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EXISTENCE AND UNIQUENESS THEOREM FOR LINEAR FUZZY DIFFERENTIAL EQUATIONS

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Abstract. The introduction of fuzzy differential equation is to deal with fuzzy dynamic systems. As classical differential equations, it is difficult to find the solutions to all fuzzy differential equations. In this paper, an existence and uniqueness theorem for linear fuzzy differential equations is obtained. Moreover, the exact solution to linear fuzzy differential equation is given.

1. Introduction

In order to describe a set without definite boundary, fuzzy set was initiated by Zadeh [16] in 1965, whose membership function indicates the degree of an element belonging to it. But how to measure a fuzzy event? To solve this question, Liu and Liu [10] introduced the concept of credibility measure in 2002. Then a sufficient and necessary condition for credibility measure was given by Li and Liu [6]. Credibility theory was founded by Liu [7] and refined by Liu [8].

In order to describe the evolution of fuzzy dynamic systems, fuzzy process was proposed by Liu [9], in which, Liu process is an important and useful fuzzy process as Brownian motion in stochastic process. Based on Liu process, Liu integral and Liu formula were presented by Liu [9], which are the counterparts of Brownian motion, Ito integral and Ito formula. Some researches concerning Liu process have been done. You [15] extended Liu process, Liu integral and Liu formula to the case of multi-dimensional. Complex Liu process was considered by Qin [13]. Dai [1] deduced that Liu process is Lipschitz continuous and has finite variation. Liu process has been applied to mathematical finance. Under the assumption that stock price is modeled by geometric Liu process, a basic stock model was proposed by Liu [9], which is called Liu's stock model. Qin and Li [14] deduced the European option pricing formula for Liu's stock

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model. As the application of Liu process in fuzzy control, an optimal equation was obtained by Zhu [17].

Differential equations with fuzzy parameters were studied in many literatures, such as Kaleva [3], Kaleva [4] and Ding, Ma and Kandel [2], Lakshmikantham and Mohapatra [11], Pearson [12] and Kaleva [5]. But the study on stochastic differential equations are paid most attention to the differential equations driven by Brownian motion. In 2008, a new kind of fuzzy differential equation was defined by Liu [9] as

$$\mathrm{d}X_t = f(t, X_t)\mathrm{d}t + g(t, X_t)\mathrm{d}C_t,$$

where C_t is a standard Liu process, and f and g are some given functions. Its solution is a fuzzy process. It is just the fuzzy counterpart of stochastic differential equation. In this paper, we will study the fuzzy differential equation proposed by Liu [9].

In Section 2 of this paper, some concepts and results of credibility measure and Liu process will be given as a preliminaries. In Section 3, an existence and uniqueness theorem is obtained and the solutions of linear fuzzy differential equations will be discussed. In the end, a brief summary is given in Section 4.

2. Preliminaries

Credibility measure is a set function with properties of normality, monotonicity, self-duality and maximality. A fuzzy variable is a function from a credibility space to the set of real numbers.

The expected value of a fuzzy variable ξ was defined as

$$E[\xi] = \int_0^{+\infty} Cr\{\xi \geq r\}\mathrm{d}r - \int_{-\infty}^{0} Cr\{\xi \leq r\}\mathrm{d}r$$

provided that at least one of the two integrals is finite. The variance is $V[\xi] = E[(\xi - E[\xi])^2]$.

A fuzzy variable ξ is said to be continuous if $\mathrm{Cr}\{\xi = x\}$ is a continuous function of x.

A fuzzy process $X_t(\theta)$ is defined as a function from $T \times (\Theta, \mathrm{P}, \mathrm{Cr})$ to the set of real numbers. In other words, $X_{t^*}(\theta)$ is a fuzzy variable for each t^*. $X_t(\theta^*)$ is a function of t for any given $\theta^* \in \Theta$, such a function is called a sample path of $X_t(\theta)$. For simplicity, we use the symbol X_t to replace $X_t(\theta)$ in the following section.

A fuzzy process X_t is called continuous if the sample paths of X_t are all continuous functions of t for almost all $\theta \in \Theta$.

Definition 1. (Liu [9]) A fuzzy process C_t is said to be a Liu process if
(i) $C_0 = 0$,
(ii) C_t has stationary and independent increments,

(iii) every increment $C_{t+s} - C_s$ is a normally distributed fuzzy variable with expected value et and variance $\sigma^2 t^2$ whose membership function is

$$\mu(x) = 2\left(1+\exp\left(\frac{\pi|x-et|}{\sqrt{6}\sigma t}\right)\right)^{-1}, \quad -\infty < x < +\infty.$$

The Liu process is said to be standard if $e = 0$ and $\sigma = 1$.

Theorem 2.1. (Dai [1]) *Let C_t be a Liu process. For any given θ with* $\mathrm{Cr}\{\theta\} > 0$*, the path $C_t(\theta)$ is Lipschitz continuous.*

Definition 2. (Liu [9]) Let X_t be a fuzzy process and let C_t be a standard Liu process. For any partition of closed interval $[a, b]$ with $a = t_1 < t_2 < \cdots < t_{k+1} = b$, the mesh is written as

$$\triangle = \max_{1\le i\le k} |t_{i+1} - t_i|.$$

Then the Liu integral of X_t with respect to C_t is defined as follows,

$$\int_a^b X_t \mathrm{d}C_t = \lim_{\triangle\to 0} \sum_{i=1}^{k} X_{t_i} \cdot (C_{t_{i+1}} - C_{t_i})$$

provided that the limitation exists almost surely and is a fuzzy variable. In this case, X_t is called Liu integrable.

Theorem 2.2. (Liu Formula, Liu [9]) *Let C_t be a standard Liu process, and let $h(t, x)$ be a continuously differentiable function. If fuzzy process X_t is given by $\mathrm{d}X_t = u_t \mathrm{d}t + v_t \mathrm{d}C_t$, where u_t, v_t are absolutely integrable fuzzy process and Liu integrable fuzzy process, respectively. Define $Y_t = h(t, X_t)$. Then*

$$\mathrm{d}Y_t = \frac{\partial h}{\partial t}(t, X_t)\mathrm{d}t + \frac{\partial h}{\partial x}(t, X_t)\mathrm{d}X_t.$$

Theorem 2.3. (Multi-dimensional Liu Formula, You [15]) *Let $\boldsymbol{C}_t = (C_{1t}, C_{2t}, \cdots, C_{mt})^T$ be an m-dimensional standard Liu process, and let*

$$\begin{aligned}&\boldsymbol{h}(t, x_1, x_2, \cdots, x_n)\\ &= (h_1(t, x_1, x_2, \cdots, x_n), h_2(t, x_1, x_2, \cdots, x_n), \cdots, h_p(t, \boldsymbol{x}_1, x_2, \cdots, x_n))^T\end{aligned}$$

where $h_i(t, x_1, x_2, \cdots, x_n)$, $i = 1, 2, \cdots, p$ are multivariate continuously differentiable functions. If fuzzy process $(X_{1t}, X_{2t}, \cdots, X_{nt})^T$ is given by

$$\begin{cases} \mathrm{d}X_{1t} = u_{1t}\mathrm{d}t + v_{11t}\mathrm{d}C_{1t} + \cdots + v_{1mt}\mathrm{d}C_{mt} \\ \quad\vdots \qquad \vdots \qquad\qquad\qquad\qquad \vdots \\ \mathrm{d}X_{nt} = u_{nt}\mathrm{d}t + v_{n1t}\mathrm{d}C_{1t} + \cdots + v_{nmt}\mathrm{d}C_{mt} \end{cases}$$

where u_{it}, v_{ijt} are absolutely integrable fuzzy processes and Liu integrable fuzzy processes, respectively. Define $\boldsymbol{Y}_t = \boldsymbol{h}(t, X_{1t}, X_{2t}, \cdots, X_{nt})$. Then

$$\mathrm{d}\boldsymbol{Y}_t = \begin{pmatrix} \frac{\partial h_1}{\partial t}(t, X_{1t}, X_{2t}, \cdots, X_{nt})\mathrm{d}t + \sum_{i=1}^{n} \frac{\partial h_1}{\partial x_i}(t, X_{1t}, X_{2t}, \cdots, X_{nt})\mathrm{d}X_{it} \\ \frac{\partial h_2}{\partial t}(t, X_{1t}, X_{2t}, \cdots, X_{nt})\mathrm{d}t + \sum_{i=1}^{n} \frac{\partial h_2}{\partial x_i}(t, X_{1t}, X_{2t}, \cdots, X_{nt})\mathrm{d}X_{it} \\ \vdots \\ \frac{\partial h_p}{\partial t}(t, X_{1t}, X_{2t}, \cdots, X_{nt})\mathrm{d}t + \sum_{i=1}^{n} \frac{\partial h_p}{\partial x_i}(t, X_{1t}, X_{2t}, \cdots, X_{nt})\mathrm{d}X_{it} \end{pmatrix}.$$

Definition 3. (Liu [9]) Suppose C_t is a standard Liu process, and f and g are some given functions. Let X_t be an unknown fuzzy process. Then the equation

$$\mathrm{d}X_t = f(t, X_t)\mathrm{d}t + g(t, X_t)\mathrm{d}C_t$$

is called a fuzzy differential equation. A solution is a fuzzy process X_t that satisfies the above equation identically in t.

3. Existence and uniqueness theorem

In this section, existence and uniqueness theorem for linear fuzzy differential equations will be given.

Theorem 3.1. *The solution of linear fuzzy differential equation*

$$\mathrm{d}X_t = (u_{1t}X_t + u_{2t})\mathrm{d}t + (v_{1t}X_t + v_{2t})\mathrm{d}C_t \tag{1}$$

exists and is unique. Here C_t is a standard Liu process, $u_{1t}, u_{2t}, v_{1t}, v_{2t}$ are some given continuous fuzzy processes and X_t is an unknown fuzzy process.

Proof. We know that the integral equation

$$X_t = X_0 + \int_0^t (u_{1s}X_s + u_{2s})\mathrm{d}s + \int_0^t (v_{1s}X_s + v_{2s})\mathrm{d}C_s \tag{2}$$

is equivalent to linear fuzzy differential equation (1).

Next, a successive approximation method will be used to construct a solution of the integral equation (2).

Define $X_t^{(0)} = X_0$, and then

$$X_t^{(n+1)} = X_0 + \int_0^t (u_{1s}X_s^{(n)} + u_{2s})\mathrm{d}s + \int_0^t (v_{1s}X_s^{(n)} + v_{2s})\mathrm{d}C_s \tag{3}$$

for $n = 0, 1, 2, \cdots$.

For any given θ with $\mathrm{Cr}\{\theta\} > 0$, we write

$$D_t^{(n)}(\theta) = \max_{0 \le s \le t} \left| X_s^{(n+1)}(\theta) - X_s^{(n)}(\theta) \right|, \ n = 0, 1, 2, \cdots.$$

It follows from Theorem 2.1 that $C_t(\theta)$ is Lipschitz continuous with respect to t, that is, there exists a finite $K(\theta)$ such that $|C_t(\theta)-C_s(\theta)|\le K(\theta)|t-s|$. Since a continuous function in a closed interval is bounded, there exist finite $M(\theta)$ such that

$$\begin{aligned}
D_t^{(0)}(\theta) &= \max_{0\le r\le t}\left|\int_0^r (u_{1s}X_0+u_{2s})\mathrm{d}s+\int_0^r (v_{1s}X_0+v_{2s})\mathrm{d}C_s\right| \\
&\le \max_{0\le r\le t}\left|\int_0^r (u_{1s}X_0+u_{2s})\mathrm{d}s\right|+\max_{0\le r\le t}\left|\int_0^r (v_{1s}X_0+v_{2s})\mathrm{d}C_s\right| \\
&\le \max_{0\le r\le t}\int_0^r |u_{1s}X_0+u_{2s}|\mathrm{d}s \\
&\quad+\max_{0\le r\le t}\left|\lim_{\Delta\to 0}\sum_{i=1}^{k}(v_{1s_i}X_0+v_{2s_i})(C_{s_{i+1}}(\theta)-C_{s_i}(\theta))\right| \\
&\le \max_{0\le r\le t}\int_0^r |u_{1s}X_0+u_{2s}|\mathrm{d}s \\
&\quad+\max_{0\le r\le t}\lim_{\Delta\to 0}\sum_{i=1}^{k}|v_{1s_i}X_0+v_{2s_i}||C_{s_{i+1}}(\theta)-C_{s_i}(\theta)| \\
&\le \max_{0\le r\le t}\int_0^r |u_{1s}X_0+u_{2s}|\mathrm{d}s \\
&\quad+K(\theta)\max_{0\le r\le t}\lim_{\Delta\to 0}\sum_{i=1}^{k}|v_{1s_i}X_0+v_{2s_i}|(s_{i+1}-s_i) \\
&\le \max_{0\le r\le t}\int_0^r |u_{1s}X_0+u_{2s}|\mathrm{d}s+K(\theta)\max_{0\le r\le t}\int_0^r |v_{1s}X_0+v_{2s}|\mathrm{d}s \\
&\le \int_0^t |u_{1s}X_0+u_{2s}|\mathrm{d}s+K(\theta)\int_0^t |v_{1s}X_0+v_{2s}|\mathrm{d}s \\
&\le (1+|X_0|)M(\theta)(1+K(\theta))
\end{aligned}$$

for all times $0\le t\le T$ by the continuities of $u_{1t},\ u_{2t},\ v_{1t},\ v_{2t}$.

The inductive assumption is

$$D_t^{(n)}(\theta)\le (1+|X_0|)\frac{(M(\theta)(1+K(\theta)))^{n+1}}{(n+1)!}t^n,\ n=0,1,2,\cdots,$$

for $0\le t\le T$.

To see this note that

$$
\begin{aligned}
&D_t^{(n)}(\theta)\\
&= \max_{0\le r\le t}\left|\int_0^r u_{1s}(X_s^{(n)}-X_s^{(n-1)})\mathrm{d}s+\int_0^r v_{1s}(X_s^{(n)}-X_s^{(n-1)})\mathrm{d}C_s\right|\\
&\le \max_{0\le r\le t}\left|\int_0^r u_{1s}(X_s^{(n)}-X_s^{(n-1)})\mathrm{d}s\right|+\max_{0\le r\le t}\left|\int_0^r v_{1s}(X_s^{(n)}-X_s^{(n-1)})\mathrm{d}C_s\right|\\
&\le \max_{0\le r\le t}\int_0^r |u_{1s}(X_s^{(n)}-X_s^{(n-1)})|\mathrm{d}s\\
&\quad+\max_{0\le r\le t}\left|\lim_{\Delta\to 0}\sum_{i=1}^{k} v_{1s_i}(X_{s_i}^{(n)}-X_{s_i}^{(n-1)})(C_{s_{i+1}}(\theta)-C_{s_i}(\theta))\right|\\
&\le \int_0^t |u_{1s}(X_s^{(n)}-X_s^{(n-1)})|\mathrm{d}s\\
&\quad+\max_{0\le r\le t}\lim_{\Delta\to 0}\sum_{i=1}^{k} |v_{1s_i}(X_{s_i}^{(n)}-X_{s_i}^{(n-1)})||(C_{s_{i+1}}(\theta)-C_{s_i}(\theta))|\\
&\le \int_0^t |u_{1s}(X_s^{(n)}-X_s^{(n-1)})|\mathrm{d}s\\
&\quad+K(\theta)\max_{0\le r\le t}\lim_{\Delta\to 0}\sum_{i=1}^{k} |v_{1s_i}(X_{s_i}^{(n)}-X_{s_i}^{(n-1)})||s_{i+1}-s_i|\\
&\le \int_0^t |u_{1s}(X_s^{(n)}-X_s^{(n-1)})|\mathrm{d}s\\
&\quad+K(\theta)\max_{0\le r\le t}\int_0^r |v_{1s}(X_s^{(n)}-X_s^{(n-1)})|\mathrm{d}s\\
&\le \int_0^t |u_{1s}(X_s^{(n)}-X_s^{(n-1)})|\mathrm{d}s+K(\theta)\int_0^t |v_{1s}(X_s^{(n)}-X_s^{(n-1)})|\mathrm{d}s\\
&\le M(\theta)(1+K(\theta))\int_0^t |X_s^{(n)}-X_s^{(n-1)}|\mathrm{d}s\\
&\le M(\theta)(1+K(\theta))\int_0^t (1+|X_0|)\frac{(M(\theta)(1+K(\theta)))^n}{n!}s^{n-1}\mathrm{d}s\\
&= (1+|X_0|)\frac{(M(\theta)(1+K(\theta)))^{(n+1)}}{(n+1)!}t^n.
\end{aligned}
$$

In view of the claim, for $m \geq n$ we have

$$\max_{0\leq t\leq T} |X_t^{(m)} - X_t^{(n)}| \leq (1+|X_0|) \sum_{k=n}^{\infty} \frac{(M(\theta)(1+K(\theta)))^{(n+1)}}{(n+1)!} T^n \to 0, \text{as } n \to \infty.$$

Thus there exists a fuzzy process X_t such that $X_t^{(n)}$ converges uniformly to X_t with respect to $t \in [0,T]$ for almost every θ. It is easy to check that X_t is the solution of (2). The existence is proved.

Next we will prove that the solution of the integral differential equation (2) is unique. Assume that both X_t and X_t^* are solutions of (2). Then for any given θ with $\mathrm{Cr}\{\theta\} > 0$, we have

$$|X_t(\theta) - X_t^*(\theta)| = \left| \int_0^t u_{1s}(X_s - X_s^*)\mathrm{d}s + \int_0^t v_{1s}(X_s - X_s^*)\mathrm{d}C_s \right|$$

$$\leq \left| \int_0^t u_{1s}(X_s - X_s^*)\mathrm{d}s \right| + \left| \int_0^t v_{1s}(X_s - X_s^*)\mathrm{d}C_s \right|$$

$$\leq \int_0^t |u_{1s}(X_s - X_s^*)|\mathrm{d}s + \lim_{\Delta\to 0} \sum_{i=1}^{k} |v_{1s_i}(X_{s_i} - X_{s_i}^*)(C_{s_{i+1}}(\theta) - C_{s_i}(\theta))|$$

$$\leq \int_0^t |u_{1s}(X_s - X_s^*)|\mathrm{d}s + K(\theta) \lim_{\Delta\to 0} \sum_{i=1}^{k} |v_{1s_i}(X_{s_i} - X_{s_i}^*)||s_{i+1} - s_i|$$

$$\leq \int_0^t |u_{1s}(X_s^{(n)} - X_s^{(n-1)})|\mathrm{d}s + K(\theta) \int_0^t |v_{1s}(X_s - X_s^*)|\mathrm{d}s$$

$$\leq M(\theta)(1+K(\theta)) \int_0^t |X_s - X_s^*|\mathrm{d}s$$

It follows from Gronwall inequality that $|X_t - X_t^*| \equiv 0$ for almost all θ, i.e. $X_t \equiv X_t^*$. The uniqueness is proved. □

Since the solution of linear fuzzy differential equation exists and is unique, then what is the solution?

First, we consider the fuzzy differential equation

$$\mathrm{d}X_t = u_{1t}X_t\mathrm{d}t + v_{1t}X_t\mathrm{d}C_t, \tag{4}$$

where X_t is an unknown fuzzy process and u_{1t}, v_{1t} are some given continuous fuzzy processes.

Let $X_t = X_0 \exp\left(\int_0^t u_{1s}\mathrm{d}s + \int_0^t v_{1s}\mathrm{d}C_s\right)$. It follows from Liu formula that

$$\begin{aligned}\mathrm{d}X_t &= X_0 \exp\left(\int_0^t u_{1s}\mathrm{d}s + \int_0^t v_{1s}\mathrm{d}C_s\right)(u_{1t}\mathrm{d}t + v_{1t}\mathrm{d}C_t)\\ &= u_{1t}X_t\mathrm{d}t + v_{1t}X_t\mathrm{d}C_t,\end{aligned}$$

thus the solution of equation (4) is $X_t = X_0 \exp\left(\int_0^t u_{1s}\mathrm{d}s + \int_0^t v_{1s}\mathrm{d}C_s\right)$.

Let $X_t = U_tV_t$, where

$$\mathrm{d}U_t = u_{1t}U_t\mathrm{d}t + v_{1t}U_t\mathrm{d}C_t, \quad U_0 = 1,$$

$$\mathrm{d}V_t = a_t\mathrm{d}t + b_t\mathrm{d}C_t \quad V_0 = X_0.$$

It follows from the above discussion that

$$U_t = \exp\left(\int_0^t u_{1s}\mathrm{d}s + \int_0^t v_{1s}\mathrm{d}C_s\right).$$

Taking the differentials of both sides of $X_t = U_tV_t$, we have

$$\mathrm{d}X_t = U_t\mathrm{d}V_t + V_t\mathrm{d}U_t$$

by using multi-dimensional Liu formula, i.e.

$$\mathrm{d}X_t = (U_ta_t + u_{1t}V_tU_t)\mathrm{d}t + (U_tb_t + v_{1t}V_tU_t)\mathrm{d}C_t. \tag{5}$$

Comparing (5) with (1), we can choose coefficients a_t and b_t such that $X_t = U_tV_t$. The desired coefficients satisfy equations

$$U_ta_t = u_{2t}, \text{ and } U_tb_t = v_{2t}.$$

Thus, the solution of linear fuzzy differential equation (1) is

$$X_t = U_t\left(X_0 + \int_0^t \frac{u_{2s}}{U_s}\mathrm{d}s + \int_0^t \frac{v_{2s}}{U_s}\mathrm{d}C_s\right),$$

where

$$U_t = \exp\left(\int_0^t u_{1s}\mathrm{d}s + \int_0^t v_{1s}\mathrm{d}C_s\right).$$

4. Conclusions

This paper was mainly to discuss the solutions of linear fuzzy differential equations based on Liu process. A method was provided to solve linear fuzzy differential equation and existence and uniqueness theorem for linear fuzzy differential equation was given.

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