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On *-bimultipliers, Generalized *-biderivations and Related Mappings

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Abstract. In this paper we define the notions of left $*$-bimultiplier, $*$-bimultiplier and generalized $*$-biderivation, and to prove that if a semiprime $*$-ring admits a left $*$-bimultiplier M, then M maps $R \times R$ into $Z(R)$. In Section 3, we discuss the applications of theory of $*$-bimultipliers. Further, it was shown that if a semiprime $*$-ring R admits a symmetric generalized $*$-biderivation $G : R \times R \to R$ with an associated nonzero symmetric $*$-biderivation $B : R \times R \to R$, then G maps $R \times R$ into $Z(R)$. As an application, we establish corresponding results in the setting of C^*-algebra.

1. Introduction

Throughout the discussion, unless otherwise mentioned, R will denote an associative ring with center $Z(R)$, and A will represent a C^*-algebra. However, A may not have unity with center $Z(A)$. For any $x, y \in A$, the symbol $[x, y]$(resp. $x \circ y$) will denote the commutator $xy - yx$ (resp. the anti-commutator $xy + yx$). Recall that an algebra A is prime if $xAy = \{0\}$ implies $x = 0$ or $y = 0$, and A is semiprime if $xAx = \{0\}$ implies $x = 0$. A Banach algebra is a linear associative algebra which, as a vector space, is a Banach space with norm $\| \cdot \|$ satisfying the multiplicative inequality; $\|xy\| \leq \|x\|\|y\|$ for all x and y in A. An additive mapping $x \longmapsto x^*$ of A into itself is called an involution if the following conditions are satisfied: (i) $(xy)^* = y^*x^*$, (ii) $(x^*)^* = x$, and (iii) $(\lambda x)^* = \bar{\lambda}x^*$ for all $x, y \in A$ and $\lambda \in \mathbb{C}$, where $\bar{\lambda}$ is the conjugate of λ . An algebra(ring) equipped with an involution is called a $*$-algebra($*$-ring) or algebra with involution(ring with involution). A C^*-algebra A is a Banach $*$-algebra with the additional norm condition $\|x^*x\| = \|x\|^2$ for all $x \in A$.

Let S be a nonempty subset of R. A function $f : R \to R$ is said to be centralizing

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on S if $[f(x), x] \in Z(R)$ for all $x \in S$. In the special case when $[f(x), x] = 0$ for all $x \in S$, f is said to be commuting on S. The study of such mappings were initiated by Posner. In [14, Lemma 3], Posner proved that if a prime ring R has a nonzero commuting derivation, then R is commutative. Over the last five decades, many authors [4, 6, 7] have proved commutativity theorems for prime and semiprime rings admitting various types of additive maps like automorphisms, derivations, biderivations and generalized derivations which are centralizing or commuting on certain appropriate subsets of R.

The purpose of this paper is to prove some theorems on prime(semiprime) $*$-rings, which have independent interests and related to symmetric $*$-biadditive mappings. Moreover, in the last section we discuss the applications of the theory of $*$-bimultipliers. Finally, we establish corresponding results in the setting of C^*-algebra.

An additive mapping $d : R \longrightarrow R$ is called a derivation (resp. reverse derivation) if $d(xy) = d(x)y + xd(y)$ (resp. $d(xy) = d(y)x + yd(x)$) holds for all $x, y \in R$. Following [15], an additive mapping $T : R \longrightarrow R$ is called a left (resp. right) centralizer if $T(xy) = T(x)y$ (resp. $T(xy) = xT(y)$) holds for all $x, y \in R$. An additive mapping $F : R \longrightarrow R$ is called a generalized derivation (resp. generalized reverse derivation) if there exists a derivation (resp. reverse derivation) $d : R \longrightarrow R$ such that $F(xy) = F(x)y + xd(y)$ (resp. $F(xy) = F(y)x + yd(x)$) holds for all $x, y \in R$. A mapping $B : R \times R \longrightarrow R$ is said to be symmetric if $B(x, y) = B(y, x)$ holds for all $x, y \in R$. In [12] Muthana defined the following notions: a biadditive (i.e., additive in both arguments) mapping $B : R \times R \longrightarrow R$ is called a left (resp. right) bimultiplier if $B(xy, z) = B(x, z)y$ (resp. $B(xy, z) = xB(y, z)$) holds for all $x, y, z \in R$. A symmetric biadditive mapping $B : R \times R \longrightarrow R$ is called a symmetric biderivation if $B(xy, z) = B(x, z)y + xB(y, z)$ is fulfilled for all $x, y, z \in R$. The concept of a symmetric biderivation was introduced by Maksa in [10] (see also [11] where an example can be found).

Let R be a $*$-ring. Following [1, 3], an additive mapping $d : R \longrightarrow R$ is called a $*$-derivation (resp. reverse $*$-derivation) if $d(xy) = d(x)y^* + xd(y)$ (resp. $d(xy) = d(y)x^* + yd(x)$) holds for all $x, y \in R$. An additive mapping $T : R \longrightarrow R$ is said to be a left (resp. right) $*$-centralizer if $T(xy) = T(x)y^*$ (resp. $T(xy) = x^*T(y)$) holds for all $x, y \in R$. A symmetric biadditive mapping $B : R \times R \longrightarrow R$ is called a symmetric $*$-biderivation if $B(xy, z) = B(x, z)y^* + xB(y, z)$ holds for all $x, y, z \in R$, and B is called a symmetric reverse $*$-biderivation if $B(xy, z) = B(y, z)x^* + yB(x, z)$ holds for all $x, y, z \in R$. An additive mapping $F : R \longrightarrow R$ is called a generalized $*$-derivation (resp. generalized reverse $*$-derivation) if there exists a $*$-derivation (resp. reverse $*$-derivation) $d : R \longrightarrow R$ such that $F(xy) = F(x)y^* + xd(y)$ (resp. $F(xy) = F(y)x^* + yd(x)$) holds for all $x, y \in R$. In [5], Bresar and Vukman proved that if a prime $*$-ring R admits a $*$-derivation (resp. reverse $*$-derivation) d, then either $d = 0$ or R is commutative. Further, the first author together with Ashraf [3] extended the above mentioned result for semiprime $*$-rings. Very recently, the first author in [1] established that if a semiprime $*$-ring admits a generalized $*$-derivation (resp. generalized reverse $*$-derivation) F, then F maps R into $Z(R)$.

Now we introduce the concept of $*$-bimultiplier and generalized $*$-biderivation as follows: A symmetric biadditive mapping $M : R \times R \longrightarrow R$ is said to be a symmetric left $*$-bimultiplier (resp. symmetric right $*$-bimultiplier) if $M(xy, z) = M(x, z)y^*$ (resp. $M(xy, z) = x^*M(y, z)$) holds for all $x, y, z \in R$. If M is both symmetric left as well as right $*$-bimultiplier, then M is a symmetric $*$-bimultiplier. A symmetric biadditive mapping $G : R\times R \longrightarrow R$ is called a symmetric generalized $*$-biderivation if there exists a symmetric $*$-biderivation $B : R\times R \longrightarrow R$ such that $G(xy, z) = G(x, z)y^* + xB(y, z)$ holds for all $x, y, z \in R$. A symmetric biadditive mapping $G : R \times R \longrightarrow R$ is called a symmetric generalized reverse $*$-biderivation if there exists a symmetric reverse $*$-biderivation $B : R \times R \longrightarrow R$ such that $G(xy, z) = G(y, z)x^* + yB(x, z)$ holds for all $x, y, z \in R$. Of course the relation $G(z, xy) = G(z, y)x^* + yB(x, z)$ is fulfilled for all $x, y, z \in R$. Hence, the concept of symmetric generalized $*$-biderivations covers both the concepts of symmetric $*$-biderivations and symmetric left $*$-bimultipliers. Note that if R is a $*$-ring, and B is any symmetric $*$-biderivation of R. Consider the symmetric biadditive function $f : R \times R \to R$ such that

$$f(xy, z) = f(x, z)y^* \text{ and } f(x, yz) = f(x, y)z^* \text{ for } \quad \text{all } x, y, z \in R.$$

Then, $f + B$ is a symmetric generalized $*$-biderivation on R. Moreover, if B is any symmetric reverse $*$-biderivation of R, and $g : R\times R \to R$ is a symmetric biadditive function such that

$$g(xy, z) = g(y, z)x^* \text{ and } g(x, yz) = g(x, z)y^* \text{ for all } x, y, z \in R.$$

Then, $g + B$ is a symmetric generalized reverse $*$-biderivation on R.

2. Left (resp. right) $*$-bimultipliers

The main goal of this section is to prove the following theorem related to left $*$-bimultipliers. More precisely, we shall prove the following result:

Theorem 2.1. *Let R be a semiprime $*$-ring. If $M : R \times R \to R$ is a biadditive mapping such that $M(xy, z) = M(x, z)y^*$ for all $x, y, z \in R$, then M maps $R \times R$ into $Z(R)$.*

Proof. By the hypothesis, we have

$$M(xy, z) = M(x, z)y^* \text{ for all } x, y, z \in R. \tag{2.1}$$

Replacing y by yw in (2.1), one hand we obtain

$$M(xyw, z) = M(x(yw), z) = M(x, z)w^*y^* \text{ for all } w, x, y, z \in R. \tag{2.2}$$

On the other hand, we have

$$M(xyw, z) = M((xy)w, z) = M(x, z)y^*w^* \text{ for all } w, x, y, z \in R. \tag{2.3}$$

Subtracting (2.2) from (2.3), we obtain

$$M(x,z)[y^*,w^*]=0 \text{ for all } w,x,y,z\in R. \tag{2.4}$$

Substituting y^* for y and w^* for w in (2.4), we arrive at

$$M(x,z)[y,w]=0 \text{ for all } w,x,y,z\in R. \tag{2.5}$$

Replacing y by $yM(x,z)$ in the above expression we find that

$$M(x,z)[y,w]M(x,z)+M(x,z)y[M(x,z),w]=0 \text{ for all } w,x,y,z\in R.$$

Application of relation (2.5) forces that

$$M(x,z)y[M(x,z),w]=0 \text{ for all } w,x,y,z\in R. \tag{2.6}$$

Multiplying by w to (2.6) from left yields that

$$wM(x,z)y[M(x,z),w]=0 \text{ for all } w,x,y,z\in R. \tag{2.7}$$

Now putting wy for y in (2.6), we get

$$M(x,z)wy[M(x,z),w]=0 \text{ for all } w,x,y,z\in R. \tag{2.8}$$

Combining (2.7) with (2.8) we arrive at

$$[M(x,z),w]y[M(x,z),w]=0 \text{ for all } w,x,y,z\in R. \tag{2.9}$$

This implies that $[M(x,z),w]R[M(x,z),w]=\{0\}$ for all $w,x,z\in R$. Thus, we obtain, $[M(x,z),w]=0$ for all $w,x,z\in R$ by the semiprimeness of R. Hence, M maps $R\times R$ into $Z(R)$. This completes the proof of our first theorem. □

We now prove another theorem in this vein.

Theorem 2.2. *Let R be a semiprime $*$-ring. If $M:R\times R\to R$ is a biadditive mapping such that $M(xy,z)=x^*M(y,z)$ for all $x,y,z\in R$, then M maps $R\times R$ into $Z(R)$.*

Proof. We compute $M(xyw,z)$ in two different ways. Then, we have

$$M(x(yw),z)=x^*y^*M(w,z) \text{ for all } w,x,y,z\in R \tag{2.10}$$

and

$$M((xy)w,z)=y^*x^*M(w,z) \text{ for all } w,x,y,z\in R. \tag{2.11}$$

Comparing the expressions so obtained for $M(xyw,z)$, we arrive at

$$[x^*,y^*]M(w,z)=0 \text{ for all } w,x,y,z\in R. \tag{2.12}$$

Henceforth, using similar approach as we have used after (2.4) in the proof of the last paragraph of Theorem 2.1 with necessary variations, we find that $[M(w,z),y]=0$ for all $w,y,z \in R$. Hence, M maps $R \times R$ into $Z(R)$. $\square$

Corollary 2.1. *Let R be a semisimple $*$-ring. If $M : R \times R \to R$ is a biadditive mapping such that $M(xy,z) = M(x,z)y^*$ for all $x,y,z \in R$ or $M(xy,z) = x^*M(y,z)$ for all $x,y,z \in R$, then M maps $R \times R$ into $Z(R)$.*

Proof. As a consequence of Theorems 2.1 & 2.2, and of the fact that every semisimple $*$-ring is semiprime $*$-ring. $\square$

It is to remark that every C^*-algebra is semiprime ring (see [2] for further details), and therefore satisfies the requirements of Theorems 2.1 & 2.2. Hence, we have the following:

Theorem 2.3. *Let A be a C^* -algebra. If $M : A \times A \to A$ is a bilinear mapping such that $M(xy,z) = M(x,z)y^*$ for all $x,y,z \in A$ or $M(xy,z) = x^*M(y,z)$ for all $x,y,z \in A$, then M maps $A \times A$ into $Z(A)$.*

Next, let us consider the prime versions of Theorem 2.1 and Theorem 2.2.

Theorem 2.4. *Let R be a prime $*$-ring. If $M : R \times R \to R$ is a nonzero biadditive mapping such that $M(xy,z) = M(x,z)y^*$ for all $x,y,z \in R$, then R is commutative.*

Proof. In view of Theorem 2.1, we have $M(x,z)[y,w] = 0$ for all $w,x,y,z \in R$. Substituting yx for y, we find that $M(x,z)y[x,w] = 0$ for all $w,x,y,z \in R$, and hence $M(x,z)R[x,w] = \{0\}$ for all $w,x,z \in R$. Thus, the primeness of R forces that for each $x \in R$ either $[x,w] = 0$ or $M(x,z) = 0$ for all $w,z \in R$. The set of all $x \in R$ for which these two properties hold are additive subgroups of R whose union is R. But a group can not be the set-theoretic union of two of its proper subgroups, therefore $M(x,z) = 0$ for all $x,z \in R$ or $[x,w] = 0$ for all $w,x \in R$. But $M(x,z) \neq 0$, we conclude that $[x,w] = 0$ for all $w,x \in R$ and hence R is commutative. $\square$

Similarly, we can prove the following:

Theorem 2.5. *Let R be a prime $*$-ring. If $M : R \times R \to R$ is a nonzero biadditive mapping such that $M(xy,z) = x^*M(y,z)$ for all $x,y,z \in R$, then R is commutative.*

3. Generalized $*$-biderivations

In this section, we present some applications of theory of $*$-bimultipliers in $*$-rings. If $G : R \times R \longrightarrow R$ is a symmetric generalized $*$-biderivation of R related to a symmetric $*$-biderivation $B : R \times R \longrightarrow R$, then it is easy to see that G is a symmetric generalized $*$-biderivation of R if and only if G is of the form $G = B + M$, where B is a symmetric $*$-biderivation and M is a symmetric left $*$-bimultiplier of

R. Hence, we write $M = G - B$. In the proof of Theorem 3.1 below, we use this technique which can be regarded as a contribution to the theory of $*$-bimultipliers in $*$-rings. In fact, we prove the following result:

Theorem 3.1. *Let R be a semiprime $*$-ring. If R admits a symmetric generalized $*$-biderivation $G : R \times R \to R$ with an associated nonzero symmetric $*$-biderivation $B : R \times R \to R$, then G maps $R \times R$ into $Z(R)$.*

Proof. Let us give the proof of this theorem in the following two steps:
Step 1. We assume that G is a symmetric generalized $*$-biderivation with an associated symmetric $*$-biderivation B. If $B = 0$, then G is a left $*$-bimultiplier on R. Thus in view of Theorem 2.1, we get the required result.

Step 2. On the other hand, suppose that the associated $*$-biderivation $B \neq 0$. Then, we set $G = B + M$ and hence $M = G - B$ where M, G and B are biadditive maps on R. Therefore, we have

$$\begin{aligned} M(xy,z) &= G(xy,z) - B(xy,z) \\ &= G(x,z)y^* + xB(y,z) - B(x,z)y^* - xB(y,z) \\ &= (G(x,z) - B(x,z))y^* \\ &= (G - B)(x,z)y^* \\ &= M(x,z)y^* \quad \text{for all } x,y,z \in R. \end{aligned}$$

This implies that $M(xy,z) = M(x,z)y^*$ for all $x,y,z \in R$. That is, M is a left $*$-bimultiplier on R. Therefore, we conclude that G can be written as $G = B + M$, where B is a symmetric $*$-biderivation and M is a left $*$-bimultiplier on R. Thus, in view of Theorem 2.1 above and Theorem 3.1 of ([3], for $\alpha = \beta = I_R$, the identity map on R), we conclude that G maps $R \times R$ into $Z(R)$. This proves the theorem completely. □

Theorem 3.2. *Let R be a semiprime $*$-ring. If R admits a symmetric generalized reverse $*$-biderivation $G : R \times R \to R$ with an associated nonzero symmetric reverse $*$-biderivation $B : R \times R \to R$, then $[B(x,y),t] = 0$ for all $x,y,t \in R$.*

Proof. We are given that G is a symmetric generalized reverse $*$-biderivation with an associated nonzero symmetric reverse $*$-biderivation B, we have

$$G(x,yz) = G(x,z)y^* + zB(x,y) \text{ for all } x,y,z \in R. \tag{3.1}$$

Replacing z by zt in the above relation, we find that

$$G(x,y(zt)) = G(x,t)z^*y^* + tB(x,z)y^* + ztB(x,y) \text{ for all } x,y,z,t \in R. \tag{3.2}$$

Also, we have

$$G(x,(yz)t) = G(x,t)z^*y^* + tB(x,z)y^* + tzB(x,y) \text{ for all } x,y,z,t \in R. \tag{3.3}$$

Comparing (3.2) with (3.3), we obtain

$$[z,t]B(x,y) = 0 \text{ for all } x,y,z,t \in R. \tag{3.4}$$

Substituting $B(x,y)z$ for z in (3.4) we find that

$$B(x,y)[z,t]B(x,y) + [B(x,y),t]zB(x,y) = 0 \text{ for all } x,y,z,t \in R. \tag{3.5}$$

In view of (3.4), the above expression reduces to

$$[B(x,y),t]zB(x,y) = 0 \text{ for all } x,y,z,t \in R. \tag{3.6}$$

Taking $z = zt$ in (3.6), we get

$$[B(x,y),t]ztB(x,y) = 0 \text{ for all } x,y,z,t \in R. \tag{3.7}$$

Right multiplication by t to equation(3.6) forces that

$$[B(x,y),t]zB(x,y)t = 0 \text{ for all } x,y,z,t \in R. \tag{3.8}$$

Subtracting (3.7) from (3.8), we arrive at

$$[B(x,y),t]z[B(x,y),t] = 0 \text{ for all } x,y,z,t \in R. \tag{3.9}$$

The last equation can be rewritten in the form $[B(x,y),t]R[B(x,y),t] = \{0\}$ for all $x,y,t \in R$. It follows from the semiprimeness of R that $[B(x,y),t] = 0$ for all $x,y,t \in R$. This proves the theorem. □

Theorem 3.3. *Let R be a prime $*$-ring. If R admits a symmetric generalized reverse $*$-biderivation G with an associated nonzero symmetric reverse $*$-biderivation B, then R is commutative.*

Proof. By Theorem 3.2, we have $[B(x,y),t] = 0$ for all $x,y,t \in R$. Replace y by yz in the last expression and using the fact that B is a reverse $*$-biderivation, we obtain $B(x,z)[y^*,t] + [z,t]B(x,y) = 0$ for all $x,y,z,t \in R$. This implies that $B(x,z)[y^*,z] = 0$ for all $x,y,z \in R$ by (3.4). Substituting y^* for y in the last relation, we get $B(x,z)[y,z] = 0$ for all $x,y,z \in R$. Now replace y by wt to get $B(x,z)w[t,z] = 0$ for all $w,x,z,t \in R$. That is, $B(x,z)R[t,z] = \{0\}$ for all $x,z,t \in R$. The primeness of R yields that either $[t,z] = 0$ or $B(x,z) = 0$ for all $x,t \in R$. Now, we put $A_1 = \{z \in R \mid [t,z] = 0 \text{ for all } t \in R\}$ and $A_2 = \{z \in R \mid B(x,z) = 0 \text{ for all } x \in R\}$. Then, clearly A_1 and A_2 are additive subgroups of R. Moreover, by the discussions given, R is the set-theoretic union of A_1 and A_2. But a group can not be the set-theoretic union of two of its proper subgroups, hence $A_1 = R$ or $A_2 = R$. If $A_1 = R$, then $[t,z] = 0$ for all $z,t \in R$ and hence R is commutative. On the other hand if $A_2 = R$, then $B(x,z) = 0$ for all $x,z \in R$, a contradiction. With this the proof is complete. □

Similarly, we can prove the following:

Theorem 3.4. *Let R be a prime $*$-ring. If R admits a symmetric generalized $*$-biderivation G with an associated nonzero $*$-biderivation B, then R is commutative.*

We conclude the paper with the proof of our last two theorems in the setting of C^*-algebra.

Theorem 3.5. *Let A be a C^* -algebra. If A admits a symmetric bilinear generalized $*$-biderivation $G : A \times A \to A$ with an associated nonzero symmetric bilinear $*$-biderivation $B : A \times A \to A$, then G maps $A \times A$ into $Z(A)$.*

Proof. As a consequence of Theorem 3.1, and of the fact that every C^*-algebra is semiprime ring (viz., [2]). □

Similarly, we can establish the following:

Theorem 3.6. *Let A be a C^* -algebra. If A admits a symmetric bilinear generalized reverse $*$-biderivation $G : A \times A \to A$ with an associated nonzero symmetric bilinear reverse $*$-biderivation $B : A \times A \to A$, then G maps $A \times A$ into $Z(A)$.*

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