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주파수 수요와 주파수 재할당 비용을 고려한 무선통신 네트워크 재설계

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Redesigning Radio Networks Considering Frequency Demands and Frequency Reassignment Cost

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■ Abstract ■

In this paper, we present a frequency reassignment problem (FRP) arising from the reconfiguration of radio networks such as adding new base stations (BSs) and changing the number of frequencies assigned to BSs. For this problem, we develop an integer programming (IP) model that minimizes the sum of frequency reassignment cost and the cost for unsatisfied frequency demands, while avoiding interference among frequencies. To obtain tight lower bounds, we develop some valid inequalities and devise an objective function relaxation scheme. Also, we develop a simple but efficient heuristic procedure to solve large size problems. Computational results show that the developed valid inequalities are effective for improving lower bounds. Also, the proposed tabu search heuristic finds tight upper bounds with average optimality gap of 2.3%.

Keyword : Telecommunications, Radio network, Integer programming, Heuristic

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1. Introduction

This paper deals with a frequency reassignment problem (FRP) arising from the reconfiguration of radio access networks such as installing new base stations (BSs) and adjusting the number of radio frequency channels (in short, frequencies) allocated to BSs to expand service area or to resolve hot spots. Suppose that there exists a BS A having b_0 frequencies, and the frequency demand for BS A has increased to b_1 ($> b_0$). Then, we seek to assign to BS A b_1-b_0 new frequencies incurring minimum interference. If we cannot find b_1-b_0 new frequencies to assign to BS A not incurring interference, we may have two options. First, we assign only b_2-b_0 new frequencies to BS A, where $b_0 < b_2 < b_1$, not incurring any interference, when penalty cost for b_1-b_2 unsatisfied frequency demands should be considered. Alternatively, we may reassign frequencies for some BSs to add b_1-b_0 new frequencies not incurring interference to BS A, when frequency reassignment cost should be considered. Reassigning frequencies may degrade service quality. If a frequency f assigned to BS A becomes unavailable in the BS A due to frequency reassignment, a mobile station (MS) using the frequency f can be handed over to one of the adjacent BSs. If the MS cannot find any unoccupied frequency from the adjacent BSs, the call associated with the MS can be terminated. Even if the MS finds an unoccupied frequency from adjacent BSs, the signal quality of new frequency may not be as good as the original one. Thus, it is necessary to minimize the number of frequencies dropped or replaced with other ones when redesigning a radio network. In this paper, we address a frequency reassignment problem that finds an interference free frequency assignment to BSs incurring minimum total cost consisting of two cost components: penalty cost not satisfying frequency demands of BSs and frequency reassignment cost. The fact that commercial radio networks are almost never static motivates this study. A mobile operator typically redesigns/reassigns parts of the network perhaps every three to six months when the network is in high growth state. However, unfortunately, we often find notices and complaints on unavailability of radio network in some areas during the deployment of radio network redesign (see references 19 and 20). We expect that the proposed integer programming approach can be a viable choice for operators that seek to minimize possible service failure or degrade that may occur while redesigning radio networks.

There exist numerous studies on generic frequency assignment problem (FAP). However, there are few to deal with a frequency reassignment problem but the work by Han (2007). Han (2007) addressed a multi-period frequency reassignment problem for code division multiple access (CDMA) networks, where exactly one pseudo noise (PN) code is assigned to each BS. Also, Han (2007) developed a simple two-phase heuristic algorithm. First, it finds a PN code assignment pattern for all BSs using the branch-and-bound procedure considering both PN code reassignment cost and interference cost, from which a set of BSs to change PN codes is determined. Then, it determines the time periods to reassign PN codes for the BSs selected in the first phase considering interference cost during PN code reassignment process. Below, some of the distinguished research results on generic FAPs are summarized. The first model of generic FAP, referred to as Min-FAP, is to minimize the total number of frequencies to satisfy the frequency demands for all BSs while avoiding interference. This type of FAP was dealt by Hale (1980), Gamst and Rave (1982), Hao et al. (1990) and Sung and Wong (1997). Hale (1980) showed that this problem can be expressed as a graph coloring problem. Another type of FAP, referred to as Max-FAP, is to maximize the total number of frequencies assigned to BSs, while avoiding interference among frequencies. Gamst and Rave (1982), Marthar and Mattfeldt (1993), Chang and Kim (1997), Sung and Wong (1997) and Tiourine et al. (2000) dealt with the Max-FAP. Hao et al. (1990) developed a tabu search algorithm to solve a FAP that minimizes the total interference among frequencies, referred to as MI(Minimum Interference)-FAP. An important feature of Hao et al. (1990) is the change of cost table during the implementation of tabu search. For the MI-FAP, Tiourine et al. (2000) developed a heuristic algorithm and compared the performance with a tabu search algorithm by Hao et al. (1990). Also, Tiourine et al. (2000) obtained tight lower bounds by reformulation, and found an optimal solution using the branch-and-bound and preprocessing. Montemanni et al. (2002) developed an Ant Colony System algorithm for minimum span assignment problem (MS-FAP). Avenali et al. (2002) devised a new integer programming approach generating columns and rows as needed, which was successful to find optimal solutions to larger problems of MS-FAP. Recently, Graham et al. (2008) developed two types of algorithm; simulated annealing algorithm and a new Ant Colony System algorithm that can efficiently handle non-binary constraints expressing, for example, signal-to-interference ratio (SIR) as well as binary constraints. Besides, quite many research papers on generic FAP are well summarized in Aardal et al. (2007).

The remainder of this paper is organized as follows. In Section 2, we develop an integer programming formulation. In Section 3, we devise some valid inequalities along with an objective function relaxation to obtain tight lower bounds. In Section 4, we consider a simple heuristic procedure based on objective function relaxation, and develop an effective heuristic procedure for dealing with large size problems. Computational results are provided in Section 5, and Section 6 concludes this paper.

2. Integer Programming Model

Let N be the set of nodes (BSs), and let F be the set of frequencies. Also, let $F(i) \subseteq F$ be the set of frequencies that are currently assigned to node $i \in N$, and let $b(i)$ be the updated frequency demands of node $i \in N$. If $|F(i)| = 0$ and $b(i) > 0$, we consider that new node $i \in N$ having frequency demands $b(i)$ should be installed. Define $E = \{i, j\ (> i) \in N: r(i, j) > 0\}$, where $r(i, j)$ denotes the minimum distance to avoid interference between two frequencies assigned to nodes i and $j\ (> i) \in N$, respectively. Let $x_{if} = 1$ if frequency $f \in F$ is assigned to node $i \in N$, and 0 otherwise. Let $y_{ik} = 1$ if the number of frequencies dropped from $F(i)$ is equal to $k\ (= 1, \cdots, |F(i)|)$ for node $i \in N$, and 0 otherwise. If $y_{ik} = 1$, frequency reassignment cost p_{ik} arises. Obviously, p_{ik} should increase as k denoting the number of frequencies reassigned (or dropped) increases for $i \in N$. Let $u_{il} = 1$ if the unsatisfied frequency demands of node $i \in N$ is equal to $l\ (= 1, \cdots, b(i))$, and 0 otherwise. If $u_{il} = 1$, penalty cost q_{il} for not assigning l frequencies to node $i \in N$ arises. Penalty cost q_{il} should also increase as l increases for $i \in N$. Estimating cost factors $\boldsymbol{p}$ and $\boldsymbol{q}$ exactly can be cumbersome in practice. Rather than trying to find out optimal trade-offs between cost factors $\boldsymbol{p}$ and $\boldsymbol{q}$, it would be realistic to focus on one cost factor. When $\boldsymbol{p} >> \boldsymbol{q}$, we seek to minimize the frequency reassignments first. If there are alternatives having the same reassignment cost, we prefer the one that meets the frequency demands most. While, if $\boldsymbol{p} << \boldsymbol{q}$, we try to meet the frequency demands first. In case of tie, we prefer the one that minimizes the change of current frequency assignments. Using the notations defined above, we can formulate the frequency reassignment problem as a linear integer programming (IP) model as follows.

FRP : Minimize $\sum_{i \in N} \sum_{k \leq |F(i)|} p_{ik}\, y_{ik} + \sum_{i \in N} \sum_{l \leq b(i)} q_{il}\, u_{il}$ (1)

Subject to

$\sum_{f \in F(i)} x_{if} + \sum_{k \leq |F(i)|} k\, y_{ik} = |F(i)| \quad i \in N,$ (2)

$\sum_{k \leq |F(i)|} y_{ik} \leq 1 \quad i \in N,$ (3)

$\sum_{f \in F} x_{if} + \sum_{l \leq b(i)} l\, u_{il} = b(i) \quad i \in N,$ (4)

$\sum_{l \leq b(i)} u_{il} \leq 1 \quad i \in N,$ (5)

$x_{if} + x_{jg} \leq 1 \quad (i, j) \in E, f, g \in F: |f-g| < r(i, j),$ (6)

all the variables are binary.

Objective function (1) minimizes the sum of cost for not satisfying the frequency demands and frequency reassignment cost. Constraints (2) and (3) express the number of frequencies dropped from the set of original frequencies $F(i)$ for each node $i \in N$. Constraints (4) and (5) express unsatisfied frequency demands for each node $i \in N$. Constraint (6) prohibits interference among frequencies.

Remark 1 : The problem FRP is NP-hard since it becomes Max-FAP (see Aardal et al., 2001) by letting $p_{ik} = 0$ for all $i \in N$ and $k \in F(i)$, and $q_{il} = 1$ for all $i \in N$ and $l \in b(i)$.

3. Lower Bounds

3.1 Valid Inequalities

Remark 2 : If $|F(i)| > b(i)$ for some $i \in N$, any feasible solution satisfies that

$$\sum_{k = |F(i)|-b(i), \cdots, |F(i)|} y_{ik} = 1. \tag{7}$$

Thus, we replace constraint (3) by equation (7) for all $i \in N$ satisfying that $|F(i)| > b(i)$. Also, if $|F(i)| = b(i)$ for some $i \in N$, the following inequality is valid

$$\sum_{k \leq |F(i)|} y_{ik} \geq x_{if} \qquad f \in F \backslash F(i) \tag{8}$$

Proposition 1 : For $i \in N$ satisfying that $|F(i)| \geq b(i)$, the following inequality is valid

$$\sum_{k = |F(i)|-b(i), \cdots, |F(i)|} k\, y_{ik} \geq \sum_{l \leq b(i)} l\, u_{il} + |F(i)| - b(i) \qquad l \leq b(i). \tag{9}$$

Proof : If $u_{il} = 0$ for all $l \leq b(i)$, inequality (9) becomes $\sum_{k = |F(i)|-b(i), \cdots, |F(i)|} k\, y_{ik} \geq |F(i)|-b(i)$, which is obviously valid due to (7). Thus, assuming that $u_{il} = 1$ for some $l \leq b(i)$, we see that at least $l + |F(i)|-b(i)$ frequencies should be dropped from $F(i)$. This completes the proof.

Below, we consider some valid inequalities based on the clique subgraph in $G(N, E)$. For $(i, j) \in E$, f and $g \in F$ satisfying that $|f-g| < r(i, j)$, consider $j' \in N \backslash \{i, j\}$ and $g' \in F$ satisfying that $|f - g'| < r(i, j')$ and $|g-g'| < r(j, j')$. Then, by adding $x_{jg'}$ to the left- hand side of constraint (5), we obtain

$$x_{if} + x_{jg} + x_{jg'} \leq 1. \quad (10)$$

As an extension of inequality (10), we consider the following inequality.

Remark 3 (Fischetti et al., 2000) : For a clique inducing node set $C \subseteq N$, the following inequality is valid if $|f(i) - f(j)| < r(i, j)$ for all i and j ($> i$) $\in C$

$$\sum_{i \in C} x_{i,f(i)} \leq 1, \quad (11)$$

where $f(i)$ denotes the frequency assigned to node $i \in C$. Inequality of type (11) was first investigated by Padberg (1973) for solving a set packing problem, and was used by Fischetti et al. (2000) for solving the Max-FAP in a branch-and-cut framework. Below, we consider lifting (11).

Proposition 2 : For a clique inducing node set $C \subseteq N$, if $|F(v)| > b(v)$ for some $v \in C$ and if $|f(i) - f(j)| < r(i, j)$ for all i and j ($> i$) $\in C$, the following inequality is valid

$$\sum_{i \in C} x_{i,f(i)} \leq \sum_{k = |F(v)| - b(v), \cdots, |F(v)|} y_{vk}. \quad (12)$$

Proof : Note that $\sum_{k = |F(v)|-b(v), \cdots, |F(v)|} y_{vk} = 1$ in any feasible solution to FRP since $|F(v)| > b(v)$. Thus, inequality (12) holds true. This completes the proof.

We can separate the inequality (11) by finding a maximum-weight clique in an augmented graph $G(N', E')$, where $N' = \{(i, f) : i \in N \text{ and } f \in F\}$ and $E' = \{[(i, f), (j, g)] : (i, f), (j, g) \in N', |f - g| < r(i, j)\}$, which is a tremendous task due to the size of N' and E' (see Aardal et al., 2001). Unlike Fischetti et al. (2000) using a greedy algorithm for separating the inequality (11), we consider a two-phase procedure. First, we find a maximum clique C on a graph $G(N, E)$, which is computationally easier than solving a maximum-weight clique problem over $G(N', E')$ since $|N| = |N'|/|F|$ and $|E| << |E'|$. Then, we find $f(i)$'s for all $i \in C$ satisfying that $\sum_{i \in C} x^{\circ}_{i,f(i)} > 1$, where $(x^{\circ}, y^{\circ}, u^{\circ})$ denotes an optimal solution to the LP- relaxation of FRP, which is equivalent to finding an (optimal) solution to a generalized assignment problem (GAP) as follows.

GAP : Maximize $\sum_{i \in C} \sum_{f \in F} x^{\circ}_{if} x_{if}$

Subject to

$$\sum_{f \in F} x_{if} = 1 \qquad i \in C,$$

$$\sum_{f \in F} f x_{if} - \sum_{g \in F} g x_{jg} \leq r(i, j) - 1 \qquad i, j (> i) \in C,$$

$$\sum_{g \in F} g x_{if} - \sum_{f \in F} f x_{if} \leq r(i, j) - 1 \qquad i, j (> i) \in C,$$

$\boldsymbol{x}$ is binary.

If $\Theta(\text{GAP}) > 1$, where $\Theta(\text{P})$ denotes the optimal objective value of formulation P, we see that the solution $arg\{f \in F : x_{if} = 1\}$ for $i \in C$ violates the inequality (11). If $r(i, j) = 1$ for all i and j ($> i$) $\in C$, it is sufficient to find a frequency $f \in F$ satisfying that $\sum_{i \in C} x^{\circ}_{if} > 1$, which is simpler than solving the GAP. While, if $\Theta(\text{GAP}) \leq 1$, we attempt to solve GAP on a different maximum clique inducing node set C' ($\neq C$). In this paper, we have generated as many different clique subgraphs as possible with the cardinality of four at the root node of the branch-and bound tree. For separating the inequality (12), we can also solve the GAP optimally to examine if $\Theta(\text{GAP}) > \sum_{k = |F(v)| - b(v), \cdots, |F(v)|} y_{vk}$ for some $v \in C$.

3.2 Objective Function Relaxation

For each $i \in N$, if costs p_{ik} and q_{il} are stepwise concave non-decreasing functions over k = 1 ,⋯,$|F(i)|$ and l = 1,⋯, $b(i)$, respectively, we can linearize the cost functions $\boldsymbol{p}$ and $\boldsymbol{q}$. Let p' and q' be arbitrary cost functions satisfying the property of constant return to scale. For example, we can set $p'_i = p_{i1}$ (or $p_{i|F(i)|}/|F(i)|$) and $q'_i = q_{i1}$ (or $q_{ib(i)}/b(i)$) for all $i \in N$. Then, we can convert binary variables y_{ik} and u_{il} associated with the cost functions $\boldsymbol{p}$ and $\boldsymbol{q}$, respectively, to general integer variables y_i ($\le |F(i)|$) and u_i ($\le b(i)$) for all $i \in N$. Below, we present an objective function relaxation (OFR) of formulation FRP.

OFR : Minimize $\sum_{i \in N} p'_i\, y_i + \sum_{i \in N} q'_i\, u_i$

Subject to

$$\sum_{f \in F(i)} x_{if} + y_i = |F(i)| \qquad i \in N, \qquad (13)$$

$$\sum_{f \in F} x_{if} + u_i = b(i) \qquad i \in N, \qquad (14)$$

$$x_{if} + x_{jg} \le 1 \qquad (i, j) \in E,\ f, g \in F: |f - g| < r(i, j), \qquad (15)$$

$$x_{if} \in \{0, 1\} \qquad i \in N, f \in F,$$

$$0 \le y_i \le |F(i)| \text{ and integer} \qquad i \in N,$$

$$0 \le u_i \le b(i) \text{ and integer} \qquad i \in N.$$

Remark 4 : Due to the equality constraints (13) and (14), the objective function of OFR can be expressed as

$$\begin{aligned}
&\text{Min}\{\textstyle\sum_{i\in N} p'_i\ (|F(i)| - \sum_{f\in F(i)} x_{if}) \\
&\quad + \textstyle\sum_{i\in N} q_i(b(i) - \sum_{f\in F} x_{if})\} \\
&\quad = \textstyle\sum_{i\in N} \{p'_i|F(i)| + \sum_{i\in N} q_i b(i)\} \\
&\quad - \text{Max}\{\textstyle\sum_{i\in N} \sum_{f\in F(i)} p'_i\, x_{if} + \sum_{i\in N} \sum_{f\in F} q'_i\, x_{if}\} \\
&\quad = \textstyle\sum_{i\in N} \{p'_i\ |F(i)| + \sum_{i\in N} q'_i\ b(i)\} \\
&\quad - \text{Max}\{\textstyle\sum_{i\in N} \sum_{f\in F(i)} (p_i + q'_i) x_{if} \\
&\quad + \textstyle\sum_{i\in N} \sum_{f\in F\backslash F(i)} q'_i\ x_{if}\}.
\end{aligned}$$

Thus, we can express the formulation OFR as

OFR2 : Maximize $\sum_{i \in N} \sum_{f \in F(i)} (p'_i + q'_i)\ x_{if} + \sum_{i \in N} \sum_{f \in F\backslash F(i)} q'_i\ x_{if}$

Subject to

$$x_{if} + x_{jg} \le 1 \qquad (i, j) \in E,\ f, g \in F: |f - g| < r(i, j), \qquad (16)$$

$$\sum_{f \in F} x_{if} \le b(i) \qquad i \in N, \qquad (17)$$

$$x_{if} \in \{0, 1\} \qquad i \in N, f \in F.$$

Note that OFR2 is a maximum-weight clique problem with side constraint (17). Now, we consider some proper assumption on the cost functions of $\boldsymbol{p}$ and $\boldsymbol{q}$ to derive valid lower bounds by solving the OFR (or OFR2).

Assumption : Suppose that costs p_{ik} and q_{il} are stepwise concave non-decreasing functions over k (= 1,⋯, $|F(i)|$) and l (= 1,⋯, $b(i)$), respectively, for all $i \in N$. Under this Assumption, we have the following results.

Proposition 3 : If we set $p'_i = \min\{p_{i1}, p_{i|F(i)|}/|F(i)|\}$ and $q'_i = \min\{q_{i1}, q_{ib(i)}/b(i)\}$ for all $i \in N$ at OFR and OFR2, we have that

$$\Theta(\text{OFR}) = \sum_{i \in N} \{p'_i\ |F(i)| + \sum_{i \in N} q'_i\ b(i)\} - \Theta(\text{OFR2}) \le \Theta(\text{FRP}). \qquad (18)$$

Proof : We show that $\Theta(\text{OFR}) \le \Theta(\text{FRP})$ since the relations between $\Theta(\text{OFR})$ and $\Theta(\text{OFR2})$ are obvious from Remark 4. For any feasible solution (x^*, y^*, u^*) to FRP, we can always derive a feasible solution (x^*, y, u) to OFR by setting $y_i = \sum_{k \le |F(i)|} k\ y^*_{ik}$ and $u_i = \sum_{l \le b(i)} l\ u^*_{il}$. Also, note that $p'_i \times k \le p_{ik}$ for all $i \in N$ and $k \le |F(i)|$ and that $q'_i \times l \le q_{il}$ for all $i \in N$ and $l \le b(i)$. Thus, for any feasible solution (x^*, y^*, u^*) to FRP with objective value $\Theta(\text{FRP})$, the associated objective value $\Theta(\text{OFR}) = \sum_{i \in N} \{p'_i \times arg\{k = 1, \cdots, |F(i)| : \sum_{k \le |F(i)|} y^*_{ik} = 1\} + q'_i \times arg\{l = 1, \cdots, b(i) : \sum_{l \le b(i)}$

$u^*_{il} = 1\}\}$ provides a lower bound of Θ(FRP). This completes the proof.

Define $N' = \{i \in N : |F(i)| > b(i)\}$. If $N' \neq \{\}$, we obtain a tighter lower bound of FRP than Θ(OFR).

Corollary 1 : If we set $p'_i = (p_{i|F(i)|} - p_{i\{|F(i)|-b(i)\}})/b(i)$ for all $i \in N'$, $p'_i = \min\{p_{i1}, p_{i|F(i)|}/|F(i)|\}$ for all $i \in N \backslash N'$, and $q'_i = \min\{q_{i1}, q_{ib(i)}/b(i)\}$ for all $i \in N$ at OFR and OFR2, we have that

$$\Theta(\text{OFR}')+\zeta = \textstyle\sum_{i\in N} \{p'_i\ |F(i)| + \sum_{i \in N} q_i b(i)\} - \Theta(\text{OFR2})+\zeta \le \Theta(\text{FRP}), \quad (19)$$

where OFR′ denotes the OFR set by $|F(i)|-b(i) \le y_i \le |F(i)|$ for all $i \in N'$, and where $\zeta = \sum_{i \in N'} \{p_{i|F(i)|} - (p_{i|F(i)|} - p_{i\{|F(i)|-b(i)\}}) \times |F(i)|/b(i)\}$.

Proof : Similar to the proof of Proposition 3, we show that $\Theta(\text{OFR}')+\zeta \le \Theta(\text{FRP})$. For any feasible solution (x^*, y^*, u^*) to FRP, we can always derive a feasible solution (x^*, y, u) to OFR′ by setting $y_i = \sum_{k \le |F(i)|} k\ y^*_{ik}$ and $u_i = \sum_{l \le b(i)} l\ u^*_{il}$. Here, note that the y variables of OFR′ satisfy that $|F(i)|-b(i) \le y_i \le |F(i)|$ for all $i \in N'$ since $|F(i)|-b(i) \le arg\{k = 1, \cdots, |F(i)| : \sum_{k \le |F(i)|} y^*_{ik} = 1\} \le |F(i)|$ for all $i \in N'$, which is also satisfied by equation (7). Note that $p'_i \times k \le p_{ik}$ for all $i \in N \backslash N'$ and $k \le |F(i)|$ and that $q'_i \times l \le q_{il}$ for all $i \in N$ and $l \le b(i)$. Now, for all $i \in N'$ and $k = |F(i)|-b(i), \cdots, |F(i)|$, we see that

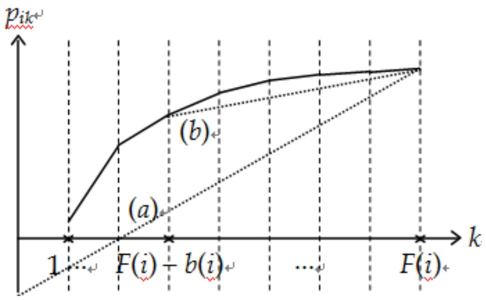
$$p'_i \times k + \{p_{i|F(i)|} - (p_{i|F(i)|} - p_{i\{|F(i)|-b(i)\}}) \times |F(i)|/b(i)\} \le p_{ik}$$
$$\Leftrightarrow (p_{i|F(i)|} - p_{i\{|F(i)|-b(i)\}})/b(i) \times k + \{p_{i|F(i)|} \times b(i) - (p_{i|F(i)|} - p_{i\{|F(i)|-b(i)\}}) \times |F(i)|\}/b(i) \le p_{ik}$$
$$\Leftrightarrow (p_{i|F(i)|} - p_{i\{|F(i)|-b(i)\}}) \times k + p_{i|F(i)|} \times b(i) - (p_{i|F(i)|} - p_{i\{|F(i)|-b(i)\}}) \times |F(i)| \le p_{ik} \times b(i)$$
$$\Leftrightarrow (p_{i|F(i)|} - p_{i\{|F(i)|-b(i)\}}) \times (k - |F(i)|) + p_{i|F(i)|} \times b(i) \le p_{ik} \times b(i)$$
$$\Leftrightarrow (p_{i|F(i)|} - p_{i\{|F(i)|-b(i)\}}) \times (k - |F(i)|) + (p_{i|F(i)|} - p_{ik}) \times b(i) \le 0$$
$$\Leftrightarrow (p_{i|F(i)|} - p_{i\{|F(i)|-b(i)\}})/b(i) \ge (p_{i|F(i)|} - p_{ik})/(|F(i)| - k). \quad (20)$$

Note that inequality (20) shows the relationship between two slopes from $(|F(i)|-b(i), p_{i\{|F(i)|-b(i)\}})$ to $(|F(i)|, p_{i|F(i)|})$ and from (k, p_{ik}) to $(|F(i)|, p_{i|F(i)|})$ for any $k = |F(i)|-b(i), \cdots, |F(i)|$. From the assumption of stepwise concave non-decreasing cost function p_{ik} over $k = 1, \cdots, |F(i)|$ for all $i \in N$, inequality (20) holds true. Thus, for any feasible solution (x^*, y^*, u^*) to FRP with Θ(FRP), the associated objective value $\Theta(\text{OFR}')+\zeta = \sum_{i \in N \backslash N'} \{p'_i \times arg\{k = 1, \cdots, |F(i)| : \sum_{k \le |F(i)|} y^*_{ik} = 1\}\} + \sum_{i \in N} \{q'_i \times arg\{l = 1, \cdots, b(i) : \sum_{l \le b(i)} u^*_{il} = 1\}\} + \sum_{i \in N'} \{p_{i|F(i)|} - (p_{i|F(i)|} - p_{i\{|F(i)|-b(i)\}}) \times |F(i)|/b(i)\}$ provides a lower bound of Θ(FRP). This completes the proof.

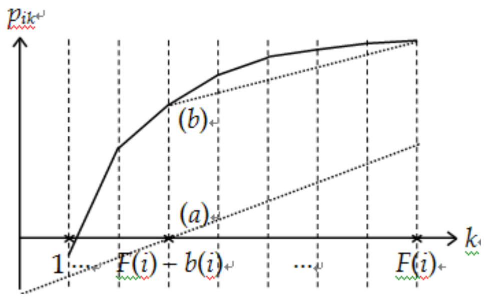
Remark 5 : Note that the lower bound (19) is always greater than or equal to the lower bound (18). This is easily verified by comparing the cost p's for each $k = |F(i)|-b(i), \cdots, |F(i)|$ in both cases 1 and 2 of [Figure 1]. That is, p's on line (b) representing the cost function of Corollary 1 is always greater than or equal to the p's on line (a) representing the cost function of Proposition 3 in both cases 1 and 2 for all $k = |F(i)|-b(i), \cdots, |F(i)|$. Also, if $y_i < |F(i)|$ for some $i \in N'$ in an optimal solution of OFR and OFR′, the lower bound (18) is strictly greater than the lower bound (18).

4. Solution Procedure

We consider two heuristic procedures. In Section



case 1 : when $p_{i1} \geq 0$



case 2 : when $p_{i1} < 0$

[Figure 1] Illustration of concave non- decreasing function of cost p for OFR

4.1, we find an optimal solution to OFR, which is feasible to FRP, and calculate the total cost based on the original cost table. In Section 4.2, we develop a local improvement procedure based on a tabu memory combined with a restarting strategy.

4.1 Objective Function Relaxation

Suppose that we have a feasible solution (x', y', u') to OFR. Then, we can always derive a feasible solution (x', y°, u°) to FRP from (x', y', u') by letting $y^{\circ}_{i,k(i)} = 1$, where $k(i) = y'_i$ and $y^{\circ}_{ik} = 0$ for $k \in F(i) \backslash k(i)$ and $i \in N$, and $u^{\circ}_{i,l(i)} = 1$, where $l(i) = u'_i$ and $u^{\circ}_{il} = 0$ for $1\ (\neq l(i)) \leq b(i)$ and $i \in N$.

4.2 Local Improvement

We develop a local improvement heuristic algorithm that can handle large size problems. The proposed heuristic algorithm begins with an initial feasible solution, and attempts to improve the initial solution by adding some frequencies to BSs and by exchanging the frequencies assigned to a pair of adjacent BSs. If we cannot escape from a local optimal solution, which happens frequently since we do not allow any interference, we delete some frequencies. Deleting some frequencies increases the possibility of finding updated feasible solutions when adding frequencies. Another important feature of the proposed heuristic algorithm is that it manages a list of elite solutions, which is a subset of high quality feasible solutions. If we cannot improve the current solution, we resume the above procedure with one of the feasible solutions in the elite solution list (ESL). Also, we employ a tabu list, a collection of recently generated feasible solutions (not a collection of moves), to generate a feasible solution that has not been evaluated. Restarting strategy based on ESL and incorporating a tabu list improves the overall quality of ESL as computing time increases. For describing the heuristic algorithm in detail, we define some notations. Let $F^{\circ}(i)$ be the set of frequencies assigned to node $i \in N$, and let $\delta(i) = \{j \in N : r(i, j) > 0 \text{ or } r(j, i) > 0\}$ for node $i \in N$.

4.2.1 Initial Procedure

Step 1 : Set $F^{\circ}(i) = F(i)$ for each $i \in N$.

Step 2 : Pick an arbitrary node $i \in N$ such that $|F^{\circ}(i)| > b(i)$, and delete $|F^{\circ}(i)| - b(i)$ frequencies from $F^{\circ}(i)$ at random.

Step 3 : If $|F^{\circ}(i)| \leq b(i)$ for all $i \in N$, stop. Otherwise, go to Step 2.

4.2.2 Improving Procedure

For $i \in N$ and $f \in F^{\circ}(i)$, define *del_move*(i, f) as deleting a frequency f from $F^{\circ}(i)$. Also, define *add_move*(i, f) as adding a frequency f to $F^{\circ}(i)$ for $i \in N$ such that $|F^{\circ}(i)| < b(i)$ and $f \notin F^{\circ}(i)$. Note that *add_move*(i, f) may cause interference with frequency $g \in F^{\circ}(j)$ assigned to node $j \in \delta(i)$ if $|f - g| < r(i, j)$. Thus, after executing *add_move*(i, f), we execute *del_move* (j, g)'s for $j \in \delta(i)$ and $g \in F^{\circ}(j)$ such that $|f-g| < r(i, j)$ at random until the solution becomes interference free. In particular, we denote *add_move* not requiring any *del_move* by *improving_add_move*. We also define *swap_move*(i, f, f') for $i \in N, f \in F^{\circ}(i)$ and $f' \in F \backslash F^{\circ}(i)$ such that $|f - g| \geq r(i, j)$ for all $j \in \delta(i)$ and $g \in F^{\circ}(j)$ as deleting a frequency f from $F^{\circ}(i)$ and adding a new frequency $f' \in F \backslash F^{\circ}(i)$ to $F^{\circ}(i)$. We denote *swap_move* that improves the current solution by *improving_swap_move*. Identifying an *improving_swap_move* is simple. That is, for a given $i \in N$, executing a *swap_move*(i, f, f') for $f \in F^{\circ}(i) \backslash F(i)$ and $f' \in F(i) \backslash F^{\circ}(i)$ improves the current solution if $|f-g| \geq r(i, j)$ for all $j \in \delta(i)$ and $g \in F^{\circ}(j)$. Below, we describe the overall procedure for improving the solution.

Initialize : Set ESL = {}. Let S denote the current solution.

Step 1 (Improving phase I) : Execute arbitrary *improving_add_move*'s until we cannot improve the current solution. If the current solution has been updated, update ESL with S. ESL is updated as follows. Add S to ESL. If |ESL| = *size_ESL*+1, delete a solution having the largest objective value from ESL.

Step 2 (Improving phase II) : Execute arbitrary *improving_swap_move*'s until we cannot improve the current solution. If the current solution has been updated, update ESL with S and go to Step 1. Otherwise, go to Step 3.

Step 3 (Diversifying phase I) : Execute arbitrary *swap_move*'s until a new solution S' satisfying that $Z(S') \leq Z(S) \times$threshold is generated, where $Z(\cdot)$ denotes the objective value of a feasible solution ·. If a new solution S' satisfying that $Z(S') \leq Z(S) \times$threshold is not generated within max_ move executions of *swap_move*, go to Step 4. Otherwise, update the tabu list consisting of up to *size_tabu* recently generated solutions with S', and go to Step 1. To identify if S' is a new one, we examine the tabu list. At every execution of *swap_move*, we perform next swap_move based on S not S' if $Z(S') > Z(S) \times$threshold (%).

Step 4 (Diversifying phase II) : Execute arbitrary *del_move*'s until a new solution S' satisfying that $Z(S') \leq Z(S) \times$*threshold* (%) is generated. If a new solution S' satisfying that $Z(S') \leq Z(S) \times$*threshold* is not generated within *max_move* executions of *del_move*, go to Step 5. Otherwise, update the tabu list with S', and go to Step 1. If $Z(S') > Z(S) \times$ *threshold*, we perform next *del_move* based on S not S'.

Step 5 (Restarting with an elite solution) : If time limit expired, stop. Otherwise, pick a solution S from ESL. Delete all the solutions in the tabu list with objective value greater than $Z(S) \times$ threshold (%), and go to Step 1.

Although the improving procedure is simple, it has some features that are effective for improving the solution. For example, deleting frequencies as in Step 4 enables us to add and swap frequencies avoiding interference in Steps 1 and 2, respectively. This strategy is quite effective when compared with just swapping frequencies. Also, restarting strategy in Step 5 helps escape from local optima.

In particular, since we delete the worst solution in ESL when |ESL| exceeds a threshold, we can improve the overall quality of solutions in ESL. Managing ESL in this way enables us to restart with a high quality solution. Deleting tabu elements with objective value greater than $Z(T) \times threshold$ (%) reduces the computing time required to examine the tabu list.

4.2.3 Parameter Calibration

There are four parameters affecting the performance of our tabu search algorithm : *threshold*, *max_move*, *size_tabu* and *size_ESL*. Adequate values for them were determined in consideration of both solution quality and computation time with preliminary experiments.

- *threshold* and *max_move* : We perform arbitrary *swap_move*'s and *del_move*'s in order to diversify search space hoping that these random moves followed by *improving_add_move*'s and by *improving_swap_move*'s enable us to improve the solution. However, we need to prevent the current solution from being deviated too far from the optimal solution. For this purpose, we have set *threshold* = 110%. That is, we seek to find a new solution with objective value not greater than 110% of the current solution. If max_move is large, we may consume long computing time to evaluate moves at diversification phases. Thus, we have set *max_move* = $|N|/2$.
- *size_tabu* : We incorporate a tabu list to evaluate if a solution generated by moves at diversification phases is a new one. Thus, size_tabu needs to be greater than *max_move*. We set *size_tabu* = $2 \times |N|$. Also, in order to reduce unnecessary computing time, we delete all the solutions with objective value greater than $Z(T) \times$threshold (%) from the tabu list when restarting with a solution T.
- *size_ESL* : If *size_ESL* is too large, both the worst and best solutions in ESL are slowly updated. While, if *size_ESL* is too small (for example, 1), the best solution is rarely updated. Thus, we set *size_ESL* = $|N|/2$.

5. Computational Results

In this section, we report computational results of the proposed solution procedure. Test problems are generated as follows.

Step 1 (Generate a network) : Generate $|N|$ BSs on a square with dimension of 1000 by 1000 at random, and calculate the distance $D(i, j)$ for all pairs of BSs i and j $(> i) \in N$. If $D(i, j) > min\_dist$, where min_dist is the minimum distance to define $r(i, j)$ that guarantees the connectivity of the resulting network, set $r(i, j) = 0$. The min_dist is set by trial-and-error. Otherwise, set $r(i, j) = \lceil 3 \times (min\_dist - D(i, j))/min\_dist \rceil$ for all i and j $(> i) \in N$.
Step 2 (Generate $F(i)$) : Pick a node at random, and assign the lowest index frequency in F not incurring any interference with other frequencies. Repeat this step until frequency assignment is not possible. Let $F(i)$ denote the set of frequencies assigned to BS $i \in N$.
Step 3 (Set $b(i)$) : Set $b(i) = |F(i)|/2 \times (1+U[0, 1])$ if $|F(i)| > 0.9 \times AvgF$, where $AvgF$ is the average of $|F(i)|$ over $i \in N$. Otherwise, set $b(i) = |F(i)| + AvgF \times U[0, 1]$.
Step 4 (Set cost factors) : Set $p_{i1} = 10 \times |F| + \lceil 10 \times |F|/|F(i)| \rceil$ and $p_{i,|F(i)|} = 0.8 \times p_{i1} \times |F(i)|$ for all $i \in N$. Set $p_{ik} = p_{i1} + (p_{i,|F(i)|} - p_{i1})/\log(|F(i)|) \times \log(k)$ for $k = 2, \cdots, |F(i)|-1$ and $i \in N$. Also, set $q_{i1} = 10 \times |F| + \lceil 10 \times |F| \times wgt\_q\_over\_p/|b(i)| \rceil$ and $q_{i,b(i)} = 0.8 \times q_{i1} \times b(i)$ for all $i \in N$, where $wgt\_q\_over\_p$ is

a parameter indicating the integer multiple of cost factor $\boldsymbol{q}$ over cost factor $\boldsymbol{p}$. Set $q_{il} = q_{i1}+(q_{ib(i)}- q_{i1})/\log(b(i))\times\log(l)$ for $l = 2, \cdots, b(i)-1$ and $i \in N$.

The coding was done in C, and all runs were made on a Pentium IV 3.2 GHz PC, 2GByte RAM with CPLEX version 10.0 as a LP/MIP solver. We report computational results of 80 test problems in <Table 1>~<Table 4>. "EFRP" denotes the FRP enhanced by inequalities (7)~(11). LP-relaxation bound "Best" denotes the best lower bound obtained by running the CPLEX optimization procedure for 7,200 seconds. Lower bound "OFR" denotes the optimal objective value to the formulation OFR obtained by Proposition 3, and lower bound "EOFR" denote the optimal objective value to the relaxed formulation OFR enhanced by Corollary 1. "OFR" and "EOFR" in the column "Upper bound" represent the objective values obtained by recovering the relaxed cost factors to the original ones for given optimal solutions to OFR and EOFR, respectively. "Tabu" denotes the results of the proposed tabu search algorithm. For all test problems, we interrupted the CPLEX optimization procedure in 7,200 CPU seconds if it is not terminated until then. We have run the tabu search procedure for

〈Table 1〉 Computational results : $|M|$ = 30, $|F|$ = 30, *wgt_q_over_p* = 1

No	Lower bound					Upper bound (Gap)					Elapsed time			
	LP-relaxation at root node			Objective function relaxation		FRP	EFRP	OFR	EOFR	Tabu	FRP	EFRP	EOFR	Tabu
	FRP	EFRP	Best	OFR	EOFR									
1	23174	30066	32480	25065	31164	33656(3.6%)	33656(3.6%)	36537(12.4%)	35513(9.3%)	33457(3.0%)	7200	7200	1476	600
2	17125	24019	25759	18282	24955	25759(0%)	25759(0%)	26569(3.1%)	26569(3.1%)	25759(0%)	72	87	11	600
3	12520	18485	19045	13878	16964	19045(0%)	19045(0%)	19152(0.5%)	19152(0.5%)	19045(0%)	7	9	98	600
4	17658	23562	27437	18322	24236	29078(5.9%)	29078(5.9%)	28062(2.2%)	27528(0.3%)	29099(6.0%)	7200	7200	45	600
5	15137	20066	23309	16652	21319	23629(1.3%)	23608(1.2%)	24255(4.0%)	24255(4.0%)	23501(0.8%)	7200	7200	105	600
6	18200	24703	26219	19597	24970	27498(4.8%)	27498(4.8%)	26453(0.8%)	26383(0.6%)	27310(4.1%)	7200	7200	2675	600
7	14863	20106	22305	16775	21324	22478(0.7%)	22478(0.7%)	22955(2.9%)	22955(2.9%)	22478(0.7%)	7200	7200	163	600
8	19579	26331	27799	20741	27004	27799(0%)	27799(0%)	29060(4.5%)	29060(4.5%)	27799(0%)	310	435	24	600
9	12668	16935	19047	14580	17941	19047(0%)	19047(0%)	19573(2.7%)	19573(2.7%)	19078(0.1%)	7200	5921	483	600
10	15699	20772	23024	17157	21321	23024(0%)	23024(0%)	24160(4.9%)	23936(3.9%)	23211(0.8%)	1172	970	36	600
11	14943	21133	22786	16216	21739	22786(0%)	22786(0%)	22957(0.7%)	22957(0.7%)	22786(0%)	40	31	5	600
12	16733	21695	23487	18690	22816	25096(6.8%)	24977(6.3%)	26252(11.7%)	25652(9.2%)	24538(4.4%)	7200	7200	661	600
13	17342	24113	26225	19475	25122	26432(0.7%)	26432(0.7%)	27216(3.7%)	27216(3.7%)	26413(0.7%)	7200	7200	228	600
14	15560	20919	22950	17595	22338	24709(7.6%)	24428(6.4%)	25157(9.6%)	24428(6.4%)	24289(5.8%)	7200	7200	2927	600
15	21170	28574	30319	22488	28986	30319(0%)	30319(0%)	31104(2.5%)	31142(2.7%)	30611(0.9%)	389	332	7	600
16	13899	20656	22666	15460	21726	22965(1.3%)	22965(1.3%)	23192(2.3%)	22994(1.4%)	22894(1.0%)	7200	7200	465	600
17	14003	18564	20578	15768	19260	21106(2.5%)	21235(3.1%)	21761(5.7%)	21817(6.0%)	21034(2.2%)	7200	7200	338	600
18	14944	22579	25495	16505	23610	25495(0%)	25495(0%)	25607(0.4%)	25625(0.5%)	25552(0.2%)	66	44	21	600
19	20306	27170	29615	22117	28208	29615(0%)	29615(0%)	30911(4.3%)	30986(4.6%)	29618(0.1%)	2011	1934	126	600
20	14785	20631	21677	15667	21292	21677(0%)	21677(0%)	21835(0.7%)	21835(0.7%)	21737(0.2%)	19	18	5	600
Statistics of Gap (Min : Average : Max)						0 : 1.8 : 7.6	0 : 1.7 : 6.4	0.4 : 4.0 : 12.4	0.3 : 3.9 : 9.3	0 : 1.6 : 6.0				

600 seconds. The "Gap" in the column "Upper bound" is calculated as follows :

Gap = (Upper bound-Best lower bound) /Best lower bound×100%.

From <Table 1>, we see that the LP-relaxation lower bounds "EFRP" enhanced by valid inequalities (7)~(11) are quite tight. Also, note that lower bounds "EOFR" are better than the lower bounds "EFRP" except one (see problem 3). Comparing the best upper bounds found within 7,200 seconds for "FRP" and "EFRP", we cannot find significant difference between them. Also, we cannot find significant difference between the best upper bounds "OFR" and "EOFR." However, note that for only two problems (4 and 6), either "OFR" or "EOFR" provides better upper bounds than "FRP" or "EFRP" although "OFR" or "EOFR" consumes far less computing time. Also, the proposed tabu search algorithm found better, equally good and worse solutions for eight, five and seven problems, respectively, in 600 seconds compared with the upper bounds "EFRP" using 4,089 seconds on the average.

In <Table 2>, we present computational results

〈Table 2〉 Computational results : $|M|$ = 30, $|F|$ = 30, *wgt_q_over_p* = 3

No	Lower bound					Upper bound (Gap)					Elapsed time			
	LP-relaxation at root node			Objective function relaxation		FRP	EFRP	OFR	EOFR	Tabu	FRP	EFRP	EOFR	Tabu
	FRP	EFRP	Best	OFR	EOFR									
1	15965	21877	24421	18537	23272	25024(2.4%)	25024(2.4%)	25260(3.4%)	25260(3.4%)	25002(2.3%)	7200	7200	382	600
2	15789	19912	21862	17854	20776	22112(1.1%)	22017(0.7%)	22112(1.1%)	22112(1.1%)	21923(0.2%)	7200	7200	440	600
3	16735	23513	26080	19351	25296	27333(4.8%)	27333(4.8%)	27636(5.9%)	27636(5.9%)	27373(4.9%)	7200	7200	510	600
4	18034	23524	25356	19312	24173	25356(0%)	25356(0%)	25732(1.4%)	25732(1.4%)	25356(0%)	195	223	11	600
5	13230	18002	19318	14259	18842	19318(0%)	19318(0%)	19377(0.3%)	19377(0.3%)	19343(0.1%)	205	180	106	600
6	11877	17190	18540	13019	17959	18540(0%)	18540(0%)	18680(0.7%)	18680(0.7%)	18562(0.1%)	52	80	27	600
7	13400	18705	20994	14892	19752	20994(0%)	20994(0%)	21383(1.8%)	21383(1.8%)	21302(1.4%)	263	165	26	600
8	13325	17180	19182	15205	18454	19816(3.3%)	19816(3.3%)	20226(5.4%)	20273(5.6%)	19816(3.3%)	7200	7200	863	600
9	17658	22969	25829	19838	23960	25986(0.6%)	25986(0.6%)	28516(10.4%)	28908(11.9%)	25983(0.5%)	7200	7200	193	600
10	18413	25449	28588	21514	27665	29824(4.3%)	29824(4.3%)	30132(5.4%)	30162(5.5%)	29672(3.7%)	7200	7200	1589	600
11	17012	21877	23948	19091	23275	25501(6.4%)	25398(6.0%)	26076(8.8%)	26076(8.8%)	25027(4.5%)	7200	7200	465	600
12	13187	18618	20112	13940	19183	20112(0%)	20112(0%)	20149(0.1%)	20112(0%)	20196(0.4%)	5	8	2	600
13	14109	19130	20517	15344	19936	20517(0%)	20517(0%)	20917(1.9%)	20917(1.9%)	20961(2.1%)	138	112	27	600
14	13314	17490	19634	15657	19357	21482(9.4%)	21212(8.0%)	21202(7.9%)	21562(9.8%)	21108(7.5%)	7200	7200	2540	600
15	16097	20225	22249	18384	21689	23014(3.4%)	23014(3.4%)	23418(5.2%)	23418(5.2%)	23100(3.8%)	7200	7200	1318	600
16	14074	18701	21441	16288	20171	22485(4.8%)	22485(4.8%)	23531(9.7%)	23531(9.7%)	22110(3.1%)	7200	7200	704	600
17	13928	19820	21879	16027	21201	21879(0%)	21879(0%)	23057(5.3%)	23057(5.3%)	21897(0.1%)	963	423	244	600
18	8881	11630	13593	10666	12951	13806(1.5%)	13806(1.5%)	13820(1.6%)	13820(1.6%)	13793(1.4%)	7200	7200	951	600
19	15565	21178	23684	17224	22029	23684(0%)	23684(0%)	24081(1.6%)	24081(1.6%)	23902(0.9%)	2588	2330	63	600
20	17264	23875	26142	18546	24874	26142(0%)	26142(0%)	26376(0.8%)	26376(0.8%)	26893(2.8%)	1790	1280	290	600
Statistics of Gap (Min : Average : Max)						0 : 2.1 : 9.4	0 : 2.0 : 8.0	0.1 : 3.9 : 10.4	0 : 4.1 : 11.9	0 : 2.1 : 7.5				

for twenty test problems with more emphasis on interference cost q against frequency drop cost p. Similar to the results of <Table 1>, the LP-relaxation lower bounds "EFRP" are quite tight compared with those of "FRP." Also, note that lower bounds "EOFR" are always better than the lower bounds "EFRP." However, "EOFR" (or "OFR") upper bounds are not as good as those obtained by "EFRP" within 7,200 seconds. Observe that "EFRP" always provides better (or equally good) feasible solutions than "FRP" does. Tabu search algorithm found better, equally good and worse solutions for eight, two and ten problems, respectively. Average gap of "Tabu" upper bounds is 2.1%, while the average gap of "EFRP" is 2.0%.

In <Table 3> and <Table 4>, we provide computational results for some larger test problems. Similar to the results in <Table 1> and <Table 2>, we see that valid inequalities (7)~(11) improved the LP-relaxation bounds significantly. For the problems in <Table 3> and <Table 4>, upper bounds by "OFR" or by "EOFR" are not satisfactory although "EOFR" provides better lower bounds than "EFRP" does. Note that there are five test problems in <Table 3> and <Table 4> that are not optimally solved within 7,200 seconds using

〈Table 3〉 Computational results : $|M|$ = 40, $|F|$ = 30, *wgt_q_over_p* = 1

No	Lower bound					Upper bound (Gap)					Elapsed time			
	LP-relaxation at root node			Objective function relaxation		FRP	EFRP	OFR	EOFR	Tabu	FRP	EFRP	EOFR	Tabu
	FRP	EFRP	Best	OFR	EOFR									
1	17755	25037	26923	20005	25971	27706(2.9%)	27576(2.4%)	27737(3.0%)	27737(3.0%)	27556(2.3%)	7200	7200	921	600
2	19819	26516	28504	21194	27339	28504(0%)	28504(0%)	28680(0.6%)	28570(0.2%)	28521(0.1%)	176	162	4	600
3	19060	27385	29118	21523	28538	30322(4.1%)	30274(3.9%)	30782(5.7%)	30937(6.2%)	30302(4.0%)	7200	7200	297	600
4	22148	30767	33217	23801	31493	33457(0.7%)	33457(0.7%)	34949(5.2%)	34949(5.2%)	33457(0.7%)	7200	7200	89	600
5	19393	26941	28714	21659	28581	31091(8.2%)	31064(8.1%)	32050(11.6%)	31541(9.8%)	30062(4.6%)	7200	7200	7200	600
6	22519	30423	32378	24840	31720	34212(5.6%)	34212(5.6%)	35347(9.1%)	35347(9.1%)	34212(5.6%)	7200	7200	5441	600
7	20807	28008	29929	22530	29102	30633(2.3%)	30633(2.3%)	30839(3.0%)	30839(3.0%)	31166(4.1%)	7200	7200	576	600
8	21124	30196	32316	23500	31458	32679(1.1%)	32622(0.9%)	33064(2.3%)	33064(2.3%)	32622(0.9%)	7200	7200	4211	600
9	17342	23578	25381	19631	24715	26036(2.5%)	26388(3.9%)	26498(4.4%)	26498(4.4%)	26021(2.5%)	7200	7200	1789	600
10	20637	28254	30302	22454	29290	30302(0%)	30302(0%)	30761(1.5%)	30823(1.7%)	30302(0%)	1069	1023	86	600
11	19913	25218	27255	22097	26273	27980(2.6%)	27980(2.6%)	28798(5.6%)	28840(5.8%)	27988(2.6%)	7200	7200	2877	600
12	22801	30979	33145	25007	32037	34666(4.5%)	34289(3.4%)	36322(9.5%)	36127(8.9%)	34276(3.4%)	7200	7200	1784	600
13	9098	12367	14830	11382	14148	15182(2.3%)	15228(2.6%)	15256(2.8%)	15256(2.8%)	15134(2.0%)	7200	7200	7200	600
14	15513	20776	22813	18047	21955	23738(4.0%)	23738(4.0%)	24626(7.9%)	24626(7.9%)	23730(4.0%)	7200	7200	1713	600
15	19881	28505	30563	21785	29541	30563(0%)	30563(0%)	31001(1.4%)	31064(1.6%)	30563(0%)	7200	7200	3560	600
16	13334	19007	21203	15032	20066	21207(0.1%)	21207(0.1%)	21237(0.2%)	21237(0.2%)	21211(0.1%)	7200	7200	224	600
17	21153	29834	32371	23633	31350	34019(5.0%)	33746(4.2%)	35368(9.2%)	35140(8.5%)	33564(3.6%)	7200	7200	5444	600
18	23348	33070	36210	24631	33881	36210(0%)	36210(0%)	36210(0%)	36210(0%)	36222(0.1%)	2880	1372	3	600
19	18341	25526	27275	20268	26693	27825(2.0%)	27815(1.9%)	28043(2.8%)	27941(2.4%)	27798(1.9%)	7200	7200	5843	600
20	21898	31689	34275	23462	32734	34275(0%)	34275(0%)	34624(1.0%)	34624(1.0%)	342750%)	7200	5570	231	600
Statistics of Gap (Min:Average:Max)						0:2.4:8.2	0:2.3:8.1	0:4.3:11.6	0:4.2:9.8	0:2.1:5.6				

"EOFR." In this case, we cannot be sure if the best upper bound of EOFR provides a lower bound. Thus, for these test problems, we marked "NA" in the column lower bound by "EOFR." For test problems even in <Table 3> and <Table 4>, tabu search algorithm found quite good feasible solutions.

From the computational results displayed in <Table 1>~<Table 4>, we see that

(1) the developed valid inequalities (7)~(11) are effective for improving the LP-relaxation bound,

(2) the optimal objective value to EOFR provides tighter lower bounds than those by valid inequalities (7)~(11), while the upper bounds recovered from the optimal solution to EOFR are not as good as those obtained by EFRP, while

(3) the proposed tabu search algorithm exhibits better performance in terms of both solution quality and computing time when compared with the CPLEX optimization procedure using EFRP (or FRP).

6. Conclusions

In this paper, we considered a frequency re-

〈Table 4〉 Computational results : $|M|$ = 40, $|F|$ = 30, *wgt_q_over_p* = 3

No	Lower bound					Upper bound (Gap)					Elapsed time			
	LP-relaxation at root node			Objective function relaxation		FRP	EFRP	OFR	EOFR	Tabu	FRP	EFRP	EOFR	Tabu
	FRP	EFRP	Best	OFR	EOFR									
1	11096	15928	17654	13326	17279	18670(5.7%)	18626(5.5%)	18626(5.5%)	18626(5.5%)	18626(5.5%)	7200	7200	713	600
2	20453	28306	29981	21987	29109	29981(0%)	29981(0%)	30526(1.8%)	30526(1.8%)	29990(0.1%)	683	660	73	600
3	17824	25106	27020	19443	25868	27081(0.2%)	27081(0.2%)	27081(0.2%)	27081(0.2%)	27027(0.1%)	7200	7200	116	600
4	20410	28142	30860	22183	29346	30860(0%)	30860(0%)	31613(2.4%)	31613(2.4%)	30889(0.1%)	7200	4183	2716	600
5	21724	31300	33310	24410	32606	34290(2.9%)	34240(2.7%)	35440(6.4%)	35509(6.6%)	34178(2.6%)	7200	7200	6257	600
6	14822	20622	22250	16742	21864	22610(1.6%)	22610(1.6%)	23166(4.1%)	23166(4.1%)	22810(2.5%)	7200	7200	3026	600
7	17395	23879	25787	20075	25523	27472(6.5%)	27557(6.8%)	27492(6.6%)	27492(6.6%)	27208(5.5%)	7200	7200	7200	600
8	19356	27283	29071	22903	29814	32024(10.1%)	31850(9.5%)	33023(13.6%)	33215(14.2%)	31362(7.9%)	7200	7200	7200	600
9	22954	31631	34238	24558	32700	34864(1.8%)	34864(1.8%)	35783(4.5%)	35676(4.2%)	34677(1.2%)	7200	7200	108	600
10	25047	34279	36972	26232	34905	37508(1.4%)	37504(1.4%)	38178(3.2%)	38301(3.6%)	37311(0.9%)	7200	7200	35	600
11	18801	27544	29775	21099	28869	30261(1.6%)	30261(1.6%)	31202(4.8%)	31202(4.8%)	30082(1.0%)	7200	7200	820	600
12	24454	33693	36576	26826	35284	38563(5.4%)	38410(5.0%)	40956(11.9%)	40956(11.9%)	37790(3.3%)	7200	7200	7200	600
13	15420	21054	22631	17289	21842	22841(0.9%)	22841(0.9%)	23232(2.6%)	23232(2.6%)	23121(2.1%)	7200	7200	329	600
14	21246	28149	30788	22789	29303	31139(1.1%)	31139(1.1%)	32106(4.2%)	32106(4.2%)	31007(0.7%)	7200	7200	204	600
15	23981	33527	35704	26236	34879	37050(3.7%)	36910(3.3%)	37166(4.1%)	37166(4.1%)	36690(2.7%)	7200	7200	1196	600
16	17603	23232	25330	20226	24616	26685(5.3%)	26662(5.2%)	28043(10.7%)	27521(8.6%)	26431(4.3%)	7200	7200	4758	600
17	23173	31549	34060	25084	32816	35633(4.6%)	35633(4.6%)	37099(8.9%)	37204(9.2%)	35603(4.5%)	7200	7200	7200	600
18	20386	27850	30522	22363	29282	31608(3.5%)	31305(2.5%)	32519(6.5%)	32514(6.5%)	31160(2.1%)	7200	7200	1613	600
19	23186	30875	32829	24681	31586	32829(0%)	32829(0%)	33227(1.2%)	33227(1.2%)	32845(0.1%)	519	460	36	600
20	23897	33472	35687	25945	34515	35975(0.8%)	35975(0.8%)	36794(3.1%)	36794(3.1%)	35800(0.3%)	7200	7200	147	600
Statistics of Gap (Min : Average : Max)						0 : 2.8 : 10.1	0 : 2.7 : 9.5	0.2 : 5.3 : 13.6	0.2 : 5.2 : 14.2	0.1 : 2.3 : 7.9				

assignment problem arising from the reconfiguration of radio networks. For this problem, we developed an IP model. Also, we developed some valid inequalities and an objective function relaxation scheme to derive tight lower bounds. For solving large problem instances, we developed an effective heuristic procedure. Computational results show that the developed valid inequalities are effective for reducing the computation time to find an optimal solution for small size problems. Also, the proposed heuristic procedure finds feasible solutions of good quality within reasonable time bound for large size problems.

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