigated.<sup>3,4)</sup>

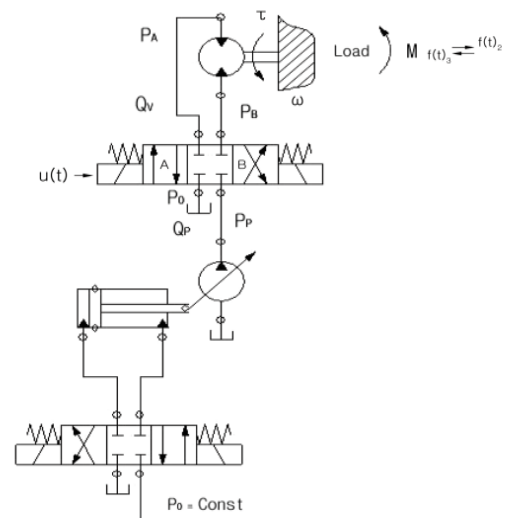


Fig. 1 A hydrostatic driver.

In this paper the given system jointly connected by rotating shaft and the payloads will at first be described into the ALS and transformed into BLS, and give a whole description how the method of nonlinearity estimation and compensation is used to design a robust decentralized controller for the considered system. Then simulation's results of a rotating shaft and the heavy payload are given, which will confirm the high performance of this

control concept, even if the nonlinear coupling effects are considered. Comparisons between the method of exact linearization for ALS and the way of BLS are also made, which will demonstrate the advantages of the proposed approach.

## 2. Model of driver dynamics

The given physical model Fig. 1 can be written mathematical form generally as follows.

$$\dot{x}(t) = f(x(t), u(t)), \quad x_0 = x(t_0) \quad (1)$$

$$y(t) = h(x(t)) \quad (2)$$

where  $x \in R^n$  is state vector, and  $y(t)$  is vector of measurement.

From Eq. (1), the dynamic behavior of joint system can be modeled by analogy to Spong<sup>5)</sup> as

$$M(q)\ddot{q} + h(q, \dot{q}) = u_v \quad (3)$$

where  $q$  is the vector of rotating shaft coordinates,  $M(q)$  the non-diagonal, positive definite mass matrix. Since all nonlinear effects will be considered as a whole by following control design methods, the friction and the backlash as well as the gravitational forces are summarized in one vector  $h(q, \dot{q})$ . The vector  $u_v$  represents the valve-room displacement. The equation can be written in a general form as follows.

$$\dot{x}(t) = a(x(t)) + b(x(t))u(t), \quad x_0 = x(t_0) \quad (4)$$

$$y(t) = c(x(t)) \quad (5)$$

Here,  $a$  denotes system matrix,  $b$  presents input matrix, and  $c$  illustrates measurement matrix. The vectors  $x(t)$ ,  $\dot{x}(t)$ ,  $y(t)$  show the displacement, velocity and output, respectively.

## 3. Control design methods

### 3.1 Exact linearization

According to Beater<sup>1)</sup>, multi-variables controller

can be designed with the method of bilinearization based on Eq. (3). The driver torques  $u$  is chosen herewith as

$$u = M(q)\nu + h(q, \dot{q}) \quad (6)$$

Considering the regularity of the mass matrix  $M(q)$  a decoupled linear system

$$\ddot{q} = \nu \quad (7)$$

can be obtained using Eq. (3). Here  $\nu$  is a new input vector. For Eq. (4), simple linear controllers can be designed. If, for example, a desired trajectory  $q_d(t)$  with the desired velocity  $\dot{q}_d(t)$  and acceleration  $\ddot{q}_d(t)$  is to be realized, it can thus be reached by the feedback

$$\nu = \ddot{q}_d + k_{\dot{q}}(\dot{q}_d - \dot{q}) + k_q(q_d - q) \quad (8)$$

i.e.

$$u_f = M(q)(\ddot{q}_d + k_{\dot{q}}(\dot{q}_d - \dot{q}) + k_q(q_d - q)) + h(q, \dot{q}) \quad (9)$$

where  $k_{\dot{q}}$  and  $k_q$  are diagonal matrices with positive diagonal elements which determine the dynamics of the controllers. The Eq. (3) can be written in the general form

$$\sum_{ALS} \begin{cases} \dot{x}(t) = a x(t) + b x(t)u(t), & x_0 = x(t_0) \\ y(t) = c^T x(t) \end{cases} \quad (10)$$

where  $x \in R^n$

### 3.2 Bilinearization approximation

There are two ways to the bilinearization approximation. One is the mathematical bilinearization of a system by Carleman<sup>6)</sup> that causes the fast increase of system order due to the small model errors and results in high cost, the other is the method of model adaption. In this category, Beater<sup>1)</sup> showed bilinearization with linear model, which is treated with the various operating points. Dorissen<sup>7)</sup> developed a way based on the discrete time system identification which needs impulse excitation at an input so as to create a Markov parameter at an output as a very

useful substitute model. For this process a very useful substitute model is needed. The last model is the application of parameter identification.<sup>2)</sup> As a disadvantage, this process demands an experience of a recursive algorithm for the selection of start value. But it has advantage in structure characteristics such as observer, observability, control, and canonical structure. Those ways let the Eq. (9) write down

$$\sum_{BLS} \begin{cases} \dot{x}(t) = Ax(t) + [Nx(t) + b]u(t), & x_0 = x(t_0) \\ y(t) = c^T x(t) \end{cases} \quad (11)$$

where  $A, N, b, c^T$  are constant matrices of appropriate dimension.

### 3.3 Stability for BLS

For the BLS the stability must be considered. There are some analysis processes for the stability: linear feedback, square state feedback and linearized state space feedback. Now with

$$u(t) = -k^T x(t) \quad (12)$$

where  $k^T$  is a constant matrix, it is convenient to rewrite BLS Eq. (11) as follows.

$$\dot{x}(t) = Ax(t) - (Nx(t) + b)k^T x(t), \quad x_0 = x(t_0) \quad (13)$$

$$y(t) = c^T x(t) \quad (14)$$

For a Lyapunov function of Eq. (13), choose the simple quadratic form given by

$$V(x) = x^T(t) Q x(t) \quad (15)$$

where  $Q$  is a real symmetric matrix. Then

$$\begin{aligned} \dot{V}(x) &= [Ax(t) - (Nx(t) + b)k^T x(t)]^T Q x(t) + \\ &\quad x^T(t) Q [Ax(t) - (Nx(t) + b)k^T x(t)] \\ &= x^T(x) A^T Q x(t) + x^T(x) Q A x(t) \\ &\quad - x^T(t) k^T (Nx(t) + b)^T Q x(t) \\ &\quad - x^T(x) Q (Nx(t) + b) k^T x(t) \end{aligned} \quad (16)$$

$$\dot{V}(x) = x^T(t) [(A - bk^T)^T Q + Q(A - bk^T)] x(t) - 2x^T(t) Q N x(t) k^T x(t), \quad x_0 = x(t_0) \quad (17)$$

The linear feedback of the system Eq. (10) must

content the requirement of Lyapunov function like in linear theory, i.e. eigen values of

$$\bar{A} = (A - bk^T) \quad (18)$$

have to exhibit the negative real parts as follows :  $Re(\lambda_i(\bar{A})) < 0, i = 1, 2, \dots, n$  in terms of the proof of the region of the unit circle. Additionally, the terms of  $2x^T(t) Q N x(t) k^T x(t)$  must be positive.

Unfortunately, it is usually impossible to stabilize globally the BLS with linear feedback control. Therefore, we limit this case to local asymptotic stability. Another way to guarantee the globally asymptotical stability for the system is the quadratic state feedback<sup>8)</sup> for

$$u(t) = -[Nx(t) + b]^T Q x(t) \quad (19)$$

$$Re(\lambda_i(\bar{A})) < 0, i = 1, 2, \dots, n$$

### 3.4 Observability for BLS

The method of linear approximation was able to prove separation principle for the well-known nonlinear system which is known from linear system theory since 30 years but the results is theoretical and hard to use in the practical application because the statement over the region of inside of unit circle is unclear for stability. In terms of this problem, Schwarz<sup>9)</sup> presented a special nonlinear observer for BLS from Keller<sup>10)</sup> as Fig. 2 if the BLS has a sufficient space state model as follows. It is constructed like Fig. 2.

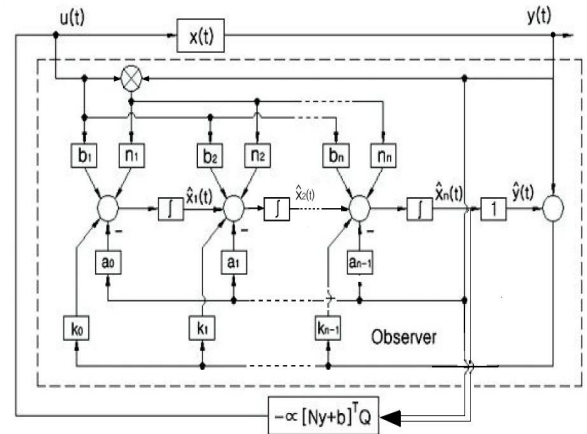


Fig. 2 BLS from Keller<sup>10)</sup>

$$\begin{cases} \dot{x}(t) = [E - ac^T]x(t) + [nc^T x(t) + b]u(t), & x_0 = x(t_0) \\ y(t) = c^T x(t) \end{cases} \quad (20)$$

where  $n_i = b_i = 0, \forall i = 2, 3, \dots, n$ .  $E$  represents identity matrix. From the above statement an observer with linear error dynamic can be designed as follows.

$$\begin{cases} \dot{\hat{x}}(t) = E\hat{x}(t) - ac^T x(t) + [nc^T x(t) + b]u(t) \\ \quad + k[y(t) - \hat{y}(t)], & x_0 = \hat{x}(t_0) \\ \dot{\epsilon}(t) = [E - kc^T]\epsilon(t), & \epsilon_0 = x(t_0) - \hat{x}(t_0) \\ u(t) = -\alpha[nc^T x(t) + b]Q\hat{x}(t) \end{cases} \quad (21)$$

where  $\alpha$  is a feedback control factor for stability

### 3.5 Nonlinearity Estimation and Compensation

Starting point for the design of decentralized controllers with the method of nonlinearity estimation and compensation is still the coupled system Eq. (3). At first it is written down in a decoupled form relative to the single axis. For this the mass matrix  $M(q)$  is divided into a constant diagonal matrix  $M_0$ , the mean values of the moments of inertia, and a remaining position dependent part  $\Delta M(q)$

$$M(q) = M_0 + \Delta M(q) \quad (22)$$

These mean values can be chosen, for example, with regard to a typical working-space of the rotating shaft or along the desired trajectory. Summarizing further for each axis  $i$  all nonlinear terms in  $n_i$ ,

$$n_i = \sum_{j=1}^N \Delta M_{ij}(q)\ddot{q}_j + h_i(q, \dot{q}), \quad i = 1, \dots, N \quad (23)$$

where  $N$  is the number of joints, Eq. (3) with Eq. (22-23) can thus be separately considered for a single axis :

$$\ddot{q}_i = \frac{1}{M_{0i}}(u_i - n_i), \quad i = 1, \dots, N \quad (24)$$

Leaving out index  $i$  for the shake of brevity, a state space description of this one-axis model can be obtained with the state vector  $x = [q \ \dot{q}]^T$

$$\dot{x} = Ax + Nn + Bu \quad (25)$$

$$y = Cx \quad (26)$$

With the matrices

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = -N = \begin{bmatrix} 0 \\ \frac{1}{M_0} \end{bmatrix} \quad (27)$$

$$C = [1 \ 0] \quad (28)$$

Here the measurement of the joint coordinate  $q$  is assumed. The objective of the position control of a robot is often that the joint coordinate  $q$  has to track along a desired trajectory  $r(t) = q_d(t)$  determined by path planning. The control error is thus defined as  $q_d(t) - q(t)$  or in state space form:

$$z = Fx + Rr \quad (29)$$

where

$$F = [-1 \ 0], \quad R = 1 \quad (30)$$

The design of each decentralized controller is based on the state space model Eq. (9-14). On the one hand, the coupling effects and other nonlinearities contained in  $n$  are compensated with the method of nonlinearity estimation and compensation. On the other hand the tracking control is reached by a feed forward, which is determined similar to the design of a disturbance rejection control.<sup>11)</sup> Altogether an asymptotically stable control with

$$z(t) \rightarrow 0, \quad \text{for } t \rightarrow \infty \quad (31)$$

is the whole design objective.

The time signal  $n$  of the nonlinearities and coupling is approximated appropriately by a time function, which is itself solution of a fictitious linear dynamic system

$$n(t) \approx H_1 w_1(t) \quad (32)$$

$$\dot{w}_1(t) = W_1 w_1(t) \quad (33)$$

It was shown in Muller<sup>3)</sup> that this approximation can be at best carried out with harmonic function, so that a model is chosen in Eq. (32-33)

$$H_1 = w, \quad W_1 = 0 \quad (34)$$

The desired trajectory  $r(t) = q_d(t)$  is known. For the feed forward control the variables  $\dot{q}_d(t)$  and  $\ddot{q}_d(t)$  are additionally needed. Although they are theoretically available too, but usually they don't exactly correspond to the derivatives of the actually requested trajectory due to possible disturbances. Therefore, they are estimated by an observer which is constructed like Eq. (32-33) as well, see <sup>3,4,11)</sup>

$$r(t) \approx H_2 w_2(t) \quad (35)$$

$$\dot{w}_2(t) = W_2 w_2(t) \quad (36)$$

The approximation is based on step and ramp functions, here

$$H_2 = [1 \ 0 \ 0], \quad W_2 = \begin{bmatrix} 0 & w & 0 \\ 0 & 0 & w \\ 0 & 0 & 0 \end{bmatrix} \quad (37)$$

is selected. The design of observers to estimate the signal such as  $n$ ,  $\dot{q}_d(t)$ ,  $\ddot{q}_d(t)$  and  $\dot{q}(t)$  if needed, is based on a linear system, which is obtained by inserting Eq. (32-37) in Eq. (25-30);

$$\begin{bmatrix} \dot{x} \\ \dot{w}_1 \\ \dot{w}_2 \end{bmatrix} = \begin{bmatrix} A & NH_1 & 0 \\ 0 & W_1 & 0 \\ 0 & 0 & W_2 \end{bmatrix} \begin{bmatrix} x \\ w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u \quad (38)$$

$$\begin{bmatrix} y \\ r \end{bmatrix} = \begin{bmatrix} C & 0 & 0 \\ 0 & 0 & H_2 \end{bmatrix} \begin{bmatrix} x \\ w_1 \\ w_2 \end{bmatrix} \quad (39)$$

$$z = [F \ 0 \ RH_2] \begin{bmatrix} x \\ w_1 \\ w_2 \end{bmatrix} \quad (40)$$

This extended system with the given system matrices is completely observable. Therefore, an observer can be designed according to one of usual methods, a quadratic optimal identity observer is determined here. It is shown at the same time that this observer can be separated into two parts, one for  $q, \dot{q}, n$  and the other for  $q_d, \dot{q}_d, \ddot{q}_d$

$$\begin{bmatrix} \hat{x} \\ \hat{w}_1 \end{bmatrix} = \begin{bmatrix} A - L_x C & NH_1 \\ -L_{w_1} C & W_1 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{w}_1 \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} L_x \\ L_{w_1} \end{bmatrix} y \quad (41)$$

$$\dot{\hat{w}}_2 = (W_2 - L_{w_2} H_2) \hat{w}_2 + L_{w_2} r \quad (42)$$

The desired estimated values are then obtained from

$$\hat{n} = H_1 \hat{w}_1 \quad (43)$$

With the estimated variables  $\hat{x}$ ,  $\hat{w}_1$  and  $\hat{w}_2$ , a feedback controller

$$u = -K_x \hat{x} - K_{w_1} \hat{w}_1 - K_{w_2} \hat{w}_2 \quad (44)$$

is constructed. The gain matrix  $K_x$  of the state feedback can be set with standard methods such as pole assignment, where the complete controllability of the matrices  $(A, B)$  is fulfilled. The feedback gain  $K_{w_1}$  for the compensation of the nonlinearities and coupling effects and the gain matrix  $K_{w_2}$  for the feed forwarding of the desired trajectory are determined from the following equations

$$(A - BK_x)X_1 - X_1 W_1 - BK_{w_1} = NH_1 \quad (45)$$

$$FX_1 = 0 \quad (46)$$

and

$$(A - BK_x)X_2 - X_2 W_2 - BK_{w_2} = 0 \quad (47)$$

$$FX_2 = -RH_2 \quad (48)$$

which are resulted from Eq. (31). The solutions for  $K_{w_1}$  and  $K_{w_2}$  are

$$K_{w_1} = -1 \quad (49)$$

$$K_{w_2} = [-K_{x_1} \quad -K_{x_2} \quad -M_0] \quad (50)$$

$X_1$  and  $X_2$  are matrices which characterize the stationary behavior of  $x(t)$  depending on  $w_1(t)$  and  $w_2(t)$

With this the controller is finally obtained

$$u = M_0(\ddot{q}_d + K_{x_2}(\dot{\hat{q}}_d - \dot{\hat{q}}) + K_{x_1}(\hat{q}_d - \hat{q})) + \hat{n} \quad (51)$$

### 3.6 Comparison

Comparing the control law Eq. (44) with the controller Eq. (6) from the method of exact linearization in section 3.1, an formal agreement between both of them is found out. Whereas Eq. (6) needs an exact knowledge of the model parameters of  $M(q)$  and  $h(q, \dot{q})$ , Eq. (44) shows the advantage that uncertain models can be taken, here only the values of the different  $M_0$ 's have to be chosen and all the original model effects can be grasped in  $n_i$ 's by the used state observers.

The robustness of the controller Eq. (44) can easily be verified. If the parameters in the system description Eq. (1) are inaccurate, for example, due to varied payload or unknown friction torques appear, the real system behavior must then be described with a modified model

$$M'(q)\ddot{q} + h'(q, \dot{q}) = u \quad (52)$$

with a changed mass matrix  $M'(q)$  and a changed vector of nonlinear effects  $h'(q, \dot{q})$

Whereas the controller Eq. (9) is still based on the nominal model Eq. (1) so that mismatches between  $M'$  and  $M$  as well as between  $h'$  and  $h$  exist, the controller Eq. (44) has obviously regard to the changed system behavior. As in Eq. (22), let

$$M'(q) = M_0 + \Delta M'(q) \quad (53)$$

Then one has only to replace the term  $n_i$  in the description Eq. (24) by

$$n_i = \sum_{j=1}^N \Delta M'_{ij}(q)\ddot{q}_j + h'_i(q, \dot{q}), \quad i = 1, \dots, N \quad (54)$$

similar to Eq. (23). Because the design of the controller Eq. (44) is based on the fact that  $n_i$ - and thus  $n_i'$ - is interpreted as an unknown function and to be estimated by  $\hat{w}_1$ , the controller fulfills its task also in the case of changed system behavior. The controller Eq. (44) is structurally robust against parameter inaccuracies and original model effects.

## 4. Modeling and simulation on hydrostatic rotating shaft driver

The proposed control methods are demonstrated with simulation on a dynamic model of hydrostatic rotating shaft driver. For the simulation the inertia parameters reported in Schwarz<sup>9),12)</sup> are used

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{-\tau}{m} & \frac{-v_A}{m} & \frac{-v_B}{m} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -w^2 - D_v w_0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \quad (55)$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ k_u \end{bmatrix} u + \begin{bmatrix} 0 \\ \epsilon \\ \delta \\ \zeta \\ 0 \\ K_F F_{RV}(x_6) \end{bmatrix}$$

$x_1 = q$  shaft angle position

$x_2 = \dot{x}_1$  shaft angle velocity

$x_3 = P_A$  pressure in the room

$x_4 = P_B$  pressure in the room

$x_5 = s$  valve display distance

$x_6 = \dot{s}$  valve display velocity

$m$  : load mass

$v_A$  : eigen value of matrix of input in the valve

$v_B$  : eigen value of matrix of output in the valve

$\tau$  : torque

$D_v$  : viscosity of oil

And the elements in the last term denote

$$\epsilon = -\frac{\text{sgn}(x_2)}{m}(f_2 + f_3 e) \quad (56)$$

$$\delta = \frac{E(x_3)}{V_A(x_1)}[-v_A x_2 + Q_A(p_0, x_3, p_p, x_5 - Q_{li})] \quad (57)$$

$$\zeta = \frac{E_{0e}(x_4)}{V_B(x_1)}[-v_B x_2 + Q_B(p_0, x_4, x_5 - Q_{li})] \quad (58)$$

where

$Q_A$  : oil volume of the input

$Q_B$  : oil volume of the output

$k_u$  : voltage of valve in input

- $K_F F_{RV}$  : friction rates
- $V_A$  : volume in chamber A
- $V_B$  : volume in chamber B
- $E$  : oil volume in hydro motion by input
- $f_2$  : forward translating force
- $f_3$  : backward translating force

The total position error equation is defined as follows.

$$P_{err} = \sqrt{(X_d - X)^2 + (Y_d - Y)^2 + (Z_d - Z)^2} \quad (59)$$

where  $X$ ,  $Y$ , and  $Z$  represent actual position in x-axis, y-axis, and z-axis, respectively, and subscript  $d$  means the desired position

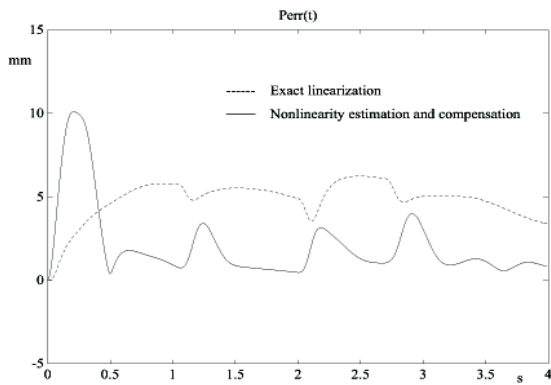


Fig. 3 Position-error compensation by BLS for real system

If the actual position tracks the desired one exactly,  $P_{err} = 0$ .

At first the control behavior is shown when the parameters of the system model are exactly known, see Fig. 3. The method of exact linearization gives no position error as expected, whereas this appears by the control design with the method of nonlinearity estimation and compensation especially at the beginning of the simulation, because the observers require a certain time until they can give right estimates. The advantage of the control method of nonlinearity estimation and compensation is then recognized, when real rotating shaft problems are considered.

The nonlinear force compensation in x-axis and y-axis for ALS and BLS is shown in the Fig. 4 and Fig. 5, respectively.

Fig. 4 gives the comparison of this method with method of exact linearization under the conditions that friction effects described in (Armstrong, 1988) and a load with 2kg are taken into account. The position error of the proposed method is much smaller as the existed one except at the very beginning. Fig. 5 illustrates the nonlinear force compensation by BLS for real system. When the rotating shaft meets the given impact from outside (0.15s) the system shows the strong force nonlinearity and compensation starts immediately

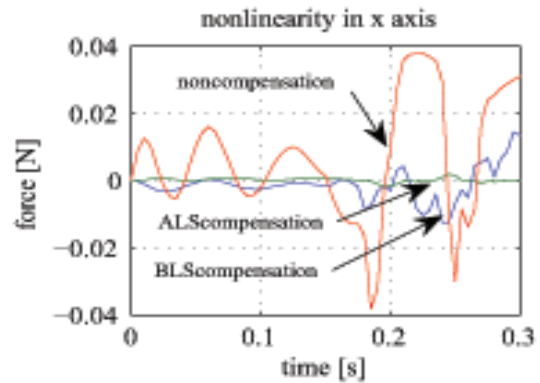


Fig. 4 Nonlinear force compensation in x-axis by ALS and BLS

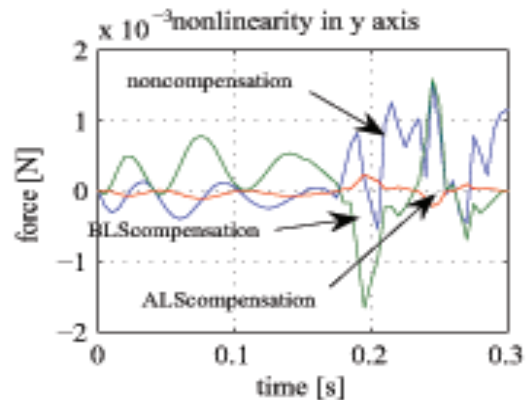


Fig. 5 Nonlinear force compensation in y-axis by ALS and BLS

## 5. Conclusions

The method of nonlinearity estimation and compensation has been proved to be a suitable



approach for the design of robust position and forces controllers for rotating shaft connected with heavy mass. In addition to its robustness, its decentralized structure offers additional advantages such that the concept of independent joint control can further be used, even in an improved manner. The requirements on the modeling of the dynamics are very low. And the control design can be easily done because it is based only on linear system theory. Simulation results have shown the efficiency of the proposed control design method.

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### Reference

1. P. Beater, 1987, Zur "Rgelung nichtlinear System mit Hilfe bilinear modelle", Diss. University Duisurg, Forschungsberichte Reihe 8, Nr. 143 Duesseldorf : VDI-Verlag.
2. X. Yin, 1992, "Bilinear Modeling and State Feedback Control of an Electro-Hydraulic Drive", IFAC Workshop, Motion Control for Intelligent Automation, Perguia, pp. 27-29.
3. P. C. Muller, 1992, "Nonlinearities In" : *Proc. First Asian Control Conf.(ASCC)*, vol.2, Tokyo, pp. 641-644.
4. P. C. Muller, 1997, "Stabilitaet und Matrizen", Berlin u.a Spinger.
5. M. Spong, 1987, "Modeling of Control of elastic Joint Robot", ASME Journal of Dynamic System, Measurement Control, vol. 109, pp. 310-317.
6. T. Carleman, 1932, Application de la theorie des equation Integralssingulieres aux equation differentielles de Dynamique, T. Ark. Mat, Astron, Phys. 22B.
7. H. T. Dorrisen, 1990, Zur Minimalrealisierung und Identifikation bilinearer Systeme durch Markovparameter, Diss, Universitaet Duisburg, VDI-Fortschittsberichte Reihe 8 Nr 221, Duesseldorf VDI-Verlag.
8. M. Zeit, 1987, "The extended Luenberger Observer or Nonlinear System", System Control Letter 9, pp. 149-156.
9. H. Schwarz, 1992, "BLS\_Beobachter in kanonischer Form", 7/90, MSRT, University Duisburg, Forschungsbericht 4/92, MSRT, FB 7. University Duisburg.
10. H. Keller, 1986, "Entwurf nichtlinear Beobachtermittels Normalform Diss", Univertaet Karlsruhe, Forchungsberichte Reihe 8, Nr. 124 Duesseldorf : VDI-Verlag.
11. P. C. Muller and J. Luckel, 1977, "Zur Theorie der Storgrobenaufschaltung in linearen Mehrgrobensystemen", Rsgelungstechnik, 25, pp. 54-59.
12. H. Schwarz, 1990, "ARS Beobachter und Filter, Forschungsbericht" 7/90, MSRT, University Duisburg.
13. S. Svoronos, G. Stephanopolos, and R. Aris, 1980, "Bilinear Approximation of general Dynamic systems with linear Input", Int J. Control, pp. 109-126.
14. P. O. Gutmam, 1981, "Stabilizing Controllers for bilinear System", IEEE TR. AC-26, pp. 917-922.

### Appendices

The data used in simulation are given as follows

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & n_1 & n_2 & n_3 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & n_4 & n_5 \end{bmatrix}$$

The nominated elements  $n_1, n_2, n_3,$  and  $n_4$  are  $-0.0084, -0.8321, -0.3747,$  and  $-0.0321$  respectively. The others are as follows.

$$\tau = 150.0 \text{ NM}, v_A = 0.0071, v_B = 0.0114$$

$$\epsilon = -\frac{\text{sgn}(x_2)}{m}(f_2 + f_3 e)$$

$$\delta = \frac{E(x_3)}{V_A(x_1)}[-v_A x_2 + Q_A(p_0, x_3, p_p, x_5 - Q_i)]$$

$$\xi = \frac{E_{0e}(x_4)}{V_B(x_1)} [-v_B x_2 + Q_B(p_0, x_4, x_5 - Q_{li})]$$

$Q_A = 5.55 \times 10^{-3} \text{ m}^3/\text{s}$ ,  $k_u = 90.0 \text{ V}$ ,  $K_F F_{RV} = 0.95$ ,  
 $V_A = 2.085$ ,  $V_B = -5.87$ ,  $D_v w_0 = 6.25 \times 10^{-6} \text{ m}^2/\text{s}$ ,  
 $f_2 = 2000 \text{ NM}$ ,  $f_3 = 2000 \text{ NM}$ ,  $x_1 = q : 220 \text{ rad}$ ,  
 $x_2 = 220 \text{ rad/s}$ ,  $x_3 = P_A : 100 \text{ N}$ ,  $x_4 = P_B : 95 \text{ N}$ ,  
 $x_5 = s : 0.2 \text{ m}$ ,  $x_6 = \dot{s} : 0.05 \text{ m/s}$