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A NOTE ON THE WEIGHTED TWISTED DIRICHLET'S TYPE q-EULER NUMBERS AND POLYNOMIALS

Serkan Araci, Nurgül Aslan and Jong Jin Seo

Abstract. We in this paper construct Dirichlet's type twisted $q-$Euler numbers and polynomials with weight α. We give some interesting identities some relations.

1. Introduction, Definitions and Notations

Let p be a fixed odd prime number. Throughout this paper we use the following notations, by $\mathbb{Z}_p$ denotes the ring of p-adic rational integers, $\mathbb{Q}$ denotes the field of rational numbers, $\mathbb{Q}_p$ denotes the field of p-adic rational numbers, and $\mathbb{C}_p$ denotes the completion of algebraic closure of $\mathbb{Q}_p$. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{N}^* = \mathbb{N} \cup \{0\}$. The p-adic absolute value is defined by $|p|_p = \frac{1}{p}$. In this paper we assume $|q-1|_p < 1$ as an indeterminate. In $[11-13]$, the fermionic *p-adic q-integral* on $\mathbb{Z}_p$ is defined by *Kim*

$$(1.1) \quad I_{-q}(f) = \int_{\mathbb{Z}_p} f(x)\, d\mu_{-q}(x) = \lim_{N\to\infty} \frac{1}{[p^N]_{-q}} \sum_{x=0}^{p^N-1} (-1)^x f(x)\, q^x$$

$[x]_q$ is a q-extension of x which is defined by

$$[x]_q = \frac{1-q^x}{1-q}, \quad \text{see[1]-[22]}$$

Note that $\lim_{q\to 1} [x]_q = x$.

If we take $f_1(x) = f(x+1)$ in (1.1), then we easily see that

$$(1.2) \qquad qI_{-q}(f_1) + I_{-q}(f) = [2]_q f(0)$$

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From (1.2), we obtain

$$(1.3)\qquad (-1)^{n-1} I_{-q}(f) + q^n I_{-q}(f_n) = [2]_q \sum_{l=0}^{n-1} (-1)^{n-1-l} q^l f(l).$$

where $f_n(x) = f(x+n)$ (see [1]-[18]).

In[18], by using *p-adic q-integral* on $\mathbb{Z}_p$, the weighted q-Euler numbers $E_{n,q}^{(\alpha)}$ and $E_{n,q}^{(\alpha)}(x)$ are defined by

$$E_{n,q}^{(\alpha)} = \int_{\mathbb{Z}_p} [x]_{q^\alpha}^n \, d\mu_{-q}(x), \text{ for } n \in \mathbb{N}^* \text{ and } \alpha \in \mathbb{Z}.$$

Let $C_{p^n} = \left\{ w \mid w^{p^n} = 1 \right\}$ be the cylic group of order p^n, and let $T_p = \lim_{n\to\infty} C_{p^n} = C_{p^\infty} = \cup_{n\geq 0} C_{p^n}$ (see [14],[17]). Note that T_p is locally constant space.

In[16], Let χ be a Dirichlet's character with conductor $d\,(= odd) \in \mathbb{N}$ and $w \in T_p$. If we take $f(x) = \chi(x) w^x e^{tx}$, then we have $f(x+d) = \chi(x) w^x w^d e^{tx} e^{td}$. From (1.3), we obtain

$$(1.4)\qquad \int_X \chi(x) w^x e^{tx} d\mu_{-q}(x) = \frac{[2]_q \sum_{i=0}^{d-1} (-1)^{d-1-i} q^i \chi(i) w^i e^{ti}}{q^d w^d e^{td} + 1}.$$

In view of (1.4) we obtain twisted Dirichlet's type q-Euler numbers as follows:

(1.5)

$$F_{w,\chi}^q(t) = \frac{[2]_q \sum_{i=0}^{d-1} (-1)^{d-1-i} q^i \chi(i) w^i e^{ti}}{q^d w^d e^{td} + 1} = \sum_{n=0}^{\infty} E_{n,\chi,w}^q \frac{t^n}{n!}, \ \ |t + \log(qw)| < \frac{\pi}{d}.$$

In this paper, we construct Dirichlet's type twisted q-Euler numbers and polynomials with weight α. By using fermionic p-adic q-integral equations on $\mathbb{Z}_p$, we investigate some interesting identities and relations on the Dirichlet's type twisted q-Euler numbers and polynomials with weight α. Furthermore, we derive the q-extensions of Dirichlet's type q-l-functions with weight α from the Mellin transformation of this generating function which interpolates the Dirichlet's type twisted q-Euler polynomials with weight α.

2. On the weighted twisted Dirichlet's type $q-$Euler numbers and polynomials

In this section, by using fermionic p-adic q-integral equations on $\mathbb{Z}_p$, some interesting identities and relation on the twisted Dirichlet's type q-Euler numbers and polynomials with weight α.

Definition 1. *Let χ be a Dirichlet's character with conductor $d\,(=odd)\in\mathbb{N}$. For each $\alpha, n\in\mathbb{N}^*$ and $w\in T_p$. Then the generating function of twisted Dirichlet's type q-Euler polynomials with weight α defined by as follows:*

$$\mathcal{F}^{(\alpha)}\left(t,x,q,w\mid\chi\right)=\sum_{m=0}^{\infty}\widetilde{E}_n\left(\alpha,x,w,q\mid\chi\right)\frac{t^n}{n!} \tag{2.1}$$

where

$$\mathcal{F}^{(\alpha)}\left(t,x,q,w\mid\chi\right)=[2]_q\sum_{m=0}^{\infty}(-1)^m q^m w^m\chi\left(m\right)e^{t[x+m]_{q^\alpha}}. \tag{2.2}$$

From (2.1) and (2.2) we obtain,

$$\sum_{n=0}^{\infty}\widetilde{E}_n\left(\alpha,w,q\mid\chi\right)\frac{t^n}{n!}=\sum_{n=0}^{\infty}\left([2]_q\sum_{m=0}^{\infty}(-1)^m q^m w^m\chi\left(m\right)[x+m]_{q^\alpha}^n\right)\frac{t^n}{n!}$$

Therefore we obtain the following theorem:

Theorem 1. *Let χ be a Dirichlet's character with conductor $d\,(=odd)\in\mathbb{N}$. For each $\alpha, n\in\mathbb{N}^*$ and $w\in T_p$. Then we have*

$$\widetilde{E}_n\left(\alpha,x,w,q\mid\chi\right)=[2]_q\sum_{m=0}^{\infty}(-1)^m q^m w^m\chi\left(m\right)[x+m]_{q^\alpha}^n \tag{2.3}$$

By using (2.3),

$$\begin{aligned}
&\widetilde{E}_n\left(\alpha,x,w,q\mid\chi\right)\\
&=[2]_q\sum_{m=0}^{\infty}\sum_{l=0}^{d-1}(-1)^{l+md}q^{l+md}w^{l+md}\chi\left(l+md\right)[x+l+md]_{q^\alpha}^n\\
&=\frac{[2]_q}{(1-q^\alpha)^n}\sum_{l=0}^{d-1}(-1)^l q^l w^l\chi\left(l\right)\sum_{m=0}^{\infty}(-1)^m\left(q^d\right)^m\left(w^d\right)^m\sum_{k=0}^{n}\binom{n}{k}(-1)^k q^{\alpha k(x+l+md)}\\
&=\frac{[2]_q}{(1-q^\alpha)^n}\sum_{l=0}^{d-1}(-1)^l q^l w^l\chi\left(l\right)\sum_{k=0}^{n}\binom{n}{k}(-1)^k q^{\alpha k(x+l)}\sum_{m=0}^{\infty}(-1)^m\left(q^d\right)^m\left(w^d\right)^m\left(q^{\alpha kd}\right)^m\\
&=\frac{[2]_q}{(1-q^\alpha)^n}\sum_{l=0}^{d-1}(-1)^l q^l w^l\chi\left(l\right)\sum_{k=0}^{n}\frac{\binom{n}{k}(-1)^k q^{\alpha k(x+l)}}{q^{\alpha kd}w^d q^d+1}.
\end{aligned}$$

So, we obtain the following corollary:

Corollary 1. *Let χ be a Dirichlet's character with conductor $d\,(=odd)\in\mathbb{N}$. For each $\alpha,n\in\mathbb{N}^*$ and $w\in T_p$. Then we have*

$$\begin{aligned}\widetilde{E}_n\left(\alpha,x,w,q\mid\chi\right) &= [2]_q\sum_{m=0}^{\infty}(-1)^m q^m w^m\chi(m)\,[x+m]_{q^\alpha}^n\\ &= \frac{[2]_q}{(1-q^\alpha)^n}\sum_{l=0}^{d-1}(-1)^l q^l w^l\chi(l)\sum_{k=0}^{n}\frac{\binom{n}{k}(-1)^k q^{\alpha k(x+l)}}{q^{\alpha kd}w^d q^d+1}.\end{aligned}$$

In (1.1), we take $f(y)=\chi(y)\,w^y\,[x+y]_{q^\alpha}^n$,

$$\begin{aligned}&\int_{\mathbb{Z}_p}\chi(y)\,w^y\,[x+y]_{q^\alpha}^n\,d\mu_{-q}(y)\\ &=\frac{1}{(1-q^\alpha)^n}\sum_{k=0}^{n}\binom{n}{k}(-1)^k q^{\alpha kx}\int_{\mathbb{Z}_p}\chi(y)\,w^y q^{ayk}d\mu_{-q}(y)\end{aligned}\tag{2.4}$$

where from (1.3), we easily see that,

$$\int_{\mathbb{Z}_p}\chi(y)\,w^y q^{y\alpha k}d\mu_{-q}(y)=\frac{[2]_q\sum_{l=0}^{d-1}(-1)^l q^l w^l\chi(l)}{q^d q^{\alpha kd}w^d+1}\tag{2.5}$$

by using (2.4) and (2.5) we obtain

$$\begin{aligned}&\int_{\mathbb{Z}_p}\chi(y)\,w^y\,[x+y]_{q^\alpha}^n\,d\mu_{-q}(y)\\ &=\frac{1}{(1-q^\alpha)^n}\sum_{l=0}^{n}\binom{n}{k}(-1)^k q^{\alpha kx}\frac{[2]_q\sum_{l=0}^{d-1}(-1)^l q^l w^l\chi(l)}{q^d q^{\alpha kd}w^d+1}\\ &=\widetilde{E}_n\left(\alpha,x,w,q\mid\chi\right)\end{aligned}\tag{2.6}$$

From (2.6) above we obtain twisted Dirichlet's type q-Euler polynomials with weight α witt's type formula as follows theorem:

Theorem 2. *Let χ be a Dirichlet's character with conductor $d\,(=odd)\in\mathbb{N}$. For each $\alpha,n\in\mathbb{N}^*$ and $w\in T_p$. Then we obtain*

$$\widetilde{E}_n\left(\alpha,x,w,q\mid\chi\right)=\int_{\mathbb{Z}_p}\chi(y)\,w^y\,[x+y]_{q^\alpha}^n\,d\mu_{-q}(y)\,.\tag{2.7}$$

By (2.2), we obtain *functional equation* as follows:

$$\mathcal{F}^{(\alpha)}\left(t,x,q,w\mid\chi\right)=e^{t[x]_{q^\alpha}}\mathcal{F}^{(\alpha)}\left(q^x t,q,w\mid\chi\right)$$

By using the definition of the generating function $\mathcal{F}^{(\alpha)}(t,x,q,w \mid \chi)$ as follows:

$$\sum_{n=0}^{\infty} \widetilde{E}_n(\alpha,x,w,q \mid \chi)\frac{t^n}{n!} = \left(\sum_{n=0}^{\infty}[x]_{q^\alpha}^n \frac{t^n}{n!}\right)\left(\sum_{n=0}^{\infty} q^{n\alpha x}\widetilde{E}_n(\alpha,w,q \mid \chi)\frac{t^n}{n!}\right)$$

By using the Cauchy product in the above equation, we have

$$\sum_{n=0}^{\infty} \widetilde{E}_n(\alpha,x,w,q \mid \chi)\frac{t^n}{n!} = \sum_{n=0}^{\infty}\left(\sum_{l=0}^{n}\binom{n}{l} q^{\alpha l x}\widetilde{E}_l(\alpha,w,q \mid \chi)[x]_{q^\alpha}^{n-l}\right)\frac{t^n}{n!}$$

Comparing the coefficients of $\frac{t^n}{n!}$ on the both sides of the above equation, we arrive at the following theorem:

Theorem 3. *Let χ be a Dirichlet's character with conductor $d\,(=odd) \in \mathbb{N}$. For each $\alpha, n \in \mathbb{N}^*$ and $w \in T_p$. Then*

$$\widetilde{E}_n(\alpha,x,w,q \mid \chi) = \sum_{l=0}^{n}\binom{n}{l} q^{\alpha x l}\widetilde{E}_l(\alpha,w,q \mid \chi)[x]_{q^\alpha}^{n-l}. \tag{2.8}$$

By (2.8), and the *umbral calculus* convention, we obtain

$$\widetilde{E}_n(\alpha,x,w,q \mid \chi) = \left(q^{\alpha x}\widetilde{E}(\alpha,w,q \mid \chi) + [x]_{q^\alpha}\right)^n \tag{2.9}$$

with usual convention about replacing $\left(\widetilde{E}(\alpha,w,q \mid \chi)\right)^n$ by $\widetilde{E}_n(\alpha,w,q \mid \chi)$.

From (1.3) we arrive at the following theorem:

Theorem 4. *Let χ be a Dirichlet's character with conductor $d\,(=odd) \in \mathbb{N}$. For each $\alpha, n \in \mathbb{N}^*$ and $w \in T_p$. We get*

$$q^n\widetilde{E}_m(\alpha,w,q \mid \chi) + (-1)^{n-1}\widetilde{E}_m(n,\alpha,w,q \mid \chi) = [2]_q \sum_{l=0}^{n-1}(-1)^l q^l \chi(l) w^l [l]_{q^\alpha}^m.$$

From (1.1), we can easily derive the following (2.10)

(2.10)

$$\begin{aligned}
&\int_{\mathbb{Z}_p} \chi(y) w^y [x+y]_{q^\alpha}^n \, d\mu_{-q}(y) \\
&= \frac{[d]_{q^\alpha}^n}{[d]_{-q}} \sum_{a=0}^{d-1}(-1)^a \chi(a) w^a q^a \int_{\mathbb{Z}_p} w^{dy}\left[\frac{x+a}{d}+y\right]_{q^{d\alpha}}^n d\mu_{(-q)^d}(y) \\
&= \frac{[d]_{q^\alpha}^n}{[d]_{-q}} \sum_{a=0}^{d-1}(-1)^a w^a q^a \chi(a) E_{n,q^d,w^d}^{(\alpha)}\left(\frac{x+a}{d}\right).
\end{aligned}$$

Therefore, by (2.10), we obtain the following theorem:

Theorem 5. *Let χ be a Dirichlet's character with conductor $d\,(=odd)\in\mathbb{N}$. For each $\alpha,n\in\mathbb{N}^*$ and $w\in T_p$. We get*

$$\widetilde{E}_n\left(\alpha,x,w,q\mid\chi\right)=\frac{[d]_{q^\alpha}^n}{[d]_{-q}}\sum_{a=0}^{d-1}(-1)^a\,w^aq^a\chi\left(a\right)E_{n,q^d,w^d}^{(\alpha)}\left(\frac{x+a}{d}\right).$$

3. Interpolation function of the polynomials $\widetilde{E}_n\left(\alpha,x,w,q\mid\chi\right)$

In this section, we give interpolation function of the generating functions of twisted Dirichlet's type q-Euler polynomials with weight α. For $s\in\mathbb{C}$, $w\in T_p$ and χ be a Dirichlet's character with conductor $d(=odd)\in\mathbb{N}$, by applying the Mellin transformation to (2.2), we obtain

$$\begin{aligned}\boldsymbol{l}_q\left(x,\alpha,w\mid s\right) &= \frac{1}{\Gamma\left(s\right)}\oint t^{s-1}\mathcal{F}^{(\alpha)}\left(t,x,q,w\mid\chi\right)dt\\ &= [2]_q\sum_{m=0}^{\infty}(-1)^m\,w^mq^m\chi\left(m\right)\frac{1}{\Gamma\left(s\right)}\int_0^\infty t^{s-1}e^{-t[m+x]_{q^\alpha}}dt\end{aligned}$$

so we have

$$\boldsymbol{l}^q\left(s,x,\alpha,w\mid\chi\right)=[2]_q\sum_{m=0}^{\infty}\frac{(-1)^m\,\chi\left(m\right)w^mq^m}{[m+x]_{q^\alpha}^s}$$

We define $q-$extension Dirichlet's type q-l-function as follows theorem:

Theorem 6. *Let χ be a Dirichlet's character with conductor $d\,(=odd)\in\mathbb{N}$. For each $\alpha,n\in\mathbb{N}^*$ and $w\in T_p$. We have*

$$\boldsymbol{l}^q\left(s,x,\alpha,w\mid\chi\right)=[2]_q\sum_{m=0}^{\infty}\frac{(-1)^m\,\chi\left(m\right)w^mq^m}{[m+x]_{q^\alpha}^s}\tag{3.1}$$

for all $s\in\mathbb{C}$. We note that $\boldsymbol{l}^q\left(s,x,\alpha,w\mid\chi\right)$ is analytic function in the whole complex s-plane.

By subsituting $s=-n$ into (3.1) we easily get

$$\boldsymbol{l}^q\left(-n,x,\alpha,w\mid\chi\right)=\widetilde{E}_n\left(\alpha,x,w,q\mid\chi\right).$$

we obtain the following theorem:

Theorem 7. *Let χ be a Dirichlet's character with conductor $d\,(=odd)\in\mathbb{N}$. For each $\alpha,n\in\mathbb{N}^*$ and $w\in T_p$. Then we define*

$$\boldsymbol{l}^q\left(-n,x,\alpha,w\mid\chi\right)=\widetilde{E}_n\left(\alpha,x,w,q\mid\chi\right)\tag{3.2}$$

$\boldsymbol{l}^q(s,0,\alpha,w\mid\chi)=\boldsymbol{l}^q(s,\alpha,w\mid\chi)$ which is the twisted Dirichlet's type q-l-function. We now consider the function $\boldsymbol{l}^q(s,\alpha,w\mid\chi)$ as follows:

$$\begin{aligned}
\boldsymbol{l}^q(s,\alpha,w\mid\chi) &= [2]_q\sum_{m=1}^{\infty}\frac{(-1)^m\chi(m)w^mq^m}{[m]_{q^\alpha}^s}\\
&= [2]_q\sum_{m=1}^{\infty}\sum_{a=0}^{d-1}\frac{(-1)^{a+dm}\chi(a+dm)w^{a+dm}q^{a+dm}}{[a+dm]_{q^\alpha}^s}\\
&= [d]_{q^\alpha}^{-s}\sum_{a=0}^{d-1}(-1)^a\chi(a)w^aq^a[2]_q\sum_{m=1}^{\infty}\frac{(-1)^m\left(w^d\right)^m\left(q^d\right)^m}{\left[\left(\frac{a}{d}+m\right)\right]_{q^{d\alpha}}^s}\\
&= [d]_{q^\alpha}^{-s}\sum_{a=0}^{d-1}(-1)^a\chi(a)w^aq^a\zeta_{q^d,w^d}^{(\alpha)}\left(s,\frac{a}{d}\right) \qquad (3.3)
\end{aligned}$$

From (3.3), we obtain the following theorem:

Theorem 8. *Let χ be a Dirichlet's character with conductor $d\,(=odd)\in\mathbb{N}$. For each $\alpha,n\in\mathbb{N}^*$ and $w\in T_p$. Then we have*

$$\boldsymbol{l}^q(s,\alpha,w\mid\chi)=[d]_{q^\alpha}^{-s}\sum_{a=0}^{d-1}(-1)^a\chi(a)w^aq^a\zeta_{q^d,w^d}^{(\alpha)}\left(s,\frac{a}{d}\right). \qquad (3.4)$$

where $\zeta_{q,w}^{(\alpha)}(s,x)$ twisted Hurwitz q-Euler zeta functions.

We now consider the function $\Im_q^{(\alpha)}(s,a,w\mid F)$ as follows:

$$\Im_q^{(\alpha)}(s,a,w\mid F)=[2]_q\sum_{m\equiv a(\mathrm{mod}F)}\frac{(-1)^mw^mq^m}{[m]_{q^\alpha}^s} \qquad (3.5)$$

If F *odd* number then we have

$$\begin{aligned}
\Im_q^{(\alpha)}(s,a,w\mid F) &= [2]_q\sum_{m\equiv a(\mathrm{mod}F)}\frac{(-1)^mw^mq^m}{[m]_{q^\alpha}^s}\\
&= [2]_q\sum_{m=0}^{\infty}\frac{(-1)^{mF+a}w^{mF+a}q^{mF+a}}{[mF+a]_{q^\alpha}^s}\\
&= \frac{(-1)^aw^aq^a}{[F]_{q^\alpha}^s}\sum_{m=0}^{\infty}\frac{(-1)^m\left(w^F\right)^m\left(q^F\right)^m}{\left[m+\frac{a}{F}\right]_{q^{\alpha F}}^s}\\
&= \frac{(-1)^aw^aq^a}{[F]_{q^\alpha}^s}\zeta_{q^F,w^F}^{(\alpha)}\left(s,\frac{a}{F}\right) \qquad (3.6)
\end{aligned}$$

From (3.2) and (3.6) we obtain the following theorem:

Theorem 9. *Let χ be a Dirichlet's character with conductor $d(=$ odd$) \in \mathbb{N}$. For each $\alpha, n \in \mathbb{N}^*$ and $\in T_p$. We get*

$$\Im_q^{(\alpha)}(-n, a, w \mid F) = (-1)^a w^a q^a [F]_{q^\alpha}^n \widetilde{E}_{n,w^F,q^F}^{(\alpha)} \left(\frac{a}{F}\right) \tag{3.7}$$

By (3.4) and (3.7), we obtain the following corollary:

Corollary 2. *Let χ be a Dirichlet's character with conductor $d\,(=$ odd$)$ $\in \mathbb{N}$. For each $\alpha, n \in \mathbb{N}^*$ and $w \in T_p$. We get*

$$\boldsymbol{l}^q(s, \alpha, w \mid \chi) = \sum_{a=0}^{d-1} \chi(a) \Im_q^{(\alpha)}(s, a, w \mid d)$$

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Serkan Aracı
Department of Mathematics, University of Gaziantep, Faculty of Science and Arts,
Gaziantep 27310, Turkey.
E-mail: mtsrkn@hotmail.com

Nurgül Aslan
Department of Mathematics, University of Gaziantep, Faculty of Science and Arts,
Gaziantep 27310, Turkey.
E-mail: guzguzelim27@hotmail.com

Jong Jin Seo
Department of Applied Mathematics, Pukyong National University,

Busan 608-737, Korea.
E-mail: seo2011@pknu.ac.kr