Journal of Institute of Control, Robotics and Systems (2011) 17(6):573-578
DOI:10.5302/J.ICROS.2011.17.6.573 ISSN:1976-5622 eISSN:2233-4335

피드포워드와 비피드포워드 비선형성이 혼재된 비선형 시스템의 적응 제어

Adaptive Control of a Class of Feedforward and Non-feedforward Nonlinear Systems

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Abstract: We propose a switching-based adaptive state feedback controller for a class of nonlinear systems that have uncertain nonlinearity. The base of the proposed conditions on the nonlinearity is the feedforward form, then it is extended via a nonlinear function containing all the states and the control input. As a result, more generalized systems containing feedforward and nonfeedforward terms are allowed as long as the ratio condition of the nonlinear function is satisfied. Moreover, the information on the growth rate of nonlinearity is not required a priori in our control scheme.

Keywords: adaptive regulation, feedforward and non-feedforward, switching control

I. INTRODUCTION

There have been various control results on the stabilization/regulation problems of the feedforward systems and many related results in either state or output feedback form still can be found in very recent years [1,4,6-10,13-16]. However, in most of these results, the considered systems and control methods are naturally limited to a class of feedforward systems only. Thus, if the systems contain some additional 'non-feedforward' terms, most of the existing results become non-applicable. Notably, in [7], the authors developed a dynamic-gain state feedback controller which allows some non-feedforward and non-triangular terms in the nonlinearity. Thus, the system under consideration is extended into some class of systems that contain both feedforward and some non-feedforward terms. However, the method of [7] is applicable when the growth rates of nonlinear terms are known a priori. The purpose of this paper is to develop a regulating state feedback controller under the newly suggested conditions, which allow feedforward and some non-feedforward terms in the nonlinearity at once. Moreover, in our control scheme, we don't need to know the growth rates of nonlinear terms. With the new conditions, some nonlinear terms, which cannot be covered by any of aforementioned existing results, will be allowed to consider the regulation problem.

In this paper, we will consider the following class of nonlinear systems

$$\dot{x} = Ax + Bu + \delta(t,x,u) \tag{1}$$

where $x \in R^n$ and $u \in R$ are the state and input of the system, respectively. The system matrices (A, B) is a Brunovsky canonical pair and the nonlinearity is an $n\times 1$ vector such as $\delta(t,x,u) = [\delta_1(t,x,u),\cdots,\delta_n(t,x,u)]^T$. We assume that the origin is the only equilibrium point of the system (1) when $u = 0$. Define a positive definite matrix $E_{\gamma(t)} = \mathrm{diag}[1,\gamma(t),\cdots,\gamma(t)^{n-1}]$, $\gamma(t) > 0$. The mappings $\delta_i(t,x,u): R\times R^n \times R \to R$, $i = 1,\cdots,n$, are C^1 and satisfy the following conditions.

Assumption 1: There exist an unknown constant $L \ge 0$ and a nonnegative function $\phi(x,u,\gamma(t))$ such that

$$\begin{aligned}\left\|E_{\gamma(t)}\delta(t,x,u)\right\|_1 &\le L\left(1+\phi(x,u,\gamma(t))\right)\\ &\times\left(\sum_{i=1}^{n-2}\gamma(t)^{i-1}\,|\,x_{i+2}\,|+\gamma(t)^{n-2}\,|\,u\,|\right)\end{aligned} \tag{2}$$

for all $x \in R^n$ and $u \in R$.

Assumption 2: There exist functions in the $\phi_i(x,u,\gamma(t))$ form of

$$\phi_i(x,u,\gamma(t)) = \prod_{j=1}^{n} |x_j|^{a_{(i,j)}}\,|\,u\,|^{\mu_i}\ \gamma(t)^{\nu_i} \tag{3}$$

where $a_{(i,j)}$, $\mu_i \ge 0$, and ν_i is an any real number for $i = 1,\cdots,m$, and $j = 1,\cdots,n$ such that

$$\phi(x,u,\gamma(t)) \le \sum_{i=1}^{m}\phi_i(x,u,\gamma(t)) \tag{4}$$

for all $x \in R^n$ and $u \in R$.

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논문접수: 2011. 3. 16., 수정: 2011. 4. 14., 채택확정: 2011. 4. 24.
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※ 이 논문은 2011년도 한국과학기술원 BK21 전자 통신기술사업단에 의하여 지원되었음. 이 논문은 2009년도 정부(교육과학기술부)의 재원으로 한국연구재단의 기초연구사업 지원을 받아 수행된 것임(2009-0063942).

Using $|x_i| \le \gamma(t)^{n-i}\|E_{\gamma(t)}x\|$, we have, for $i=1,\cdots,m$, $\prod_{j=1}^{n}|x_j|^{a_{(i,j)}} \le \gamma(t)^{-\sum_{j=1}^{n}(j-1)a_{(i,j)}}\|E_{\gamma(t)}x\|$. From Assumption 2 and the above inequality, the following inequality can be directly derived by using $E_{\gamma(t)}$.

$$\phi_i(x,u,\gamma(t)) \le \frac{\|E_{\gamma(t)}x\|^{r_i}|u|^{\mu_i}}{\gamma(t)^{q_i}} \tag{5}$$

where $r_i=\sum_{j=1}^{n}a_{(i,j)}$ and $q_i=-v_i+\sum_{j=1}^{n}(j-1)a_{(i,j)}$ for all $x\in R^n$ and $u\in R$.

Assumption 3: The following inequality holds

$$(n-1)r_i-q_i-\mu_i<1 \tag{6}$$

for all $i=1,\cdots,m$.

The control problem is to adaptively regulate the system (1) under Assumptions 1 through 3. The proposed conditions are quite extensible since a large choices of $\phi(x,u,\gamma(t))$ are possible. Here, we provide some observations on how to apply the proposed conditions and how the non-feedforward terms are included throughout some specific examples for simplicity and clarity.

Example A: (Application of the conditions) We let $n=3$, $\delta_1(t,x,u)=x_2^2u^3$, $\delta_2(t,x,u)=0$, and $\delta_3(t,x,u)=0$. By Assumption 1, we have

$$|x_2^2u^3| \le \gamma(t)^{-1}|x_2|^2|u|^2\gamma(t)|u| \le (1+\gamma(t)^{-1}|x_2|^2|u|^2)\gamma(t)|u| \tag{7}$$

Thus, we obtain $\phi(x,u,\gamma(t))=\gamma(t)^{-1}|x_2|^2|u|^2$. Next, in applying Assumption 2, we give two ways and show that one is satisfactory while the other is not. (i) First, we simply choose $m=1$, i.e., $\phi(x,u,\gamma(t))=\phi_1(x,u,\gamma(t))$. (ii) Second, we may further divide $\phi(x,u,\gamma(t))\le\gamma(t)^{-1}|x_2|^4+\gamma(t)^{-1}|u|^4$ so that $\phi_1(x,u,\gamma(t))=\gamma(t)^{-1}|x_2|^4$ and $\phi_2(x,u,\gamma(t))=\gamma(t)^{-1}|u|^4$. Now, with Assumption 3, it is easy to check that the choice of (i) is satisfied with $r_1=2$, $\mu_1=2$, and $q_1=3$. However, with the choice of (ii), the term $\phi_1(x,u,\gamma(t))=\gamma(t)^{-1}|x_2|^4$ does not satisfy Assumption 3 because $r_1=4$, $\mu_1=0$, and $q_1=5$. Thus, it is important to obtain the proper functions $\phi_i(x,u,\gamma(t))$ because the same system can be viewed completely different depending on the selection.

Example B: (Norm-bound feedforward form) We consider the example in [12] such that $n=3$, $\delta_1(t,x,u)=\sin x_3+4x_3^2$, $\delta_2(t,x,u)=u^6$, and $\delta_3(t,x,u)=0$. By assumption 1, it is easy to obtain that

$$\begin{aligned}\|E_{\gamma(t)}\delta(t,x,u)\|_1 &\le |\sin x_3|+4|x_3|^2+\gamma(t)|u|^6\\ &\le 4(1+|x_3|+|u|^5)(|x_3|+\gamma(t)|u|)\end{aligned} \tag{8}$$

From (8), we set $\phi_1(x,u,\gamma(t))=|x_3|$ and $\phi_2(x,u,\gamma(t))=|u|^5$ for Assumption 2. Now, with Assumption 3, we have $r_1=1$, $\mu_1=0$, $q_1=2$, $r_2=0$, $\mu_2=5$ and $q_2=0$, i.e., all conditions are satisfied. In fact, this particular example can be easily generalized into the following feedforward form. For $i=1,\cdots,n-2$

$$\delta_i(t,x,u)\le L_i(1+|x_n|^{p_{i,1}}+|u|^{p_{i,2}})(|x_{i+2}|+\cdots+|x_n|+|u|) \tag{9}$$

with $\delta_{n-1}(t,x,u)\le L_{n-1}(1+|x_n|^{p_{n-1,1}}+|u|^{p_{n-1,2}})|u|$ and $\delta_n(t,x,u)=0$. Here, $p_{i,1},p_{i,2}\ge 1$, $i=1,\cdots,n-1$. From Assumption 1 and 2, we have $\phi(x,u,\gamma(t))=\sum_{i=1}^{n}|x_n|^{p_{i,1}}+|u|^{p_{i,2}}$ and $\phi_1(x,u,\gamma(t))=|x_n|^{p_{1,1}}$. Then, we obtain $r_1=p_{1,1}$, $\mu_1=0$ and $q_1=(n-1)p_{1,1}$. Therefore, it is easy to check that the conditions in Assumption 3 are satisfied. Similarly, for $\phi_j(x,u,\gamma(t))$, $j=2,\cdots,2(n-1)$, we can check that this form satisfies Assumptions 1 through 3 via simple algebraic manipulation. Some non-feedforward terms are limitedly allowed because the feedforward form is expressed in terms of norm-bound. For example, let $\delta_1(t,x,u)=x_3\sin x_1\sin x_2$, which violates the conditions used in [4,11-14] due to x_1 and x_2. However, in view of norm-bound, it is obvious that $\delta_1(t,x,u)\le|x_3|$ which belongs to (9). Thus, this norm bound feedforward form itself has some extensions over the existing feedforward conditions.

Example C: (Non-feedforward form) Let $n=3$, $\delta_1(t,x,u)=x_3^2+x_1^{\frac{1}{5}}x_2^2u^3\sin x_1$, $\delta_2(t,x,u)=x_2^{\frac{1}{3}}u^4$, and $\delta_3(t,x,u)=x_1u^4$. In this case, it clearly contains both feedforward and non-feedforward terms. By Assumptions 1 and 2, we can obtain that $\phi_1(x,u,\gamma(t))=|x_3|$, $\phi_2(x,u,\gamma(t))=\gamma(t)^{-1}|x_1|^{\frac{1}{5}}|x_2|^2|u|^2$, $\phi_3(x,u,\gamma(t))=|x_2|^{\frac{1}{3}}|u|^3$, and $\phi_4(x,u,\gamma(t))=\gamma(t)|x_1||u|^3$. Then, Assumption 3 is satisfied by taking $r_1=1$, $\mu_1=0$, $q_1=2$, $r_2=\frac{11}{5}$, $\mu_2=2$, $q_2=3$, $r_3=\frac{1}{3}$, $\mu_3=3$, $q_3=\frac{1}{3}$, $r_4=1$, $\mu_4=3$, and $q_4=-1$.

II. ADAPTIVE FEEDBACK CONTROLLER

To solve our control problem for any given initial conditions $x(0)\in R^n$, we introduce an adaptive controller which can tune the gain parameter by using the suitably defined monitoring signals.

On-line tuning feedback controller:

$$u=K(\gamma(t))x \tag{10}$$

where $K(\gamma(t))=[k_1/\gamma(t)^n,\cdots,k_n/\gamma(t)]$, $\gamma(t)>0$ is to be tuned according to the switching logic.

Monitoring signals:

$$\theta_1(t)=\sum_{i=1}^{m}\frac{\|E_{\gamma(t)}x\|^{r_i+\mu_i}}{\gamma(t)^{q_i+n\mu_i}} \tag{11}$$

$$\theta_2(t) = \int_{t_k}^{t} \left\| E_{\gamma(\tau)} x(\tau) \right\| d\tau \tag{12}$$

for $t > t_k$ where t_k, $k = 0,1,2\cdots$, is the moment that switching occurs.

Remark 1: Logic (11) is proposed for preventing the finite time escape phenomenon of controlled system with high-order nonlinearity. Logic (12) is presented for the regulation of the system.

Initialization:

• Set the positive constant $0 < c < 1$ such as $c > (n-1)r_i - q_i - \mu_i$ for all $i = 1,\cdots,m$.

• Set positive constants $S_0 \geq 1$, $\alpha > 1$ and $\gamma_0 \geq 1$ such as $\gamma_0^c > \dfrac{\left\| E_{\gamma_0} x(0) \right\|^{r_i+\mu_i}}{\gamma_0^{q_i+n\mu_i}}$ for all $i = 1,\cdots,m$.

• Set $k = 0$ at $t_0 = 0$.

Switching logic:

Step 1: Set $\gamma(t) = \gamma_k$.

Step 2: If $\theta_1(t) \geq m\gamma_k^c$ or $\theta_2(t) > k\gamma_k^2 \left\| E_{\gamma_k} x(t_k) \right\|$

$\rightarrow S_{k+1} = \alpha S_k$;

$\rightarrow \gamma_{k+1} = S_{k+1}\gamma_k$;

$\rightarrow t_{k+1} = t$;

$\rightarrow k = k+1$; (Switching)

Go to Step 1;

Else

$\rightarrow \gamma(t) = \gamma_k$; (No Switching)

Go to Step 2;

Here, we address some mathematical notations and setups for Theorem 1 and its proof. Let $A_{K(\gamma(t))} = A + BK(\gamma(t))$. Then, we define $K = K(1)$ and $A_K = A_{K(1)}$. If given that A_K is Hurwitz, from [2], we can obtain a Lyapunov equation of $A_{K(\gamma(t))}^T P_{K(\gamma(t))} + P_{K(\gamma(t))} A_{K(\gamma(t))} = -\gamma(t)^{-1} E_{\gamma(t)}^2$ with $P_{K(\gamma(t))} = E_{\gamma(t)} P_K E_{\gamma(t)}$ from $A_K^T P_K + P_K A_K = -I$ where I denotes an $n \times n$ identity matrix. We note that the proposed controller (10) is equivalently expressed as $u = \gamma(t)^{-n} K E_{\gamma(t)} x$.

Now, we state the main result.

Theorem 1: Under Assumptions 1 through 3, select K such that A_K is Hurwitz. Then, the controller (10) with the switching logic asymptotically regulates the closed-loop system (1) for any given initial conditions $x(0) \in R^n$. Also, $\gamma(t) \to \gamma_{ss} < \infty$ as $t \to \infty$.

Proof: From (1) and (10), the closed-loop system is

$$\dot{x} = A_{K(\gamma(t))} x + \delta(t,x,u) \tag{13}$$

where $\gamma(t)$ is nondecreasing and $\gamma(t) \geq 1$ for all $t \geq 0$ according to the switching logic. Thus, the system (13) does not suffer from any singularity problem. The vector field of (13) is C^1 with respect to its arguments. So, the system (13) satisfies a local Lipschitz condition in a neighborhood of the initial condition $x(0) \in R^n$. Moreover, it is certain that $\gamma(t)$ exists and is finite as long as $\|x\|$ remains bounded. Thus, the solution of (13) exists and is unique on $[0, T_f)$ for some $T_f \in (0,\infty)$ [5,9]. Without loss of generality, we assume now on that $[0, T_f)$ is the maximally extended interval of the solution of (13).

The gain function $\gamma(t)$ evolves as a piecewise constant function on $t \in [0, T_f)$ and there are time-intervals $\Delta t_k := \{t \mid t \in (t_k, t_{k+1}]\}$, $k = 0,1,2\cdots$, such that $\gamma(t)$ remains as a constant value γ_k for each time-interval Δt_k. Obviously, there are associated switching time denoted as t_k, $k = 0,1,2,\cdots$, i.e., switching occurs at $t = t_k$. As a whole, the closed-loop system can be viewed as a switched system $\dot{x} = f_k(x,u,\gamma(t))$, $k = 0,1,2\cdots$, and each subsystem $\dot{x} = f_k(x,u,\gamma_k)$ is engaged for each time-interval Δt_k where $\gamma(t)$ is replaced by γ_k for each Δt_k. Now, for each subsystem $\dot{x} = A_{K(\gamma_k)} x + \delta(t,x,u)$, we set a Lyapunov function $V_k(x) = x^T P_{K(\gamma_k)} x$ where $P_{K(\gamma_k)} = E_{\gamma_k} P_K E_{\gamma_k}$ and $E_{\gamma_k} = E_{\gamma(t)}|_{\gamma(t)=\gamma_k}$. Then, we have

$$\lambda_1 \left\| E_{\gamma_k} x \right\|^2 \leq V_k(x) \leq \lambda_2 \left\| E_{\gamma_k} x \right\|^2 \tag{14}$$

where $\lambda_1 = \lambda_{\min}(P_K)$ and $\lambda_2 = \lambda_{\max}(P_K)$, which are independent of γ_k.

Along the trajectory of each subsy stem, we have

$$\begin{aligned} \dot{V}_k(x) &= -\gamma_k^{-1} \left\| E_{\gamma_k} x \right\|^2 + 2x^T P_{K(\gamma_k)} \delta(t,x,u) \\ &\leq -\gamma_k^{-1} \left\| E_{\gamma_k} x \right\|^2 \\ &\quad + 2\|P_K\| \left\| E_{\gamma_k} x \right\| \left\| E_{\gamma_k} \delta(t,x,u) \right\|_1 \end{aligned} \tag{15}$$

The inequality of (15) is obtained by using $A_{K(\gamma_k)}^T P_{K(\gamma_k)} + P_{K(\gamma_k)} A_{K(\gamma_k)} = -\gamma_k^{-1} E_{\gamma_k}^2$. Regarding the last term $\left\| E_{\gamma_k} \delta(t,x,u)_1 \right\|$ of (15), by Assumption 1, we have (recall that $u = \gamma_k^{-n} K E_{\gamma_k} x$)

$$\left\| E_{\gamma_k} \delta(t,x,u) \right\|_1 \leq L\sqrt{n}(1 + \|K\|)\gamma_k^{-2}(1 + \phi(x,u,\gamma_k)) \left\| E_{\gamma_k} x \right\| \tag{16}$$

Using (15)-(16), for $t \in \Delta t_k$, we have

$$\dot{V}_k(x) \leq -\gamma_k^{-2} \left(\gamma_k - \sigma\left(1 + \phi(x,u,\gamma_k)\right) \right) \left\| E_{\gamma_k} x \right\|^2 \tag{17}$$

where $\sigma = 2L\sqrt{n}(1 + \|K\|)\|P_K\|$.

From Assumption 2, using $u = \gamma_k^{-n} K E_{\gamma_k} x$, for $t \in \Delta t_k$, we obtain

$$\phi(x,u,\gamma_k) \leq \sum_{i=1}^{m} \frac{\left\| E_{\gamma_k} x \right\|^{r_i} |u|^{\mu_i}}{\gamma_k^{q_i}} \leq \sum_{i=1}^{m} \rho_i \frac{\left\| E_{\gamma_k} x \right\|^{r_i+\mu_i}}{\gamma_k^{q_i+n\mu_i}} \tag{18}$$

where $\rho_i = \|K\|^{\mu_i}$, $i = 1,\cdots,m$.

Note that, for each time interval, $\sum_{i=1}^{m} \dfrac{\left\| E_{\gamma_k} x \right\|^{r_i+\mu_i}}{\gamma_k^{q_i+n\mu_i}} \leq m\gamma_k^c$ holds by the proposed switching rule. That is, the condition (2) becomes the linear growth condition for each time interval. Thus, there is no finite escape time for each time interval, which means overall finite escape phenomenon does not occur with our switching rule under Assumptions 1 through 3. The remaining part is to show that (i) only a finite number of switching occurs, (ii) the system regulation is followed afterward.

(I) Finite switching: We show that switching occurs only finitely. It is clear that γ_k and S_k are nondecreasing and $c<1$ from the switching logic and the initialization. Let k^* be the smallest number of switchings such that the following inequalities are satisfied

$$\gamma_{k^*} - \sigma(1+\bar{\rho} m \gamma_{k^*}^{c}) \ge 1 \tag{19}$$

$$S_{k^*} \ge \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{\bar{r}}{c-\bar{c}}} \tag{20}$$

$$k^* \ge \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{2}} \tag{21}$$

where $\bar{\rho} = \max_{i\in[1,m]}\{\rho_i\}$, $\bar{r} = \frac{1}{2}\max_{i\in[1,m]}\{r_i+\mu_i\}$, and $\bar{c} = \max_{i\in[1,m]}\{(n-1)r_i - q_i - \mu_i\}$.

By the switching logic, for $t\in\Delta t_{k^*}$, we obtain

$$\theta_1(t) = \sum_{i=1}^{m} \frac{\left\|E_{\gamma_{k^*}} x\right\|^{r_i+\mu_i}}{\gamma_{k^*}^{q_i+n\mu_i}} \le m\gamma_{k^*}^{c} \tag{22}$$

Using (17)-(19), (22) and the continuity of $x(t)$, for $t\in\Delta t_{k^*}$, we obtain

$$\begin{aligned}\left\|E_{\gamma_{k^*}} x\right\| &\le \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{2}} \left\|E_{\gamma_{k^*}} x(t_{k^*}^{+})\right\| e^{-\gamma_{k^*}^{-2}(t-t_{k^*}^{+})} \\ &= \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{2}} \left\|E_{\gamma_{k^*}} x(t_{k^*})\right\| e^{-\gamma_{k^*}^{-2}(t-t_{k^*})}\end{aligned} \tag{23}$$

For $t\in\Delta t_{k^*+1}$, similarly to (23), we have

$$\left\|E_{\gamma_{k^*+1}} x\right\| \le \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{2}} \left\|E_{\gamma_{k^*+1}} x(t_{k^*+1})\right\| e^{-\gamma_{k^*+1}^{-2}(t-t_{k^*+1})} \tag{24}$$

From (24), $t\in\Delta t_{k^*+1}$, we have

$$\begin{aligned}\theta_1(t) &= \sum_{i=1}^{m} \frac{\left\|E_{\gamma_{k^*+1}} x\right\|^{r_i+\mu_i}}{\gamma_{k^*+1}^{q_i+n\mu_i}} \\ &\le \sum_{i=1}^{m} \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{r_i+\mu_i}{2}} \frac{\left\|E_{\gamma_{k^*+1}} x(t_{k^*+1})\right\|^{r_i+\mu_i}}{\gamma_{k^*+1}^{q_i+n\mu_i}} \times e^{-\frac{r_i+\mu_i}{\gamma_{k^*+1}^{2}}(t-t_{k^*+1})}\end{aligned} \tag{25}$$

By the properties of $\gamma_{k^*+1} = S_{k^*+1}\gamma_{k^*}$ and $S_{k^*+1}\ge 1$,

$$\begin{aligned}\left\|E_{\gamma_{k^*+1}} x(t_{k^*+1})\right\| &= \sqrt{\sum_{i=1}^{n} \gamma_{k^*+1}^{2(i-1)} x_i(t_{k^*+1})^2} \\ &\le \sqrt{S_{k^*+1}^{2(n-1)} \sum_{i=1}^{n} \gamma_{k^*}^{2(i-1)} x_i(t_{k^*+1})^2} \\ &\le S_{k^*+1}^{(n-1)} \left\|E_{\gamma_{k^*}} x(t_{k^*+1})\right\|\end{aligned} \tag{26}$$

From (26), the term $\frac{\left\|E_{\gamma_{k^*+1}} x(t_{k^*+1})\right\|^{r_i+\mu_i}}{\gamma_{k^*+1}^{q_i+n\mu_i}}$ for $i=1,\cdots,m,$ in the right term of the inequality (25), is bounded as

$$\begin{aligned}\frac{\left\|E_{\gamma_{k^*+1}} x(t_{k^*+1})\right\|^{r_i+\mu_i}}{\gamma_{k^*+1}^{q_i+n\mu_i}} &\le \frac{\left\|E_{\gamma_{k^*}} x(t_{k^*+1})\right\|^{r_i+\mu_i}}{\gamma_{k^*}^{q_i+n\mu_i} S_{k^*+1}^{q_i+n\mu_i}} S_{k^*+1}^{(n-1)(r_i+\mu_i)} \\ &\le \frac{\left\|E_{\gamma_{k^*}} x(t_{k^*+1})\right\|^{r_i+\mu_i}}{\gamma_{k^*}^{q_i+n\mu_i}} S_{k^*+1}^{((n-1)r_i-q_i-\mu_i)}\end{aligned} \tag{27}$$

By substituting (27) into (25), for $t\in\Delta t_{k^*+1}$, we obtain

$$\begin{aligned}\theta_1(t) &\le \sum_{i=1}^{m} \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{r_i+\mu_i}{2}} S_{k^*+1}^{((n-1)r_i-q_i-\mu_i)} \frac{\left\|E_{\gamma_{k^*}} x(t_{k^*+1})\right\|^{r_i+\mu_i}}{\gamma_{k^*}^{q_i+n\mu_i}} \\ &\le \left(\frac{\lambda_2}{\lambda_1}\right)^{\bar{r}} S_{k^*+1}^{\bar{c}} \sum_{i=1}^{m} \frac{\left\|E_{\gamma_{k^*}} x(t_{k^*+1})\right\|^{r_i+\mu_i}}{\gamma_{k^*}^{q_i+n\mu_i}}\end{aligned} \tag{28}$$

From (22) and (28), for $t\in\Delta t_{k^*+1}$, we obtain

$$\theta_1(t) \le \left(\frac{\lambda_2}{\lambda_1}\right)^{\bar{r}} S_{k^*+1}^{\bar{c}} m\gamma_{k^*}^{c} \tag{29}$$

Since $c-\bar{c}>0$ and $\left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{\bar{r}}{c-\bar{c}}} S_{k^*+1}^{-1} < 1$ from (20), we have

$$\begin{aligned}&\left(\frac{\lambda_2}{\lambda_1}\right)^{\bar{r}} S_{k^*+1}^{\bar{c}}\gamma_{k^*}^{c} \\ &\le \left(\frac{\lambda_2}{\lambda_1}\right)^{\bar{r}} S_{k^*+1}^{-(c-\bar{c})} \left(S_{k^*+1}\gamma_{k^*}\right)^{c} < \gamma_{k^*+1}^{c}\end{aligned} \tag{30}$$

Therefore, from (29) and (30), we obtain, for $t\in\Delta t_{k^*+1}$

$$\theta_1(t) < m\gamma_{k^*+1}^{c} \tag{31}$$

Also, from (21) and (24), we have, for $t\in\Delta t_{k^*+1}$

$$\begin{aligned}\theta_2(t) &\le \int_{t_{k^*+1}}^{t} \left\|E_{\gamma_{k^*+1}} x(\tau)\right\| d\tau \\ &\le \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{2}} \gamma_{k^*+1}^{2} \left\|E_{\gamma_{k^*+1}} x(t_{k^*+1})\right\| \\ &\le (k^*+1)\gamma_{k^*+1}^{2} \left\|E_{\gamma_{k^*+1}} x(t_{k^*+1})\right\|\end{aligned} \tag{32}$$

From (31)-(32) with our switching logic, it is clear that no further switching occurs for $t>t_{k^*+1}$.

(II) System regulation: We will show that if the switching is finite, then the closed-loop system (13) is asymptotically regulated. Let t_{k_f} be the final switching time where k_f is the finite number of switchings for $t\in[0,T_f)$. From the finite number of switchings, it is clear that $\gamma(t)$ converges to a finite value γ_{k_f} for $t\in[0,T_f)$. Also, by the switching logic, for $t\in(t_{k_f},T_f)$, we obtain

$$\left\| E_{\gamma_{k_f}} x \right\|^{r_i} \le m\gamma_{k_f}^{q_i+c},\ i\in[1,m] \tag{33}$$

$$\int_{t_{k_f}}^{t} \left\| E_{\gamma_{k_f}} x(\tau) \right\| d\tau \le k_f \gamma_{k_f}^2 \left\| E_{\gamma_{k_f}} x(t_{k_f}) \right\| \tag{34}$$

From (2) and (18), we have

$$\begin{aligned} \left\| \frac{dE_{\gamma_{k_f}} x}{dt} \right\| &= \left\| E_{\gamma_{k_f}} \dot{x} \right\| = \left\| \gamma_{k_f}^{-1} A_K E_{\gamma_{k_f}} x + E_{\gamma_{k_f}} \delta(t,x,u) \right\| \\ &\le \gamma_{k_f}^{-1} \|A_K\| \left\| E_{\gamma_{k_f}} x \right\| \\ &\quad + L\sqrt{n}(\|K\|+1)(1+\phi(x,u,\gamma(t)))\gamma_{k_f}^{-2} \left\| E_{\gamma_{k_f}} x \right\| \\ &\le \gamma_{k_f}^{-1} \|A_K\| \left\| E_{\gamma_{k_f}} x \right\| \\ &\quad + L\sum_{i=1}^{m} \rho_i \frac{\left\| E_{\gamma_{k_f}} x \right\|^{r_i+\mu_i}}{\gamma_{k_f}^{q_i+n\mu_i}} \gamma_{k_f}^{-2} \left\| E_{\gamma_{k_f}} x \right\| \end{aligned} \tag{35}$$

Thus, from (33)-(35), it is obvious that $\left\| E_{\gamma_{k_f}} x \right\|_{L_\infty}$, $\left\| \frac{dE_{\gamma_{k_f}} x}{dt} \right\|_{L_\infty}$, and $\int_{t_{k_f}}^{t} \left\| E_{\gamma_{k_f}} x(\tau) \right\| d\tau$ are bounded as T_f becomes larger. Letting $T_f \to \infty$, this yields $E_{\gamma_{k_f}} x \to 0$ as $t\to\infty$ by the Lemma 7 [3]. Then, we have $x\to 0$ as $t\to\infty$. Therefore, the regulation is achieved.

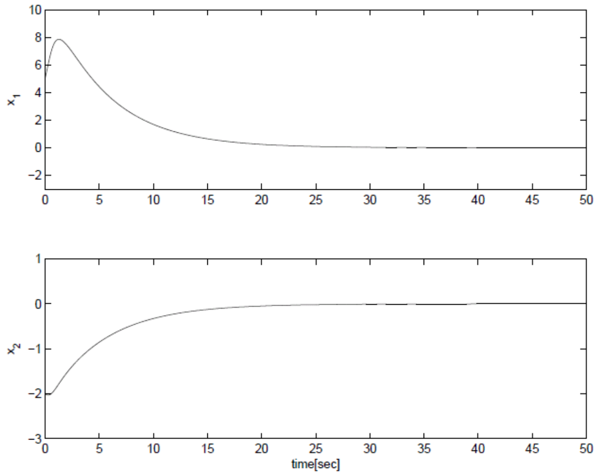
Remark 2: The inequalities (19)-(21) are defined for showing that $\theta_1(t)$ and $\theta_2(t)$ are finite after some point of the switching. Using these inequality, we show that switching occurs only finitely.

III. ILLUSTRATIVE EXAMPLES

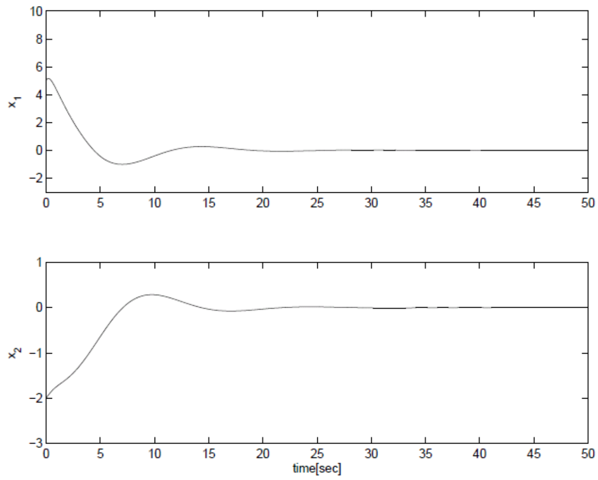
Example 1 (Known L)**:** We consider the following system as

$$\begin{aligned} \dot{x}_1 &= x_2 + (x_1^2 + x_2^4)u^4 \\ \dot{x}_2 &= u + 2x_2^3 u^6 \end{aligned} \tag{36}$$

The system (36) contains some non-feedfoward terms. In fact, it is a neither triangular nor feedforward system. Thus, it cannot be regulated by the controllers of [1,6,8-10,13-16]. Meanwhile, since L is known, the method of [7] is applicable. From $\left\| E_{\gamma(t)}\delta(t,x,u) \right\|_1 \le L(|x_1|^2|u|^3 + |x_2|^4|u|^3 + \gamma(t)|x_2|^3|u|^5)\ \ |u|$ with $L=3$, the uncertain nonlinearity satisfies Assumption 1. By Assumptions 1 and 2, we can obtain that $\phi_1(x,u,\gamma(t)) = |x_1|^2|u|^3$, $\phi_2(x,u,\gamma(t)) = |x_2|^4|u|^3$, and $\phi_3(x,u,\gamma(t)) = \gamma(t)|x_2|^3|u|^5$. Then, Assumption 3 is satisfied by taking $r_1=2$, $\mu_1=3$, $q_1=0$, $r_2=4$, $\mu_2=3$, $q_2=4$, $r_3=3$, $\mu_3=5$, and $q_3=2$. We set the parameters as $K=[-16,-8]$, $\gamma_0=10$, $S_0=1$, $\alpha=2$ and $c=0.8$ with the initial states $[x_1(0), x_2(0)]^T=[5,-2]^T$. Then, we apply the proposed adaptive switching controller to the system (36). In Figs. 1(a) and 1(b), it is shown that both methods yield control results which are not much different from each other in the performance standpoint.



(a) The proposed method.



(b) The method of [7].

그림 1. (a) 제어된 시스템 (36)의 상태 궤적: (a) 제안한 방법과 (b) [7]의 방법.

Fig. 1. State trajectories of the controlled system (36): (a) the proposed method and (b) the method of [7].

Example 2 (Unknown L)**:** We reconsider *Example C* with additional uncertainties as

$$\begin{aligned} \dot{x}_1 &= x_2 + \eta_1(t)(x_3^2 + x_1^{\frac{1}{5}} x_2^2 u^3 \sin x_1) \\ \dot{x}_2 &= x_3 + \eta_2(t) x_2^{\frac{1}{3}} u^4 \\ \dot{x}_3 &= u + \eta_3(t) x_1 u^4 \end{aligned} \tag{37}$$

where $\eta_1(t)$, $\eta_2(t)$, and $\eta_3(t)$ are only known to be finite. Similar to Example 1, from structural viewpoint, the results of [1], [6,9,10,12-14] are not applicable. Since $\eta_1(t)$, $\eta_2(t)$, and $\eta_3(t)$ are unknown, the method of [7] is not applicable. It is certain that $\eta_1(t)\le L_1$, $\eta_2(t)\le L_2$, and $\eta_3(t)\le L_3$ where $L_1,L_2,L_3>0$ due to the boundedness of $\eta_1(t)$, $\eta_2(t)$, and $\eta_3(t)$. Then, from $\left\| E_{\gamma(t)}\delta(t,x,u) \right\|_1 \le L(|x_3| + \gamma(t)^{-1}|x_1|^{\frac{1}{5}}|x_2|^2|u|^2 + |x_2|^{\frac{1}{3}}|u| + \gamma(t)|x_1||u|^3)(|x_3| + \gamma(t)|u|)$ with $L\ge L_1+L_2+L_3$, the uncertain nonlinearity satisfies Assumption 1. The remaining parts with Assumptions 2 and 3 are shown in *Example C*, thus it is not

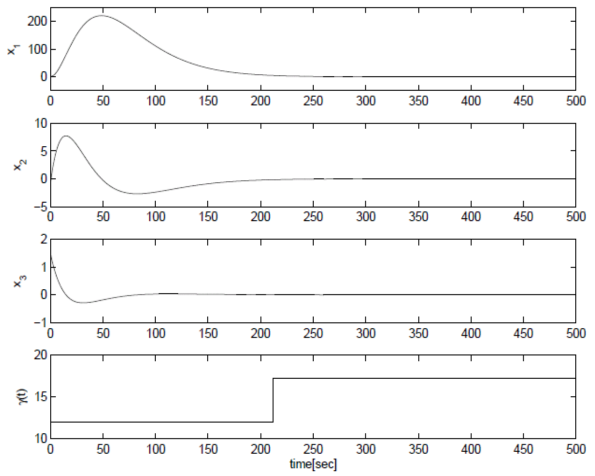


그림 2. 제어된 시스템 (37)의 상태궤적과 $\gamma(t)$의 변화.

Fig. 2. State trajectories of the controlled system (37) and evolution of $\gamma(t)$.

repeated here. We set the parameters as $K=\left[-\frac{1}{8},-\frac{3}{4},-\frac{3}{2}\right]$, $\gamma_0=10$, $S_0=1$, $\alpha=1.2$ and $c=0.8$ with the initial states $[x_1(0),x_2(0),x_3(0)]^T=[2,-1,1.5]^T$. For the simulation, we set $\eta_1(t)=\sin t$, $\eta_2(t)=2\cos t$, and $\eta_3(t)=1$. Then, we apply the proposed controller for the system (37). It is shown in Fig. 2 that the system is regulated by our controller without a priori knowledge on the growth rate of the nonlinearity. From the switching logic, there exists the moment that $\theta_1(t)\ge m\gamma_k^c$ or $\theta_2(t)>k\gamma_k^2\left\|E_{\gamma_k}x(t_k)\right\|$ is satisfied.

IV. CONCLUSION

We have presented a switching-based state feedback controller for a class of nonlinear systems with uncertain nonlinearity. The considered nonlinearity is not restricted to feedforward forms, but it is largely extended by a function containing the full states and the input. As a result, some nontriangular and nonfeedforward systems are shown to be included. Moreover, the proposed control scheme has an adaptive function such that the growth rate of nonlinearity is not needed to be known in the controller design. Through the examples, we show the improved and generalized features of our result over the existing ones.

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