East Asian Mathematical Journal
Vol. 27 (2011), No. 1, pp. 23–33

PERFORMANCE OF MYOPIC POLICY FOR OPPORTUNISTIC SPECTRUM SHARING

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Abstract. Due to underutilization of spectrum under current inefficient and static spectrum management policy, various kinds of opportunistic spectrum access (OSA) strategies have appeared. Myopic policy is a simple and robust OSA strategy with reduced complexity that maximizes immediate throughput. In this paper, we propose mathematical models to evaluate the throughput and the MAC delay of a myopic policy under saturation traffic conditions. Using the MAC delay distribution, we evaluate the packet delay of secondary users under nonsaturation conditions. Numerical results are given to show the performance of the myopic policy in cognitive radio networks.

1. Introduction

With exponential growth in wireless services to share the wireless spectrum, there is an increasing demand for more capacity to be used. On the other hand, most spectrum bands suitable for terrestrial wireless communication have already been allocated to existing licensees [5]. This expected shortage in spectrum supply, which is a major issue for service providers interested in integrating new wireless services into existing communication infrastructure enhancing the capacity for existing applications, is shown to be due not to spectrum scarcity, but to current inefficient and static spectrum management policy [18]. Recent extensive measurement-based studies [17, 3] indicate that a large portion of the licensed spectrum of deployed wireless communication network lies unused in space and time. Even when a channel is actively used, many bursty applications result in abundant spectrum opportunities at slot level [19].

As a solution for the inefficient spectrum usage, Federal Communications Commission (FCC) promotes the so-called opportunistic spectrum access (OSA), which improves spectrum efficiency by allowing secondary users not having a license for spectrum usage to opportunistically occupy an idle spectrum owned by licensee named primary user in a manner that limits interference to primary

Received March 4, 2010; Accepted December 10, 2010.
2000 *Mathematics Subject Classification.* 60K25, 68M20.
Key words and phrases. Queueing theory, opportunistic spectrum access, myopic policy.
This work was financially supported by Dongeui University Grant(2010AA174).

users [7]. To take transmission opportunities left by the primary users and to limit the level of interference perceived by the primary users, secondary users need to sense before transmitting. With limited sensing and access capability, secondary users may not be able to sense all the channels in licensed spectrum simultaneously. So, they need to decide on which channel to sense and which channel to access based on the sensing outcomes [19]. Therefore, for the design of efficient OSA we need to optimize the sensing and access protocol [11, 12, 15, 16]. The design of optimal sensing strategies can be formulated in the framework of partially observable Markov decision process [6, 13]. In [7], the author modeled primary channels as discrete-time Markov chains and developed an OSA scheme to minimize completion time of secondary users based on the theory of finite-horizon partially observable Markov decision process. Unfortunately, the solution to optimal partially observable Markov decision process has exponential complexity with respect to the number of channels [19]. A common approach for tractable solutions is to consider myopic policies, which aim at maximizing only immediate reward. In [19] the throughput of a myopic sensing policy for two channels has been derived in closed-form. The authors also discussed the lower and upper bounds on the throughput of the myopic sensing policy for the case of N channels. They show that the myopic policy is a simple and robust OSA strategy that maximizes immediate throughput and achieves optimality under certain conditions.

In this paper, to evaluate the MAC delay and the packet delay as well as the throughput of secondary users, we present a simple and efficient mathematical model for the myopic policy. The MAC delay and the packet delay are important performance measures when we consider the quality of service of secondary users. Based on our model, we derive the throughput performance and the MAC delay distribution of secondary users under saturation traffic conditions. Using the MAC delay distribution, we also present a queueing model to evaluate the packet delay of secondary users under nonsaturation traffic conditions.

The rest of the paper is organized as follows. We first study the structure of the myopic policy in Section 2 before presenting our mathematical model in Section 3. We also analyze the saturation and nonsaturation performance in Section 3. Numerical results are given in Section 4. We conclude the paper in Section 5.

2. Myopic policy

We consider a spectrum consisting of two primary channels. These channels are modeled as independent and stochastically identical Gilbert-Elliot channels [4, 20]. As illustrated in Figure 1, the occupancy $S_i(k)$, $k \geq 1$, of channel i, $i = 1, 2$, in slot k follows a discrete-time two-state Markov chain on state space $\{0, 1\}$ with one-step transition probability matrix $\{p_{ij}\}_{i,j=0,1}$. State 0

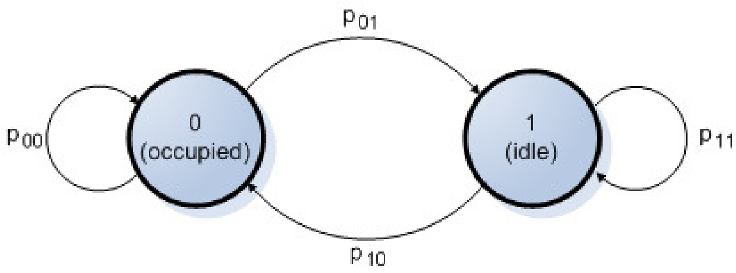

FIGURE 1. Gilbert-Elliot channel model

represents that the channel is occupied by primary users while state 1 represents that the channel is idle.

We consider a secondary user seeking spectrum opportunities. Limited by its hardware constraints and energy supply, the full spectrum is not observable to the secondary user. However, it can infer the state from its decision and observation history. Immediately before the beginning of slot k, $k \geq 1$, its knowledge of the channels' state is given by belief vector $(\lambda_1(k), \lambda_2(k))$, where $\lambda_i(k)$, $i = 1, 2$, is the conditional probability that channel i is available immediately before slot k, given all past decision and observations. In each slot, the secondary user with packets to transmit chooses a channel to sense. For myopic policy, the action in slot k is chosen to maximize expected immediate throughput, i.e., the index $a(k)$ of the channel the user selects at slot k is simply given by

$$a(k) = \arg\max_{i=1,2} \lambda_i(k) \tag{1}$$

for $k \geq 1$. If the chosen channel is sensed to be idle, the secondary user transmits its packet. Otherwise, the secondary user does not transmit the packet. Immediately before the beginning of slot $k+1$, $k \geq 1$, the belief vector is updated based on the action $a(k)$ with observation outcome $\Theta_a(k)$ (indicating the availability of channel $a(k)$) as follows:

$$\lambda_i(k+1) = \begin{cases} p_{01}, & \text{if } a(k) = i \text{ and } \Theta_a(k) = 0, \\ p_{11}, & \text{if } a(k) = i \text{ and } \Theta_a(k) = 1, \\ \lambda_i(k)p_{11} + (1-\lambda_i(k))p_{01}, & \text{if } a(k) \neq i \end{cases}$$

for $i = 1, 2$. We set $\lambda_i(1)$ as the stationary probability $\omega \equiv p_{01}/(p_{01} + p_{10})$ of the channel being idle and choose the initial action randomly.

Note that the belief vector forms a two-dimensional Markov process with an uncountable state space. In general, obtaining the myopic action in each slot requires the comparison between the entries of the belief vector with an uncountable state space and the recursive update of the belief vector, which make its performance evaluation difficult. However, the myopic policy has a

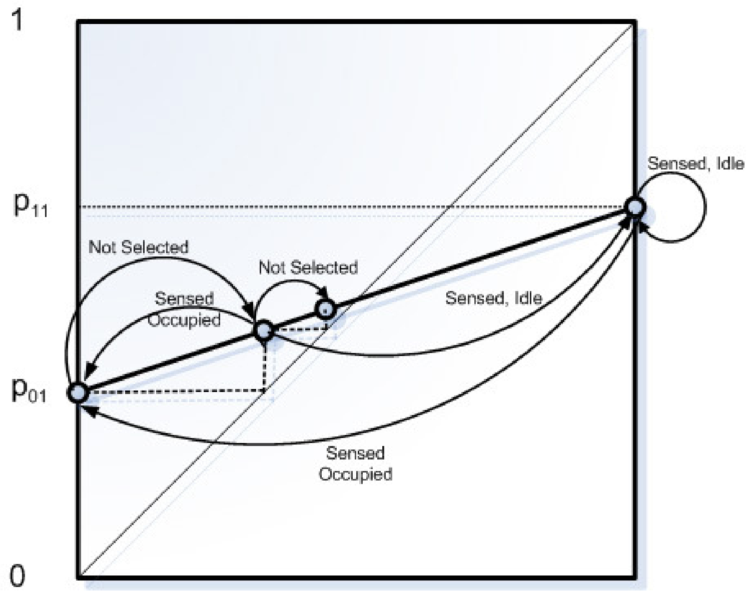

FIGURE 2. Evolution of the belief vector when $p_{01} < p_{11}$

simple structure that does not need the update of the belief vector or the comparison between the entries of the belief vector [19].

First, we consider the case of $p_{01} < p_{11}$. If channel i, $i = 1, 2$, is observed to be idle in slot k, $k \geq 1$, the probability $\lambda_i(k+1)$ becomes p_{11} (see in Figure 2). Since $\lambda_j(k+1) < p_{11}$ for $j \neq i$, channel i is sensed again in slot $k+1$. On the other hand, if channel i is observed to be busy in slot k, the probability $\lambda_i(k+1)$ becomes p_{01}. Since $\lambda_j(k+1) > p_{01}$ for $j \neq i$, the other channel is sensed in slot $k+1$. Thus, if the chosen channel is observed to be idle, the channel is chosen again in the next slot; otherwise, the other channel is chosen in the next slot.

Second, we consider the case of $p_{01} > p_{11}$. If channel i, $i = 1, 2$, is observed to be idle in slot k, $k \geq 1$, the probability $\lambda_i(k+1)$ becomes p_{11} (see in Figure 3). Since $\lambda_j(k+1) > p_{11}$ for $j \neq i$, the other channel is sensed in slot $k+1$. On the other hand, if channel i is observed to be busy in slot k, the probability $\lambda_i(k+1)$ becomes p_{01}. Since $\lambda_j(k+1) < p_{01}$ for $j \neq i$, channel i is also sensed again in slot $k+1$. Thus, if the chosen channel is observed to be busy, the channel is chosen again in the next slot; otherwise, the other channel is chosen in the next slot.

If $p_{01} = p_{11}$, myopic policy implies random channel selection. So, in this paper we assume $p_{01} \neq p_{11}$.

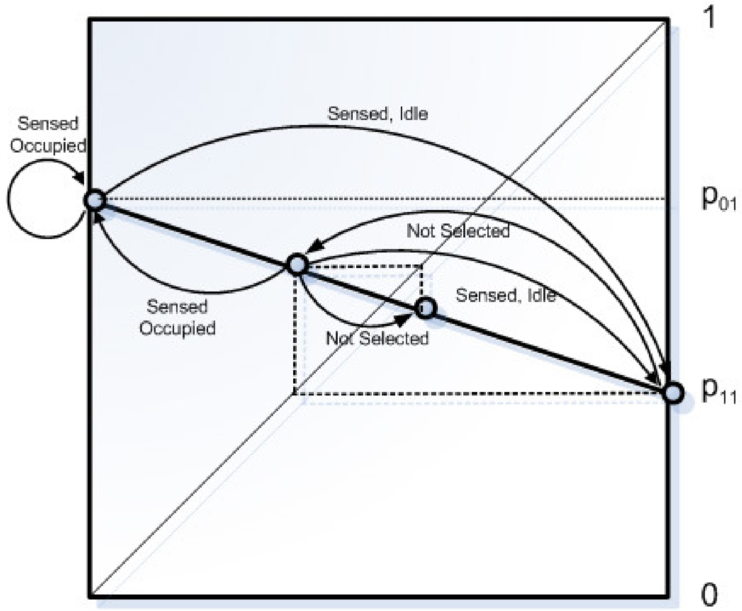

FIGURE 3. Evolution of the belief vector when $p_{01} > p_{11}$

3. Mathematical analysis

3.1. Saturation analysis

To analyze the performance of a secondary user under saturation condition, we assume that the secondary user always has another packet immediately after the successful completion of a packet transmission. We also assume that the secondary user performs perfect sensing. Then, under the assumptions, the process $\{(S_1(k), S_2(k), a(k)), k \geq 1\}$ becomes a tri-dimensional discrete-time Markov chain with finite state space $\{(0,0,1), (0,1,1), (0,0,2), (1,0,2), (1,0,1), (1,1,1), (0,1,2), (1,1,2)\}$. We analyze the stationary probability distribution for the Markov chain, defined by

$$\begin{aligned}
\pi_{i,j,l} &\equiv \lim_{k\to\infty} P\{S_1(k) = i, S_2(k) = j, a(k) = l\}, \\
\pi_{\mathbf{0}} &\equiv (\pi_{0,0,1}, \pi_{0,1,1}, \pi_{0,0,2}, \pi_{1,0,2}), \\
\pi_{\mathbf{1}} &\equiv (\pi_{1,0,1}, \pi_{1,1,1}, \pi_{0,1,2}, \pi_{1,1,2}), \\
\pi &\equiv (\pi_{\mathbf{0}}, \pi_{\mathbf{1}}).
\end{aligned}$$

The one-step transition probability matrix of the Markov chain is given by

$$\mathbf{P} \equiv \begin{pmatrix} \mathbf{P_{00}} & \mathbf{P_{01}} \\ \mathbf{P_{10}} & \mathbf{P_{11}} \end{pmatrix}, \tag{2}$$

where for $p_{01} < p_{11}$

$$\mathbf{P_{00}} \equiv \begin{pmatrix} 0 & 0 & p_{00}p_{00} & p_{01}p_{00} \\ 0 & 0 & p_{00}p_{10} & p_{01}p_{10} \\ p_{00}p_{00} & p_{00}p_{01} & 0 & 0 \\ p_{10}p_{00} & p_{10}p_{01} & 0 & 0 \end{pmatrix},$$

$$\mathbf{P_{01}} \equiv \begin{pmatrix} 0 & 0 & p_{00}p_{01} & p_{01}p_{01} \\ 0 & 0 & p_{00}p_{11} & p_{01}p_{11} \\ p_{01}p_{00} & p_{01}p_{01} & 0 & 0 \\ p_{11}p_{00} & p_{11}p_{01} & 0 & 0 \end{pmatrix},$$

$$\mathbf{P_{10}} \equiv \begin{pmatrix} p_{10}p_{00} & p_{10}p_{01} & 0 & 0 \\ p_{10}p_{10} & p_{10}p_{11} & 0 & 0 \\ 0 & 0 & p_{00}p_{10} & p_{01}p_{10} \\ 0 & 0 & p_{10}p_{10} & p_{11}p_{10} \end{pmatrix},$$

$$\mathbf{P_{11}} \equiv \begin{pmatrix} p_{11}p_{00} & p_{11}p_{01} & 0 & 0 \\ p_{11}p_{10} & p_{11}p_{11} & 0 & 0 \\ 0 & 0 & p_{00}p_{11} & p_{01}p_{11} \\ 0 & 0 & p_{10}p_{11} & p_{11}p_{11} \end{pmatrix},$$

and for $p_{01} > p_{11}$

$$\mathbf{P_{00}} \equiv \begin{pmatrix} p_{00}p_{00} & p_{00}p_{01} & 0 & 0 \\ p_{00}p_{10} & p_{00}p_{11} & 0 & 0 \\ 0 & 0 & p_{00}p_{00} & p_{01}p_{00} \\ 0 & 0 & p_{10}p_{00} & p_{11}p_{00} \end{pmatrix},$$

$$\mathbf{P_{01}} \equiv \begin{pmatrix} p_{01}p_{00} & p_{01}p_{01} & 0 & 0 \\ p_{01}p_{10} & p_{01}p_{11} & 0 & 0 \\ 0 & 0 & p_{00}p_{01} & p_{01}p_{01} \\ 0 & 0 & p_{10}p_{01} & p_{11}p_{01} \end{pmatrix},$$

$$\mathbf{P_{10}} \equiv \begin{pmatrix} 0 & 0 & p_{10}p_{00} & p_{11}p_{00} \\ 0 & 0 & p_{10}p_{10} & p_{11}p_{10} \\ p_{00}p_{10} & p_{00}p_{11} & 0 & 0 \\ p_{10}p_{10} & p_{10}p_{11} & 0 & 0 \end{pmatrix},$$

$$\mathbf{P_{11}} \equiv \begin{pmatrix} 0 & 0 & p_{10}p_{01} & p_{11}p_{01} \\ 0 & 0 & p_{10}p_{11} & p_{11}p_{11} \\ p_{01}p_{10} & p_{01}p_{11} & 0 & 0 \\ p_{11}p_{10} & p_{11}p_{11} & 0 & 0 \end{pmatrix}.$$

The stationary probability vector π of $\{(S_1(k), S_2(k), a(k)), k \geq 1\}$ is given by solving the equations $\pi\mathbf{P} = \pi$ and $\pi\mathbf{e} = 1$, where $\mathbf{e}$ is a column vector of 1's [2, 14].

The saturation throughput S_{sat} is given by

$$S_{sat} = \pi_{\mathbf{1}}\mathbf{e}. \tag{3}$$

Now we compute the distribution of the MAC delay of a secondary user under saturation traffic conditions. The MAC delay is defined as the time needed for a packet to be successfully transmitted, given that the packet is at the head-of-line position in the buffer. Let d_l be the probability that the MAC delay under saturation traffic condition is l slots. Then

$$d_1 = \frac{\pi_1 \mathbf{P_{11}} \mathbf{e}}{\pi_1 \mathbf{e}}, \qquad d_l = \frac{\pi_1 \mathbf{P_{10}} \mathbf{P_{00}^{l-2}} \mathbf{P_{01}} \mathbf{e}}{\pi_1 \mathbf{e}}, \quad l > 1. \tag{4}$$

3.2. Nonsaturation analysis

To analyze the secondary user of the myopic policy under nonsaturation traffic conditions, we assume that the secondary user has infinite buffer to store its packets and the packet arrivals follow a batch geometric process [8, 9]. The number of packets arriving during the consecutive slots are assumed to be i.i.d. non-negative discrete random variables with an arbitrary probability distribution and are characterized by the probability generating function (pgf) $A(z)$. When we model the secondary user as a queueing system, the MAC delay can be considered as service time. We consider two cases: a packet entering an empty system and a packet entering a non-empty system. The state of the Markov chain $\{(S_1(k), S_2(k), a(k)), k \geq 1\}$ observed by a packet entering an empty system is given by

$$\left(\frac{(1-\omega)\mathbf{x}}{2}, \frac{\omega \mathbf{x}}{2} \right)$$

by BASTA property, where ω is the stationary probability that a channel is idle, and $\mathbf{x} \equiv (1-\omega, \omega, 1-\omega, \omega)$. Thus, the probability d_l^e that the MAC delay of a packet entering an empty system is l slots is given by

$$d_1^e = \frac{\omega \mathbf{x}}{2} \mathbf{e}, \qquad d_l^e = \frac{(1-\omega)\mathbf{x}}{2} \mathbf{P_{00}}^{l-2} \mathbf{P_{01}} \mathbf{e}, \quad l > 1. \tag{5}$$

On the other hand, the MAC delay of a packet entering a non-empty system can be approximated by the saturation MAC delay obtained by (4). Therefore, we can model a secondary user as a discrete-time Geo^X/G/1 queue [10, 1] with service time $\{d_l, l \geq 1\}$, in which packets entering an empty system have the exceptional service time $\{d_l^e, l \geq 1\}$. Then, the mean packet delay D of a secondary user is given by

$$\begin{aligned} D \;=\; & \frac{[A'(1)]^2 \left[S'(1) + S''(1)\right] + A''(1)S'(1)}{2A'(1)\left[1 - A'(1)S'(1)\right]} - \frac{1}{2} \\ & + \frac{[1 - A(0)]\left[S_e''(1) - S''(1)\right] + 2S_e'(1)}{2\left[1 + \{1 - A(0)\}\{S_e'(1) - S'(1)\}\right]}, \end{aligned} \tag{6}$$

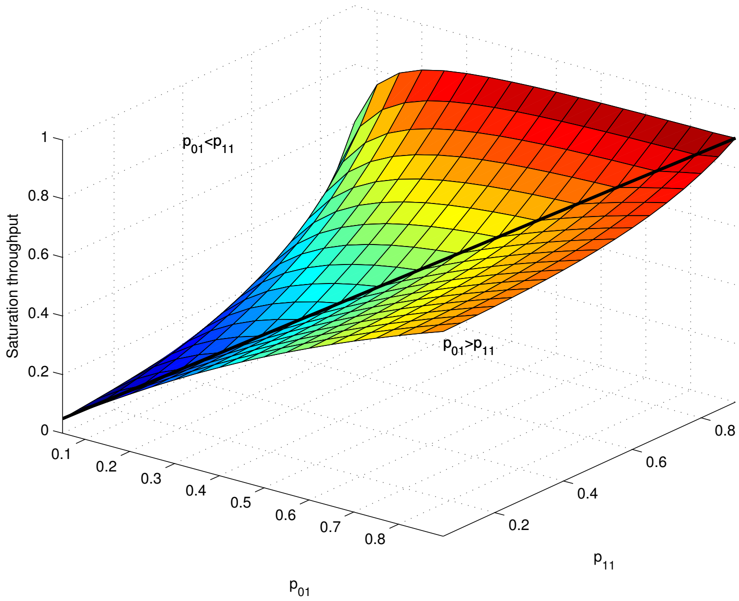

FIGURE 4. Throughput under saturation traffic condition

where $S(z)$ and $S_e(z)$ are the pgf's of the service time $\{d_l, l \geq 1\}$ and the exceptional service time $\{d_l^e, l \geq 1\}$, respectively:

$$S(z) \equiv \sum_{l=1}^{\infty} d_l z^l = \frac{\pi_1 \mathbf{P}_{11}\mathbf{e}}{\pi_1 \mathbf{e}} z + \frac{\pi_1 \mathbf{P}_{10} \left[\mathbf{I} - \mathbf{P}_{00} z\right]^{-1} \mathbf{P}_{01}\mathbf{e}}{\pi_1 \mathbf{e} z^2}, \tag{7}$$

$$S_e(z) \equiv \sum_{l=1}^{\infty} d_l^e z^l = \frac{\omega \mathbf{x}}{2}\mathbf{e} z + \frac{(1-\omega)\mathbf{x}}{2} \frac{\left[\mathbf{I} - \mathbf{P}_{00} z\right]^{-1} \mathbf{P}_{01}\mathbf{e}}{z^2}. \tag{8}$$

4. Numerical results

In this section, we present numerical results to evaluate the performance of the myopic policy.

Figure 4 shows the saturation throughput of a secondary user as the channel-state transition probabilities vary. As p_{01} and/or p_{11} increase, the probability of the channels being idle also increases. Thus, the saturation throughput increases. Figure 5 shows the analytic results of the mean MAC delay under saturation traffic conditions as the channel-state transition probabilities vary. As p_{01} and/or p_{11} increase, the probability of the channels being idle also increases. Thus, the mean MAC delay decreases. In Figure 4 and 5, the upper

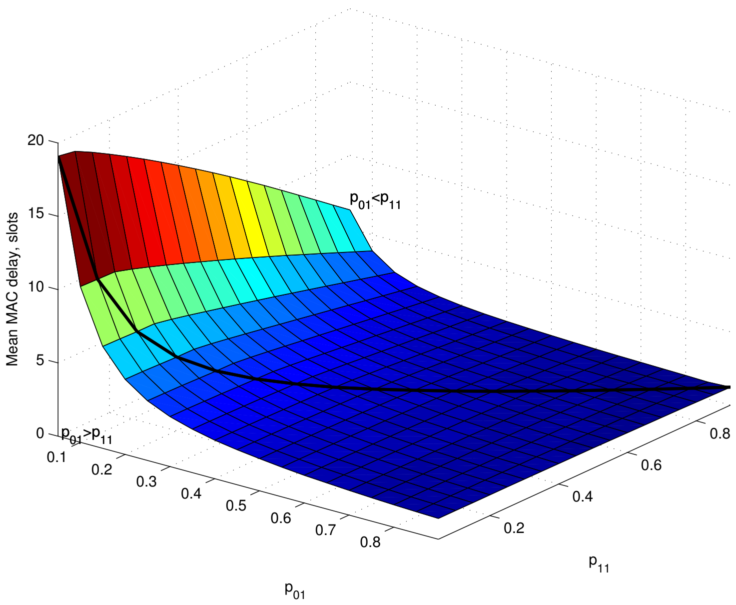

Figure 5. Mean MAC delay under saturation traffic condition

part of the bold line is for $p_{01} < p_{11}$, and the lower part of the bold line is for $p_{01} > p_{11}$.

Figure 6 shows the analytic results of the mean packet delay, including the queueing delay and the transmission time, under nonsaturation traffic conditions. In this example, we set $A(z) = 1 - \lambda + \lambda z$ for nonsaturation condition and fix the stationary probability ω of channel being idle as $1/2$. In the figure, as λ increases, the mean packet delay also increases. The solid lines are for $p_{01} < p_{11}$, and the dashed lines are for $p_{01} > p_{11}$. As p_{01} increases with fixed λ, the mean packet delay decreases, because the burstiness of channel availability decreases.

5. Conclusions

In this paper, we have developed a simple and efficient mathematical model for evaluating the performance of a myopic policy. Based on our model, we derived the saturation throughput of the myopic policy. We also analyzed the MAC delay distribution of a secondary user under saturation traffic conditions. Using the MAC delay distribution, we evaluated the packet delay of a secondary user under nonsaturation traffic conditions. The MAC delay and the packet

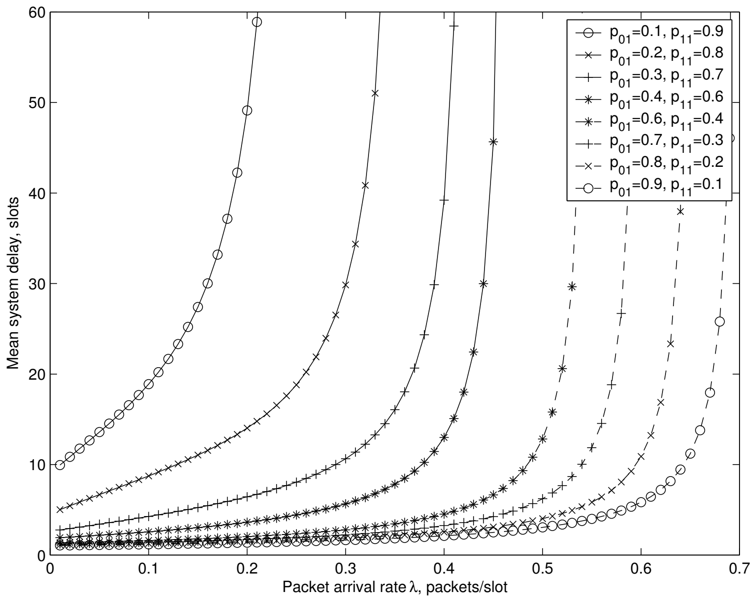

FIGURE 6. Mean packet delay under nonsaturation traffic condition

delay are important performance measures when we consider the quality of service of a secondary user.

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