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ON STOCHASTIC OPTIMAL REINSURANCE AND INVESTMENT STRATEGIES FOR THE SURPLUS UNDER THE CEV MODEL

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Abstract. It is important to find an optimal strategy which maximizes the surplus of the insurance company at the maturity time T. The purpose of this paper is to give an explicit expression for the optimal reinsurance and investment strategy, under the CEV model, which maximizes the expected exponential utility of the final value of the surplus at T. To do this optimization problem, the corresponding Hamilton-Jacobi-Bellman equation will be transformed a linear partial differential equation by applying a Legendre transform.

1. Introduction

The insurance company's risk will be reduced through reinsurance, while in addition the company invests its surplus in a financial market. Two of fundamental aims that the insurance company pursues are to minimize the ruin probability of the company and to maximize the expected utility of the final surplus at the end of the maturity time T.

In this paper we assume that, in the case of no reinsurance and no investment, the surplus process $(V(t))_{t\in[0,T]}$ is described by the following diffusion form:

$$\begin{cases} dV(t) = a_0dt + b_0dW_0(t), \\ V(0) = V_0, \end{cases} \tag{1.1}$$

where the second term of the right hand side is the stochastic integral w.r.t. a 1-dimensional standard Brownian motion $(W_0(t))_{t\geq 0}$. The constant $V > 0$ is the initial surplus, while the constants $a_0 > 0$ and $b_0 > 0$ are the exogenous parameters. The surplus described by (1.1) may be used when an insurance company deals with a large number of policyholders where an individual claim is relatively small compared with the size of the surplus.

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The proportional reinsurance level at time $t \in [0,T]$ will be associated with the value $1-\alpha(t)$, where $0 \le \alpha(t) \le 1$ is called the risk exposure. If the cedent choose the risk exposure $\alpha(t)$, then the cedent have to pay $100\alpha(t)\%$ of each claim while the rest $100(1-\alpha(t))\%$ of the claim will be paid by the reinsurer. To purchase this reinsurance, the cedent pays part of the premiums to the reinsurer at the rate of $(1-\alpha(t))\lambda$ where $\lambda \ge a_0$. Then the corresponding surplus process $(F(t))_{t\in[0,T]}$ is given by

$$\begin{cases} dV(t) = \{a_0 - (1-\alpha(t))\lambda\}dt + \alpha(t)b_0 dW_0(t), \\ V(0) = V_0. \end{cases} \tag{1.2}$$

The constants a_0 and λ can be regarded as the safety loading of the cedent and reinsurer, respectively.

In addition, assume that all of the surplus is invested in a financial market which consists of two securities, named S_1 and S_2, whose prices are given by the following stochastic differential equations:

$$dS_1(t) = a_1 S_1(t)dt + b_1 S_1(t)dW_1(t) \tag{1.3}$$

and

$$dS_2(t) = a_2 S_2(t)dt + b_2 S_2^{1+\gamma}(t)dW_2(t), \tag{1.4}$$

where a_i and b_i, $i = 1, 2$, are the constants satisfying $a_1 \le a_2$ and $b_1 < b_2$, γ is the elasticity parameter and satisfies the general condition $\gamma \le 0$, and $(W_i(t))_{t\ge 0}$, $i = 1, 2$, are independent standard Brownian motions independent of $(W_0(t))_{t\ge 0}$. We denote by $\beta(t)$ the proportion invested in the security S_2 at time $t \in [0,T]$. We disallow leverage and short-sales, which restrict $\beta(t)$ to be in 0 and 1, i.e. $0 \le \beta(t) \le 1$. Therefore, at any time $0 \le t < T$, a nominal amount $V(t)(1-\beta(t))$ is allocated to the stock S_1. We treat the risk exposure $\alpha(t)$ and the proportion $\beta(t)$ of the surplus at time t being invested in more risky stock S_2 as control parameters. Then the surplus process $(V(t))_{t\in[0,T]}$ is given by the following stochastic differential equations:

$$\begin{cases} dV(t) = [V(t)\{\beta(t)a_2 + (1-\beta(t))a_1\} + a_0 - (1-\alpha(t))\lambda]dt \\ \qquad\qquad + \alpha(t)b_0 dW_0(t) + V(t)b_1(1-\beta(t))dW_1(t) \\ \qquad\qquad + V(t)b_2\beta(t)S_2^{\gamma}(t)dW_2(t), \\ V(0) = V_0. \end{cases} \tag{1.5}$$

Given a strategy $(\alpha(\cdot), \beta(\cdot))$, the solution $(V^{\alpha,\beta}(t))_{t\in[0,T]}$ is called the surplus process corresponding to $(\alpha(\cdot), \beta(\cdot))$.

First consider the case that $\gamma = 0$. When $\beta(t) \equiv 1$ in (1.5), i.e., all of the surplus is invested in the stock S_2 only, Taksar and Markussen [12] gave an explicit expression for the optimal reinsurance policy which minimizes the ruin probability of cedent. And Luo, Taksar and Tsoi [9] extended results in [12] to the case that $b_1 = 0$ in (1.5), i.e., S_I is a riskyless asset. When $b_1 = 0$ and $\alpha(t) \equiv 1$ in (1.5), i.e., there is no reinsurance, Devolder et al. [2] found an explicit expression for the optimal asset allocation which maximizes the expected utility of the final annuity fund at retirement and at the end of the period after retirement. Kim and Lee [8] gave a stochastic optimal reinsurance and investment strategy under under only assumption that $\gamma = 0$.

Now consider the case that γ satisfies the general condition $\gamma \leq 0$. When $b_1 = 0$ and $\alpha(t) \equiv 1$ in (1.5), i.e., the security S_1 is a risk-free asset and there is no reinsurance, Gao [3] gave an explicit expression for the optimal investment strategy which maximizes the expected utility of the final value of the final annuity fund at retirement and at the end of the period after retirement. On the other hand Gu et al. [4] gave an optimal optimal proportional reinsurance and investment strategy under the assumption that $b_1 = 0$. So we can say that the work by Gu et al. [4] is an extension of results in the paper [3]. Usually, in stochastic optimal problem, the corresponding Hamilton-Jacobi-Bellman (HJB) equation is a nonlinear partial differential equation and it is difficult to solve. So Gao [3] transformed HJB equation into a linear partial differential equation by applying a Legendre transform. But Gu et al. [4] solved directly HJB equation with very complicate calculus.

In this paper we find an explicit expression for the optimal strategy $(\alpha^*(\cdot), \beta^*(\cdot))$, under the same assumption that $b_1 = 0$ as in [4], which maximizes the expected exponential utility of the final value of the surplus process given by the stochastic differential equation (1.5). To do this we use the same methods as in [3].

The structure of the paper is as follows. In Section 2 we formulate our problem and give theory background. Main results are given in Section 3. All proofs are based on stochastic optimal control theory (see Björk [1] or Øksendal [10]) and the definition and results for a Legendre transform are in [3]. The proofs are presented in Section 3.

2. Formulation of the problem and theory background

In this section we formulate our stochastic optimization problem and give main results. Let $(\Omega, \mathcal{F}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ on which two independent $\mathcal{F}_t$-adapted standard Brownian motions $(W_0(t))_{t\geq 0}$ and $(W(t))_{t\geq 0}$ are defined.

We assume that $b_1 = 0$ in (1.3). In this case S_1 is a risk-free asset like as the bank account. So we rewrite the notation of the financial market model $(S_1(t), S_2(t))$ given by (1.3) and (1.4) by $(B(t), S(t))$ such that

$$dB(t) = rB(t)dt \tag{2.1}$$

and

$$dS(t) = aS(t)dt + bS^{1+\gamma}(t)dW(t). \tag{2.2}$$

Then the dynamics (1.5) of surplus process $(V(t))_{t\in[0,T]}$ is rewritten as the following stochastic differential equations:

$$\begin{cases} dV(t) = [V(t)\{\beta(t)a + (1-\beta(t))r\} + a_0 - (1-\alpha(t))\lambda]dt \\ \qquad\qquad + \alpha(t)b_0 dW_0(t) + V(t)b\beta(t)S^{\gamma}(t)dW(t), \\ V(0) = V_0. \end{cases} \tag{2.3}$$

A control $(\alpha(\cdot), \beta(\cdot))$ is said to be admissible if $(\alpha(t))_{t\geq 0}$ and $(\beta(t))_{t\geq 0}$ are $\mathcal{F}_t$-adapted processes satisfying $0 \leq \alpha(t), \beta(t) \leq 1$ for all $t \in [0,T]$. The set of all admissible controls is denoted by $\mathcal{A}$.

We use an exponential utility function of the form:

$$U(x) = -\frac{1}{c}e^{-cx}, \ c > 0. \tag{2.4}$$

Since $U'(x) > 0$ and $U''(x) < 0$ for all $x \in [0,\infty)$, $U(x)$ may serve as the utility function of a risk-averse individual. For the surplus process $(V^{\alpha,\beta}(t))_{t\in[0,T]}$ given by (2.3), put

$$J^{\alpha,\beta}(t,s,v) = E[U(V^{\alpha,\beta}(T)) \mid S(t) = s, V^{\alpha,\beta}(t) = v] \tag{2.5}$$

for all $(t,s,v) \in [0,T] \times \mathbb{R}^1 \times \mathbb{R}^1$, where $E[X|A]$ is the conditional expectation of a random variable X given an event A. In stochastic optimal control theory it is important to find the optimal value function

$$H(t,s,v) = \sup_{(\alpha,\beta)\in\mathcal{A}} J^{\alpha,\beta}(t,s,v) \tag{2.6}$$

and the optimal strategy $(\alpha^*(\cdot), \beta^*(\cdot))$ such that

$$J^{\alpha^*,\beta^*}(t,s,v) = H(t,s,v). \tag{2.7}$$

In this paper we will give an explicit expression of $(\alpha^*(t), \beta^*(t))$. The following two theorems are essential to solve our problem. The proofs are standard and can be found in Chapter 14 of [1] or Chapter 11 of [10].

Theorem 2.1. (HJB equation) *Assume that $H(t,s,v)$ defined by (2.6) is twice continuously differentiable on $(0,\infty)$, i.e., $\in C^{1,2}$. Then $H(t,s,v)$ satisfies the following HJB equation:*

$$\begin{cases} \sup\limits_{(\alpha,\beta)\in\mathcal{A}} L^{\alpha,\beta}H(t,s,v) = 0, \\ H(T,s,v) = U(v) \end{cases} \tag{2.8}$$

for all $(t,s,v) \in [0,T) \times \mathbb{R}^1 \times \mathbb{R}^1$, where $L^{\alpha,\beta}$ is the infinitesimal generator corresponding to the diffusion process defined by the stochastic differential equation

(2.3), i.e.,

$$
\begin{aligned}
L^{\alpha,\beta} = &\frac{\partial}{\partial t} + as\frac{\partial}{\partial s} + \{v[\beta(t)a + (1-\beta(t))r] + a_0 - (1-\alpha(t))\lambda\}\frac{\partial}{\partial v} \\
&+ \frac{1}{2}\left\{\alpha(t)^2 b_0^2 + v^2\beta(t)^2 b^2 s^{2\gamma}\right\}\frac{\partial^2}{\partial v^2} + \frac{1}{2}b^2 s^{2+2\gamma}\frac{\partial^2}{\partial s^2} \\
&+ \beta(t)b^2 s^{1+2\gamma}\frac{\partial^2}{\partial s\partial v}.
\end{aligned}
$$

Theorem 2.2. (Verification theorem) *Let $G(t,s,v)$ be a solution of the HJB equation (2.8). Then the value function $H(t,s,v)$ to the control problem (2.6) is given by*

$$H(t,s,v) = G(t,s,v).$$

Moreover, if for some control $(\bar{\alpha}(\cdot),\bar{\beta}(\cdot))$

$$L^{\bar{\alpha},\bar{\beta}}G(t,s,v) = 0$$

for all $(t,s,x) \in [0,T) \times \mathbb{R}^1 \times \mathbb{R}^1$, then it holds

$$G(t,s,v) = J^{\bar{\alpha},\bar{\beta}}(t,s,v).$$

In this case $(\bar{\alpha}(t),\bar{\beta}(t)) = (\alpha^*(t),\beta^*(t))$ and $J^{\bar{\alpha},\bar{\beta}}(t,s,v) = J^{\alpha^*,\beta^*}(t,s,v)$.

Thus the HJB equation associated with our optimization problem is

$$
\begin{aligned}
0 = &H_t + asH_s + (rv + a_0 - \lambda)H_v + \frac{1}{2}bs^{2\gamma+2}H_{ss} \\
&+ \sup_{\alpha}\left\{\alpha\lambda H_v + \frac{1}{2}b_0\alpha^2 H_{vv}\right\} \\
&+ \sup_{\beta}\left\{\beta(a-r)vH_v + \beta bs^{2\gamma+1}vH_{sv} + \frac{1}{2}\beta^2 b^2 s^{2\gamma}v^2 H_{vv}\right\},
\end{aligned}
\tag{2.9}
$$

where $H_t, H_v, H_s, H_{vv}, H_{ss}, H_{sv}$ denote partial derivative of first and second orders with respect to time, stock price and wealth parameters. It is easy to show that the optimal strategy (α^*,β^*) is given by

$$\alpha^* = -\frac{\lambda H_v}{b_0^2 H_{vv}} \tag{2.10}$$

and

$$\beta^* = -\frac{(a-r)H_v + b^2 s^{2\gamma+1}H_{sv}}{vb^2 s^{2\gamma}H_{vv}}. \tag{2.11}$$

Inserting (2.10) and (2.11) into (2.9), we obtain the following second order partial differential equation for the optimal value function H:

$$\begin{aligned}0 =&H_t + asH_s + (rv + a_0 - \lambda)H_v + \frac{1}{2}b^2 s^{2\gamma+2}H_{ss}\\&- \frac{\lambda^2 H_v^2}{2b_0^2 H_{vv}} - \frac{[(a-r)H_v + b^2 s^{2\gamma+1}H_{sv}]^2}{2b^2 s^{2\gamma}H_{vv}}.\end{aligned} \tag{2.12}$$

To get an explicit expression for the optimal strategy (α^*, β^*) given by (2.10) and (2.11), we have to solve this nonlinear equation. Gu et al. [4] solved directly this equation with very complicate calculus. But, by applying a Legendre transform, we transform this equation into a linear partial differential equation of which solution give an explicit expression for (α^*, β^*).

Let $f : \mathbf{R}^n \to \mathbf{R}$ be a convex function. The Legendre transform on $\mathbf{R}$ is defined by

$$L(z) = \max_x \{f(x) - zx\}. \tag{2.13}$$

The function $L(z)$ is called the Legendre dual of the function $f(x)$. If $f(x)$ is strictly convex, the maximum in the above equation will be attened at just one point, which we denote by x_0. It is attained at the unique solution to the first order condition

$$\frac{df(x)}{dx} - z = 0. \tag{2.14}$$

So we may write

$$L(z) = f(x_0) - zx_0. \tag{2.15}$$

Following Jonsson and Sircar[7], a Legendre transform can be defined by

$$\widehat{H}(t,s,z) = \sup_{v>0}\{H(t,s,v) - zv | 0 < v < \infty\},\ 0 < t < T \tag{2.16}$$

where $z > 0$ denotes the dual variable to v. The value of v where this optimum is attained is denoted by $g(t,s,z)$, so that

$$g(t,s,z) = \inf_{v>0}\{v | H(t,s,v) \geq zv + \widehat{H}(t,s,z)\},\ 0 < t < T. \tag{2.17}$$

The function $\widehat{H}$ is related to g by

$$g = -\widehat{H}_z, \tag{2.18}$$

so we can take either one of the two function g and $\widehat{H}$ as the dual of H. As (2.14) describes, we have

$$H_v = z \tag{2.19}$$

and hence

$$\widehat{H}(t,s,z) = H(t,s,g) - zg,\ g(t,s,z) = v. \tag{2.20}$$

By differentiating (2.19) and (2.20) with respect to t, s and z, we obtain

$$\begin{aligned}&H_t = \widehat{H}_t,\ H_s = \widehat{H}_s,\ H_v = z,\ \widehat{H}_z = -g,\\&H_{ss} = \widehat{H}_{ss} - \frac{\widehat{H}_{sz}^2}{\widehat{H}_{zz}},\ H_{vv} = -\frac{1}{\widehat{H}_{zz}},\ H_{sv} = -\frac{\widehat{H}_{sz}}{\widehat{H}_{zz}}.\end{aligned} \tag{2.21}$$

At the terminal time, we denote

$$\widehat{U}(z) = \sup_{v>0}\{U(v) - zv\},$$
$$G(z) = \inf_{v>0}\{v|U(v) \geq zv + \widehat{U}(z)\}.$$

As a result, we have

$$G(z) = (U')^{-1}(z). \tag{2.22}$$

Since $H(T,s,v) = U(v)$, we can define

$$g(T,s,z) = \inf_{v>0}\{v|U(v) \geq zv + \widehat{H}(T,s,z)\}$$

and

$$\widehat{H}(T,s,z) = \sup_{v>0}\{U(v) - zv\},$$

so that

$$g(T,s,z) = (U')^{-1}(z). \tag{2.23}$$

Substituting (2.19), (2.20) and (2.21) into (2.12) and differentiating $\widehat{H}$ with respect to z, we get

$$\begin{aligned} 0 =& g_t - rg - a_0 + \lambda + rsg_s + \left(\frac{\lambda^2 z}{b_0^2} + \frac{(a-r)^2 z}{b^2 s^{2\gamma}} - rz\right) g_z \\ &+ \frac{1}{2} b s^{2\gamma+2} g_{ss} - (a-r) s z g_{sz} + \left(\frac{\lambda^2 z^2}{2b_0^2} + \frac{(a-r)^2 z^2}{2b^2 s^{2\gamma}}\right) g_{zz} \end{aligned} \tag{2.24}$$

and from (2.4) and (2.23), we can see that the boundary condition is

$$g(T,s,z) = \frac{1}{q}\ln z. \tag{2.25}$$

This is linear boundary problem that we have wanted. Moreover, we have

$$\alpha^* = -\frac{\lambda z g_z}{b_0^2} \tag{2.26}$$

and

$$\beta^* = \frac{-(a-r) z g_z + b^2 s^{1+2\gamma} g_s}{b^2 s^{2\gamma} g}. \tag{2.27}$$

3. Main results

First we solve the PDE (2.24) of which the solution give an explicit expression for the optimal strategy (α^*, β^*).

Lemma 3.1. *The solution $g(t,s,z)$ of the PDE (2.24) with a terminal condition (2.23) is given by*

$$g(t,s,z) = -\frac{1}{q} b(t)[\ln z + m(t,s)] + a(t), \tag{3.1}$$

where

$$a(t) = -\frac{a_0 - \lambda}{r}\left(1 - e^{r(t-T)}\right),$$
$$b(t) = e^{r(t-T)},$$
$$m(t,s) = C(t) + D(t)s^{-2\gamma}.$$

Here

$$C(t) = \left(\frac{(2\gamma+1)(a-r)^2}{4r} - r + \frac{\lambda^2}{2b_0^2}\right)(T-t) - \frac{(2\gamma+1)(a-r)^2}{8r^2\gamma}\left(1 - e^{-2r\gamma(T-t)}\right)$$

and

$$D(t) = \frac{(a-r)^2}{4rb^2\gamma}\left(1 - e^{-2r\gamma(T-t)}\right).$$

Proof. We try to find a solution of (2.24) in the form (3.1) with the boundary conditions given by $a(T) = 0$, $b(T) = 1$ and $m(T,s) = 0$. Then

$$g_t = -\frac{1}{q}\left[b'(t)\{\ln z + m(t,s)\} + b(t)m_t\right] + a'(t),$$
$$g_s = \frac{1}{q}b(t)m_s,\ g_z = -\frac{b(t)}{gz},$$
$$g_{ss} = \frac{1}{q}b(t)m_{ss},\ g_{sz} = 0,\ g_{zz} = \frac{b(t)}{gz^2}.$$

Substituting these derivatives in (2.24), we have

$$0 = [b'(t) - rb(t)]\ln z + [ra(t) - a'(t) + a_0 - \lambda]q + \left[m_t + rsm_s + \frac{1}{2}b^2 s^{2\gamma+2}m_{ss} + \frac{(a-r)^2}{2b^2s^{2\gamma}} - rm + \frac{b'(t)}{b(t)}m + \frac{\lambda^2}{2b_0^2} - r\right]b(t).$$

We can split this equation into the following three equations

$$b'(t) - rb(t) = 0, \tag{3.2}$$

$$a'(t) - ra(t) + \lambda - a_0 = 0 \tag{3.3}$$

and

$$m_t + rsm_s + \frac{1}{2}b^2s^{2\gamma+2}m_{ss} + \frac{(a-r)^2}{2b^2s^{2\gamma}} - rm + \frac{b'(t)}{b(t)}m + \frac{\lambda^2}{2b_0^2} - r = 0. \tag{3.4}$$

The solutions to (3.2) and (3.3) which take into account the boundary conditions $b(T) = 1$ and $a(T) = 0$ are

$$b(t) = e^{r(t-T)}, \tag{3.5}$$

$$a(t) = -\frac{a_0 - \lambda}{r}\left(1 - e^{r(t-T)}\right). \tag{3.6}$$

Now to solve (3.4), we define

$$m(t,s) = h(t,y),\ y = s^{-2\gamma},\ h(T,y) = 0. \tag{3.7}$$

Introducing these in (3.4) and combining with (3.5), we get

$$h_t + \frac{\lambda^2}{2b_0^2} + \frac{(a-r)^2}{2b^2}y - r + [(2\gamma^2+\gamma)b^2 - 2r\gamma y]h_y + 2b^2\gamma^2 y h_{yy} = 0. \tag{3.8}$$

We can try to find a solution to (3.8) in the following way:

$$h(t,y) = C(t) + D(t)y \tag{3.9}$$

with $C(T) = 0$ and $D(T) = 0$. Introducing this in (3.8), we obtain

$$\begin{aligned} &C'(t) + \gamma(2\gamma+1)b^2 D(t) + \frac{\lambda^2}{2b_0^2} - r \\ &\qquad + \left[D'(t) - 2r\gamma D(t) + \frac{(a-r)^2}{2b^2}\right] y = 0. \end{aligned}$$

We can split this equation into two ordinary differential equations as follows:

$$D'(t) - 2r\gamma D(t) + \frac{(a-r)^2}{2b^2} = 0,\ D(T) = 0.$$

$$C'(t) + \gamma(2\gamma+1)b^2 D(t) + \frac{\lambda^2}{2b_0^2} - r = 0,\ C(T) = 0.$$

The solutions are given by

$$D(t) = \frac{(a-r)^2}{4rb^2\gamma}\left(1 - e^{-2r\gamma(T-t)}\right) \tag{3.10}$$

and

$$\begin{aligned} C(t) = &\left(\frac{(2\gamma+1)(a-r)^2}{4r} + \frac{\lambda^2}{2b_0^2} - r\right)(T-t) \\ &- \frac{(2\gamma+1)(a-r)^2}{8r^2\gamma}\left(1 - e^{-2r\gamma(T-t)}\right). \end{aligned} \tag{3.11}$$

Inserting (3.10) and (3.11) into (3.9) and using the relation (3.7), we get

$$\begin{aligned} m(t,s) = &\left(\frac{(2\gamma+1)(a-r)^2}{4r} + \frac{\lambda^2}{2b_0^2} - r\right)(T-t) \\ &- \frac{(2\gamma+1)(a-r)^2}{8r^2\gamma}\left(1 - e^{-2r\gamma(T-t)}\right) \\ &+ \frac{(a-r)^2}{4rb^2\gamma}\left(1 - e^{-2r\gamma(T-t)}\right)s^{-2\gamma}. \end{aligned}$$

The proof of Lemma 3.1 is complete.

□

Our main result is as follows.

Theorem 3.2. *The optimal reinsurance and investment strategy* (α^*, β^*) *is given by*

$$\alpha^*(t) = \frac{\lambda}{b_0^2 q} e^{-r(T-t)},$$

$$\beta^*(t) = \frac{e^{-r(T-t)}}{vb^2 s^{2\gamma} q} \left[a - r + \frac{(a-r)^2}{2r} \left(1 - e^{-2r\gamma(T-t)} \right) \right].$$

Proof. The proof is clear from (2.26), (2.27) and Lemma 3.1. □

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