

# 동적 과점시장의 관리제어: 기업들은 어떻게 이윤극대화에 이를 수 있는가?

## Supervisory Control of Dynamic Oligopolistic Markets: How can Firms Reach Profit-Maximization?

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**Abstract:** In an oligopolistic market, only a few firms account for most or all of total production, e.g., automobile, steel, and computer industries. For a dynamic oligopolistic market with two firms competing in quantities, we show that supervisory control theory of discrete event systems provides a novel approach to solve the dynamic oligopoly problem with the aim of maximizing the profits of both firms. Specifically, we show that the controllability, observability, and nonblocking property (which are the core concepts in supervisory control theory) are the necessary and sufficient conditions for two oligopolistic firms in disequilibrium to eventually reach equilibrium states of maximizing the profits of both firms.

**Keywords:** discrete event systems, supervisory control, dynamic oligopoly, finite state automata

### I. INTRODUCTION

In an oligopolistic market, only a few firms account for most or all of total production. Oligopoly is a prevalent form of market structure. Examples of oligopolistic industries include automobiles, steel, and computers. In this paper, we study an oligopolistic market with a homogenous good and competition in quantities. One representative model of such oligopolistic markets is the Cournot model consisting of two firms which must decide how much to produce to maximize their profits [4]. In a Cournot equilibrium, each firm is producing an amount that maximizes its profit given what its competitor is producing, so neither would want to change its output. Thus the Cournot equilibrium is an example of Nash equilibrium. When two firms are initially producing output levels that differ from the Cournot equilibrium, is it always possible for the firms to reach the equilibrium eventually?; how do they adjust their outputs until the equilibrium is reached? These dynamic oligopoly problems have been studied by many economists [1,6,11] based on the assumption that each firm exactly knows the last strategies of its rival firms and also market demand.

In this paper, we study the dynamic oligopoly problem using supervisory control theory of discrete event systems (DESS) [10] modelled by finite state automata. In [8], it has been shown that a competitive market can be modelled as a finite state automaton. A dynamic oligopolistic market also has event-driven dynamics, e.g., the state (outputs of firms and a price) of the market changes according to an event (action or decision) of a firm such as increasing output or decreasing it.

In general, such an event occurs irregularly. In this paper, we construct a finite state automaton describing the dynamic behavior of each oligopolistic firm. The model describes how a state (comprised of output and marginal cost) of a firm changes by the events of increasing output, decreasing output, and uncontrollable fluctuations of production cost by exogenous causes. Moreover we model the behavior of a market demand to describe how a state (comprised of total output and price) of the market demand changes by the events of increasing outputs and decreasing outputs of firms, and uncontrollable demand fluctuations by exogenous causes. From these component models, we construct a composite model of the market which describes overall behaviors of firms and market demand. Based on the model, this paper presents the controllability, observability, and nonblocking property to assure that an oligopolistic market in disequilibrium can always reach an equilibrium in the presence of uncontrollable fluctuations of production cost and market demand caused by exogenous factors.

The major differences between our approach and conventional approaches by economists are as follows. (1) Conventional differential or difference equation models do not effectively capture uncontrollable changes in production cost and market demand caused by exogenous factors. They usually deal with fixed situations in which marginal costs and market demands are given as fixed (may be constants or linear curves). However, discrete event models presented in this paper are suitable for modeling such uncontrollable changes of an oligopolistic market. (2) Conventional works usually assume that each firm exactly knows the last strategies of its rival firms and a market demand. However, this paper assumes that each firm does not exactly know its rivals' outputs and marginal costs. Many economists estimate a rival's past output through current information such as price. However,

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estimation is not always exact, and to exactly know a market demand seems not to be possible in real markets.

**II. MODELLING OF AN OLIGOPOLISTIC MARKET**

1. Modelling of an oligopolistic firm

In this paper, we consider an oligopolistic market which consists of two oligopolistic firms. Such a market is called a duopoly. The results presented in this paper can be generalized to more than two firms. First, we model the behavior of each oligopolistic firm  $i$  ( $i = 1, 2$ ) as a FSA (finite state automaton)

$$F_i = (X_{F_i}, \Sigma_{F_i}, \delta_{F_i}, x_{F_i}^0)$$

of which the state transition diagram is shown in Fig. 1.

$X_{F_i} = MC \times Q$  is a set of finite states in which  $MC$  is a set of discrete marginal cost values and  $Q$  is a set of discrete output values of a good.  $\Sigma_{F_i}$  is a set of events of which descriptions are shown in Table 1.  $\delta_{F_i}$  is a state transition function defined as  $\delta_{F_i} : X_{F_i} \times \Sigma_{F_i} \rightarrow X_{F_i}$ , and  $x_{F_i}^0 (\in X_{F_i})$  is an initial state.

A state  $(MC_{i,m,n}, Q_{i,m}^f)$  of  $F_i$  means that the marginal cost of firm  $i$  at the quantity  $Q_{i,m}^f$  is  $MC_{i,m,n}$ . At  $(MC_{i,m,n}, Q_{i,m}^f)$ , when firm  $i$  increases the output from  $Q_{i,m}^f$  to  $Q_{i,m+1}^f$  ( $in\_out\_i$ ), the state of  $F_i$  changes into  $(MC_{i,m+1,n}, Q_{i,m+1}^f)$  at which firm  $i$  can produce the quantity  $Q_{i,m+1}^f$  with the marginal cost  $MC_{i,m+1,n}$ . If the firm is dominated by the law

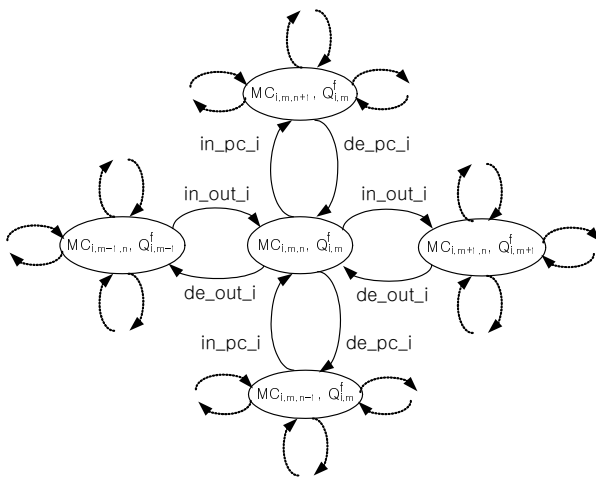


그림 1. 과점 기업  $i$  동작의 유한상태 오토마톤  $F_i$ .

Fig. 1. FSA  $F_i$  of an oligopolistic firm  $i$ 's behavior.

표 1. 유한상태 오토마톤  $F_i$ 의 사건.

Table 1. The events of the FSA  $F_i$ .

Event	Description
$in\_pc\_i$	Increase in production cost in firm $i$ .
$de\_pc\_i$	Decrease in production cost in firm $i$ .
$in\_out\_i$	Increase in output in firm $i$ .
$de\_out\_i$	Decrease in output in firm $i$ .

of diminishing returns, then as the output increases, the marginal cost also increases, i.e.,  $MC_{i,m,n} < MC_{i,m+1,n}$ .

Conversely, if the firm is dominated by the law of increasing returns, then the marginal cost decreases as the output increases. At the state  $(MC_{i,m,n}, Q_{i,m}^f)$ , when firm  $i$  decreases the output from  $Q_{i,m}^f$  to  $Q_{i,m-1}^f$  ( $de\_out\_i$ ), the state of  $F_i$  changes into a new state  $(MC_{i,m-1,n}, Q_{i,m-1}^f)$  at which firm  $i$  can produce the quantity  $Q_{i,m-1}^f$  with the marginal cost  $MC_{i,m-1,n}$ .

At a fixed output, when some external factors cause the cost of firm  $i$  to decline, the firm can produce the output with a reduced marginal cost. It is modelled in  $F_i$  as the state transition from  $(MC_{i,m,n}, Q_{i,m}^f)$  to  $(MC_{i,m,n-1}, Q_{i,m}^f)$  by the event  $de\_pc\_i$  where  $MC_{i,m,n} > MC_{i,m,n-1}$ . Conversely, as the cost of production rises by other external factors, the marginal cost rises into  $MC_{i,m,n+1} (> MC_{i,m,n})$  at the fixed output  $Q_{i,m}^f$  ( $in\_pc\_i$ ). Because the events  $in\_pc\_i$  and  $de\_pc\_i$  are caused by exogenous factors, they are uncontrollable, i.e., firm  $i$  cannot prevent the events from occurring in the market.

**Remark 1:** In this paper, we model the behavior of a firm as a deterministic FSA. It would be possible to model it as a nondeterministic FSA. For example, it would be possible that  $\delta_{F_i}((10,50), in\_pc\_i) = \{(20,50), (30,50)\}$ , i.e., the marginal cost at  $(10, 50)$  may rise to 20 or 30 by  $in\_pc\_i$ . However, we can model it as a deterministic FSA as follows:  $\delta_{F_i}((10,50), in\_pc\_i) = (20,50)$  and  $\delta_{F_i}((20,50), in\_pc\_i) = (30,50)$ . That is, if we can define the event  $in\_pc\_i$  as an increase by unit cost (10 in this example), then it is possible to model nondeterministic behaviors of a firm as a deterministic FSA. The same holds for other events.

2. Modelling of a market demand

A dynamic behavior of a market demand in an oligopolistic market can be modeled as the FSA

$$C = (X_c, \Sigma_c, \delta_c, x_c^0)$$

of which the state transition diagram is shown in Fig. 2.

$X_c$  is a set of finite states defined as  $X_c = P \times Q$  in which  $P$  is a set of discrete price values and  $Q$  is a set of discrete quantity values of a good.  $\Sigma_c$  is a set of events of which descriptions are shown in Table 2.  $\delta_c$  is a state transition function defined as  $\delta_c : X_c \times \Sigma_c \rightarrow X_c$ , and  $x_c^0 (\in X_c)$  is an initial state. A state  $(P_{m,n}, Q_m^c)$  of the FSA means that the market price of a good is  $P_{m,n}$  when the total production of two oligopolistic firms is  $Q_m^c$ . At the state  $(P_{m,n}, Q_m^c)$ , when the output of firm  $i$  increases ( $in\_out\_i$ ), the state of a market demand  $C$  changes into a new state  $(P_{m+1,n}, Q_{m+1}^c)$  at which the

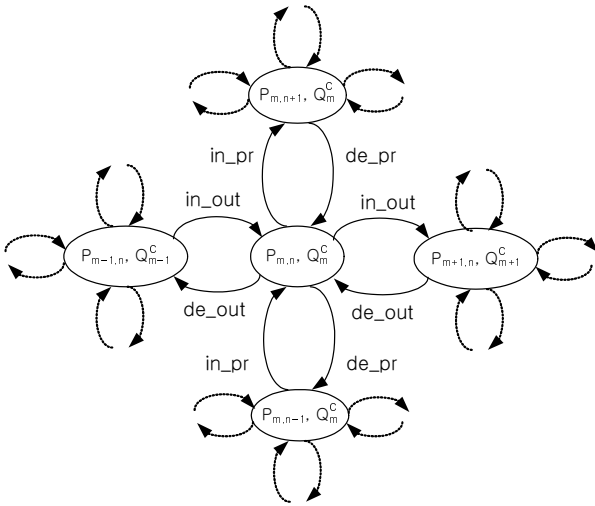


그림 2. 시장의 유한상태 오토마톤 C.  
Fig. 2. FSA C of a market.

표 2. 유한상태 오토마톤 C의 사건.

Table 2. The events of the FSA C.

Event	Description
<i>in_out</i>	<i>in_out_1</i> , <i>in_out_2</i> .
<i>de_out</i>	<i>de_out_1</i> , <i>de_out_2</i> .
<i>in_pr</i>	Increase in price.
<i>de_pr</i>	Decrease in price.

increased production of  $Q_{m+1}^c$  is sold at the price  $P_{m+1,n}$ . On the contrary, at the state  $(P_{m,n}, Q_m^c)$ , when the output of firm  $i$  decreases ( $de\_out\_i$ ), the state of  $C$  changes into a new state  $(P_{m-1,n}, Q_{m-1}^c)$  at which the decreased production of  $Q_{m-1}^c$  is sold at the price  $P_{m-1,n}$ .

According to the characteristics of goods, the behaviors of market demands may become different. Normally, they follow the law of demand, i.e., the price rises as the quantity supplied falls and falls as the quantity supplied rises. That is, as the quantity supplied increases from  $Q_m^c$  into  $Q_{m+1}^c$ , the price decreases, i.e.,  $P_{m+1,n} < P_{m,n}$ . Conversely, the price rises into  $P_{m-1,n} (> P_{m,n})$  as the market supply is reduced from  $Q_m^c$  into  $Q_{m-1}^c$  ( $de\_out\_i$ ). In contrast, market demands may sometimes violate the law of demand, i.e., the price rises as the quantity supplied rises, e.g. Giffen goods. For such goods, as the quantity supplied rises from  $Q_m^c$  into  $Q_{m+1}^c$ , the price also increases, i.e.,  $P_{m,n} < P_{m+1,n}$ .

The FSA  $C$  of a market demand also models the dynamic behaviors that some external factors may cause the market price to increase or decrease in a fixed quantity of supply. It is modelled as the transition from  $(P_{m,n}, Q_m^c)$  to  $(P_{m,n+1}, Q_m^c)$  by the event  $in\_pr$  (increase in price) and the state transition from  $(P_{m,n}, Q_m^c)$  to  $(P_{m,n-1}, Q_m^c)$  by the event  $de\_pr$  (decrease in price). In these states, the total production of two firms is  $Q_m^c$ , but the

price is changed from  $P_{m,n}$  to  $P_{m,n+1}$  with the relation of  $P_{m,n} < P_{m,n+1}$  by the event  $in\_pr$ , and conversely the price falls into  $P_{m,n-1} (< P_{m,n})$  by the event  $de\_pr$ . Because the events  $in\_pr$  and  $de\_pr$  are caused by exogenous factors, they are uncontrollable, i.e., two firms cannot prevent the events from occurring in the market.

### 3. Modelling of an oligopolistic market

In the previous sections, we have modeled individual behaviors of two oligopolistic firms and a market demand. Now it is time to find a composite model for concurrent behaviors of them. It will be a complete model of the market which fully represents the dynamics of the market.

To construct a composite model from individual component models, we employ the parallel composition for automata [3]. Given two FSAs  $G_i = (X_i, \Sigma_i, \delta_i, x_{i,0})$  ( $i=1,2$ ), the parallel composition of  $G_1$  and  $G_2$  is defined as follows:

$$G_1 \parallel G_2 := (X_1 \times X_2, \Sigma_1 \cup \Sigma_2, \delta, (x_{1,0}, x_{2,0})),$$

where  $\delta : (X_1 \times X_2) \times (\Sigma_1 \cup \Sigma_2) \rightarrow (X_1 \times X_2)$  is defined as

$$\delta((x_1, x_2), \sigma) := \begin{cases} (\delta_1(x_1, \sigma), \delta_2(x_2, \sigma)) & \text{if } \delta_1(x_1, \sigma)! \text{ and } \delta_2(x_2, \sigma)!, \\ (\delta_1(x_1, \sigma), x_2) & \text{if } \delta_1(x_1, \sigma)! \text{ and } \sigma \notin \Sigma_2, \\ (x_1, \delta_2(x_2, \sigma)) & \text{if } \sigma \notin \Sigma_1 \text{ and } \delta_2(x_2, \sigma)!, \\ \text{undefined} & \text{otherwise.} \end{cases}$$

The notation ! denotes ‘is defined’. In the parallel composition, a common event, i.e., an event in  $\Sigma_1 \cap \Sigma_2$ , can only occur if the two automata both execute it simultaneously. Thus, the two automata are synchronized on the common events. The private events, that is, those in  $(\Sigma_1 \setminus \Sigma_2) \cup (\Sigma_2 \setminus \Sigma_1)$ , are not subject to such a constraint and can occur whenever possible. In this kind of interconnection, a component can execute its private events without the participation of the other component; however, a common event can only happen if both components can execute it.

From the FSAs  $F_1$  and  $F_2$  of two oligopolistic firms and the FSA  $C$  of the market demand, the overall dynamic behavior of the oligopolistic market is modelled as a FSA

$$M := (F_1 \parallel F_2 \parallel C, X_M^m) = (X_M, \Sigma_M, \delta_M, x_M^0, X_M^m).$$

Then a state  $x \in X_M$  can be represented as follows:

$$x = ((MC_1, Q_1), (MC_2, Q_2), (P, Q)),$$

where  $(MC_i, Q_i)$  ( $i=1,2$ ) is a state of firm  $i$ 's FSA  $F_i$ , and  $(P, Q)$  is a state of the market demand FSA  $C$ . For a state  $x = ((MC_1, Q_1), (MC_2, Q_2), (P, Q))$ , let us define  $Q^i(x) = Q_i$ ,  $Q^c(x) = Q$ , and  $P(x) = P$  be the output of firm  $i$ , the market demand, and the price at the state  $x$ , respectively. We assume

that  $Q^{f1}(x) + Q^{f2}(x) = Q^c(x)$ , i.e., the total output of two firms at a state  $x$  equals the market demand at the state. The composite model  $M$  of an oligopolistic market represents all possible dynamics of the market which include both profit-maximizing behaviors and non-profit-maximizing behaviors. Also, it includes all adjustment processes that two firms can do in order to reach their profit-maximizing states.

From Fig. 1 and Fig. 2, it follows that  $\Sigma_{F_1} \cap \Sigma_{F_2} = \emptyset$  and  $\Sigma_{F_i} \cap \Sigma_C = \{in\_out\_i, de\_out\_i\} (i=1,2)$ . In other words, two oligopolistic firms are not synchronized each other, but they are synchronized with the market demand  $C$  on the common events  $in\_out\_i$  and  $de\_out\_i$ .

In the FSA  $M$ ,  $X_M^m$  is a set of marked states which are defined as follows:

**Definition 1:** A state  $x \in X_M$  is marked, i.e.,  $x \in X_M^m$ , if  $MC_1(x) = MR_1(x)$  and  $MC_2(x) = MR_2(x)$  in which  $MC_i(x)$  and  $MR_i(x)$  denote the marginal cost and the marginal revenue of firm  $i$  at the state  $x$ , respectively.

That is, a marked state means that both firms maximize their profits at the state. Two firms do not have any incentives to change their outputs at the state since they acquire maximum profits at the state. Thus, a marked state is a Nash equilibrium.

At a state  $x \in X_M$ , the marginal revenue  $MR_i(x)$  of firm  $i$  can be computed in practice as follows [9]:

$$MR_i(x) = P(x) + P(x)(1/E_i(x)),$$

where  $E_i(x)$  is the price elasticity of demand of firm  $i$  which is computed as the percentage change in quantity demanded divided by the percentage change in price. The price elasticity of demand is generally a negative number since the quantity demanded usually falls as the price of a good increases. For two states  $x, x' \in X_M$ , let  $\delta_M(x', in\_out\_i) = x$ . Then, the price elasticity at the state  $x$  of firm  $i$  can be defined as follows:

$$E_i(x) := \frac{\Delta Q^i(x)/Q^i(x)}{\Delta P(x)/P(x)} = \frac{(Q^i(x) - Q^i(x'))/Q^i(x)}{(P(x) - P(x'))/P(x)},$$

### III. SUPERVISORY CONTROL OF AN OLIGOPOLISTIC MARKET

To develop the supervisory control framework for an oligopolistic market, first we categorize the event set  $\Sigma_M$  of the market FSA  $M$  into

$$\Sigma_M = \Sigma_1^c \cup \Sigma_1^{uc} = \Sigma_2^c \cup \Sigma_2^{uc},$$

where  $\Sigma_i^c (i=1,2)$  is a set of controllable events that are enabled or disabled by firm  $i$ , and  $\Sigma_i^{uc}$  is a set of uncontrollable events that cannot be disabled by firm  $i$ . Since two oligopolistic firms in this paper compete in quantities, firm  $i$  can control only the increase ( $in\_out\_i$ ) and decrease ( $de\_out\_i$ ) of its output. Firm  $i$  cannot prevent other events from occurring in the market. For example, firm 1 cannot

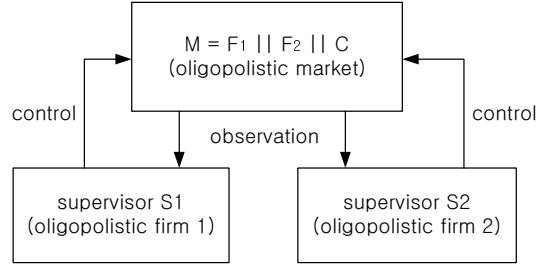


그림 3. 과점시장에 대한 제어 구조.

Fig. 3. Control structure for an oligopolistic market.

prevent firm 2 from increasing its output ( $in\_out\_2$ ), and it cannot prevent the market price from being down ( $de\_pr$ ). In addition, since we assume that the increase ( $in\_pc\_i$ ) and decrease ( $de\_pc\_i$ ) in production costs are caused by exogenous factors, these events also cannot be controlled by firm  $i$ . For example, the change in the price of some material (an input into making the oligopolistic good) may not be controlled by firms, but it may affect the cost of production. Thus, we can specify  $\Sigma_i^c$  and  $\Sigma_i^{uc}$  as follows:

$$\Sigma_i^c = \{in\_out\_i, de\_out\_i\}, \Sigma_i^{uc} = \Sigma_M \setminus \Sigma_i^c.$$

To achieve profit maximization, each oligopolistic firm decides whether it will increase or decrease the quantity of output upon the observation of a market's state. In this respect, each firm can be regarded as a supervisor (controller) with the control objective of profit maximization. The control structure for an oligopolistic market can be described as shown in Fig. 3.

Formally, the control action of an oligopolistic firm  $i$  is represented by an oligopoly supervisor  $S_i$  which is defined as a state-based feedback control map

$$S_i : \Phi_i[X_M] \rightarrow 2^{\Sigma_M},$$

where  $2^{\Sigma_M}$  denotes a power set of  $\Sigma_M$  and  $\Phi_i : X_M \rightarrow MC \times Q \times P$  is a projection mapping defined as follows: for a state  $x = ((MC_1, Q_1), (MC_2, Q_2), (P, Q)) \in X_M$ ,  $\Phi_1(x) = (MC_1, Q_1, P)$  and  $\Phi_2(x) = (MC_2, Q_2, P)$ . The projection means that when a market's state is  $x$ , firm  $i$  only observes  $\Phi_i(x)$ , i.e., its marginal cost  $MC_i$ , its output  $Q_i$ , and a market price  $P$  at the state. The firm cannot observe its competitor's marginal cost and output. Under the partial observation, the supervisor  $S_i$  issues a control command  $S_i(\Phi_i(x))$  which denotes a set of events to be enabled for next occurrence, i.e., an event  $\sigma \in \Sigma_M$  is enabled at  $x$  by  $S_i$  if  $\sigma \in S_i(\Phi_i(x))$ , and is disabled otherwise; it  $\sigma$  is always enabled if  $\sigma \in \Sigma_i^{uc}$ , i.e.,  $\Sigma_i^{uc} \subseteq S_i(\Phi_i(x))$ .

The controlled market by two oligopoly supervisors is then defined as

$$S_1 \wedge S_2 / M := (X_M^S, \Sigma_M^S, \delta_M^S, x_M^{S,0}, X_M^{S,m}),$$

where  $X_M^S \subseteq X_M$ ,  $\Sigma_M^S = \Sigma_M$ ,  $x_M^{S,0} = x_M^0$ ,  $X_M^{S,m} \subseteq X_M^m$ , and

$$\delta_M^S(x, \sigma) := \begin{cases} \delta_M(x, \sigma) & \text{if } \delta_M(x, \sigma)! \text{ and} \\ & \sigma \in S_1(\Phi_1(x)) \cap S_2(\Phi_2(x)), \\ \text{undefined} & \text{otherwise.} \end{cases}$$

It means that the controlled market at a state  $x$  reaches a new state  $\delta_M(x, \sigma)$  when the event  $\sigma$  is defined at the state  $x$  of the uncontrolled market  $M(\delta_M(x, \sigma)!)$  and additionally both the supervisors permit the event  $(\sigma \in S_1(\Phi_1(x)) \cap S_2(\Phi_2(x)))$ . The initial states of  $M$  and  $S_1 \wedge S_2/M$  are identical, and the state transitions of them are also identical for transitions permitted by the two supervisors. Thus the controlled market  $S_1 \wedge S_2/M$  is a part of the uncontrolled market  $M$ .

The aim of supervisory control is that a controlled system follows only desirable behaviors (i.e., control objective). While the uncontrolled market  $M$  includes both desirable behaviors and undesirable behaviors, the controlled market  $S_1 \wedge S_2/M$  should include only desirable behaviors. The control objective for an oligopolistic market is that the market in a non-marked state (i.e., disequilibrium state) should eventually reach a marked state (i.e., equilibrium state of maximizing the profits of both firms).

To solve this supervisory control problem, we represent the control objective using a FSA. First, we present the notion of sub-automaton of a market  $M$  as follows.

**Definition 2:** A FSA  $M' = (X_{M'}, \Sigma_{M'}, \delta_{M'}, x_{M'}^0, X_{M'}^m)$  is said to be a sub-automaton of FSA  $M = (X_M, \Sigma_M, \delta_M, x_M^0, X_M^m)$  if  $X_{M'} \subseteq X_M$ ,  $\Sigma_{M'} = \Sigma_M$ ,  $x_{M'}^0 = x_M^0$ ,  $X_{M'}^m \subseteq X_M^m$ , and for any  $x \in X_{M'}$  and  $\sigma \in \Sigma_M$  such that  $\delta_M(x, \sigma)$  is defined, the following condition is satisfied:  $\delta_{M'}(x, \sigma) = \delta_M(x, \sigma)$  or  $\delta_{M'}(x, \sigma)$  is not defined.

From this definition, it is apparent that the controlled market  $S_1 \wedge S_2/M$  is a sub-automaton of the uncontrolled market  $M$ . The control objective presented in this paper is then to achieve  $S_1 \wedge S_2/M = M'$  by two oligopoly supervisors  $S_1$  and  $S_2$  in which the sub-automaton  $M'$  is a model to specify the desirable behavior that the market in a disequilibrium state should eventually reach an equilibrium state.

In profit-non-maximizing states, firms generally make decisions in order to reach profit-maximizing states as follows: when marginal cost is larger than marginal revenue, they reduce outputs until the difference disappears, and when marginal revenue is larger than marginal cost, they raise outputs until the difference disappears. Accordingly, a sub-automaton  $M'$  representing the desirable behavior should follow this general decision manner of firms. For this purpose, the following notions are introduced.

**Definition 3:** A sub-automaton  $M'$  of an oligopolistic market  $M$  is convergent for firm  $i$  if

(1) for any state  $x_1 \in X_{M'}$  such that  $MR_i(x_1) < MC_i(x_1)$ , if there exists  $x_2 \in X_{M'}$  such that  $x_2 = \delta_{M'}(x_1, de\_out\_i)$ , then  $MC_i(x_2) - MR_i(x_2) < MC_i(x_1) - MR_i(x_1)$ , and  $\delta_{M'}(x_1, in\_out\_i)$  is not defined;

(2) for any state  $x_1 \in X_{M'}$  such that  $MR_i(x_1) > MC_i(x_1)$ , if there exists  $x_2 \in X_{M'}$  such that  $x_2 = \delta_{M'}(x_1, in\_out\_i)$ , then  $MR_i(x_2) - MC_i(x_2) < MR_i(x_1) - MC_i(x_1)$ , and  $\delta_{M'}(x_1, de\_out\_i)$  is not defined.

**Definition 4:** A sub-automaton  $M'$  of an oligopolistic market  $M$  is convergent if it is convergent for both firm 1 and firm 2.

The condition (1) means that when the marginal cost of firm  $i$  is larger than the marginal revenue at a state  $x_1$ , the decrease of output ( $de\_out\_i$ ) results in reducing the difference of marginal cost and marginal revenue. Additionally, the event of increasing output ( $in\_out\_i$ ) is not defined at the state  $x_1$ . The meaning of the condition (2) is likewise.

A convergent sub-automaton models only firms' individual profit-maximizing behaviors. In other words, firm 1 does not consider how the decision to maximize its profit may affect the profit of firm 2 in a convergent sub-automaton. Hence it is not generally guaranteed that a convergent sub-automaton always includes equilibrium states of maximizing the profits of both firms.

**Remark 2:** For a convergent sub-automaton  $M' = (X_{M'}, \Sigma_{M'}, \delta_{M'}, x_{M'}^0, X_{M'}^m)$ , if  $X_{M'}^m = \emptyset$  then  $M'$  does not include equilibrium states. It implies that even though two firms do the best to maximize their profits, they cannot achieve the goals in the sub-automaton.

The market FSA  $M$  is computed from the individual FSAs  $F_1, F_2$  and  $C$  by parallel composition of these automata, i.e.,  $M = (F_1 \parallel F_2 \parallel C, X_M^m)$ . With the computed FSA  $M$  and its initial state, we can obtain a convergent sub-automaton  $M'$  through simple searching of  $M$ , and thereby easily verify whether  $X_{M'}^m = \emptyset$  or not. This implies that through simple automata operations, we can answer to the question 'does there exist an equilibrium in the oligopolist market in which each firm does the best to maximize its profit in the absence of the exact information of its rival's cost and output?'

The next question to be addressed in this paper is that when a convergent sub-automaton has equilibrium states, is it assured that two firms in disequilibrium eventually reach equilibrium states in the sub-automaton? In other words, when each firm in a convergent sub-automaton does the best to reach its profit maximization, isn't it possible for the market to reach deadlock or livelock states? A deadlock is a state at which no further event cannot occur, and hence there is no transition path from the state to marked states. Such a deadlock may be reachable when after destructive competition of firms, one firm shuts down or exits the market and the other firm achieves its profit maximization. At the state, the shut-downed firm has nothing to do, and the other firm also has no incentive to change its output since it already achieved profit maximization. A livelock means a set of unmarked states that forms a strongly connected component, but with no transition going out of the set [3]. Such a livelock may be reachable when two firms infinitely repeat the increase and decrease of their outputs to maximize profits, but cannot reach equilibrium states. To deal with these issues, the following notions are

introduced.

**Definition 5:** A sub-automaton  $M'$  of an oligopolistic market  $M$  is nonblocking if for any state  $x \in X_{M'}$ , there exists at least one path of transitions from the state to a marked state of  $X_{M'}$ .

**Definition 6:** Two oligopoly supervisors  $S_1$  and  $S_2$  are nonblocking for an oligopolistic market  $M$  if  $S_1 \wedge S_2 / M$  is nonblocking.

Now it is time to present the final issue that for a convergent sub-automaton  $M'$  with equilibrium states, does the nonblocking property of the sub-automaton guarantee the existence of two nonblocking supervisors  $S_1$  and  $S_2$  satisfying  $S_1 \wedge S_2 / M = M'$ ? To achieve this control objective, the controllability and observability of the sub-automaton  $M'$  are required as follows.

**Definition 7:** Let  $\Sigma^{uc} := \Sigma_M \setminus (\Sigma_1^c \cup \Sigma_2^c)$ . A sub-automaton  $M'$  of an automaton  $M$  is controllable if for any  $x \in X_{M'}$ ,  $\Sigma_{M'}(x) \cap \Sigma^{uc} = \Sigma_M(x) \cap \Sigma^{uc}$ , where  $\Sigma_{M'}(x) := \{\sigma \in \Sigma_{M'} \mid \delta_{M'}(x, \sigma) \text{ is defined}\}$  and  $\Sigma_M(x) := \{\sigma \in \Sigma_M \mid \delta_M(x, \sigma) \text{ is defined}\}$ .

Since  $M'$  is a sub-automaton of  $M$ , it holds that  $\Sigma_{M'}(x) \subseteq \Sigma_M(x)$ , i.e., any event defined at a state  $x$  in  $M'$  is also defined at the state in  $M$ . Thus the controllability implies that if an uncontrollable event  $\sigma \in \Sigma^{uc}$  is defined at a state  $x$  in  $M$  ( $\sigma \in \Sigma_M(x) \cap \Sigma^{uc}$ ) but it is not defined at the state  $x$  in  $M'$  ( $\sigma \notin \Sigma_{M'}(x) \cap \Sigma^{uc}$ ), then  $M'$  is not controllable. In other words, it means that all uncontrollable events defined at a state in the automaton  $M$  should be also defined at the state in the sub-automaton  $M'$ .

**Definition 8:** A sub-automaton  $M'$  of an automaton  $M$  is observable if for any  $i \in \{1, 2\}$ ,  $x_1, x_2 \in X_{M'}$ , and  $\sigma \in \Sigma_i^c$ ,

$$[\Phi_i(x_1) = \Phi_i(x_2)] \wedge [\delta_{M'}(x_1, \sigma)!] \wedge [\delta_{M'}(x_2, \sigma)!] \Rightarrow \delta_{M'}(x_2, \sigma)!$$

When two states  $x_1$  and  $x_2$  are legal (i.e.,  $x_1, x_2 \in X_{M'}$ ) and an oligopolistic firm  $i$  cannot distinguish them (i.e.,  $\Phi_i(x_1) = \Phi_i(x_2)$ ), the observability requires that if a new state reached by a controllable event  $\sigma$  from  $x_1$  is legal (i.e.,  $\delta_{M'}(x_1, \sigma)$  is defined), then a new state reached by the event  $\sigma$  from  $x_2$  should be also legal (i.e.,  $\delta_{M'}(x_2, \sigma)$  is defined).

**Proposition 1:** For a sub-automaton  $M'$  of an oligopolistic market  $M$ , there exist two oligopoly supervisors  $S_1$  and  $S_2$  such that  $S_1 \wedge S_2 / M = M'$  if and only if  $M'$  is controllable and observable.

**Proof: (IF)** Let us consider two oligopoly supervisors  $S_i: \Phi_i[X_M] \rightarrow 2^{\Sigma^M}$  ( $i=1,2$ ) defined as follows: for any  $x \in X_M$ ,

$$S_i(\Phi_i(x)) = \{\sigma \in \Sigma_i^c \mid \delta_{M'}(x', \sigma) \text{ is defined for some } x' \in X_{M'} \text{ such that } \Phi_i(x) = \Phi_i(x')\} \cup \Sigma_i^{uc}.$$

We now prove that  $S_1 \wedge S_2 / M = M'$ . The proof is by induction on the state transition in the two automata.

• The base case is for initial states. Since  $S_1 \wedge S_2 / M$  and  $M'$  are the sub-automata of  $M$ , it holds that  $x_M^{S,0} = x_{M'}^0 = x_M^0$ . Thus the base case holds.

• Suppose that  $x \in X_M^S \cap X_{M'}$  for a state  $x \in X_{M'}$ . Then we prove that for any  $\sigma \in \Sigma_M$ ,  $\delta_M^S(x, \sigma) = \delta_{M'}(x, \sigma)$ .

- Let  $\delta_{M'}(x, \sigma)$  be defined. Then, since  $M'$  is a sub-automaton of  $M$ , it holds that  $\delta_{M'}(x, \sigma) = \delta_M(x, \sigma)$ , and the following three cases can be considered.

(1)  $\sigma \in \Sigma^{uc}$ : Since  $\Sigma_i^{uc} \subseteq S_i(\Phi_i(x))$ , it holds that  $\sigma \in S_1(\Phi_1(x)) \cap S_2(\Phi_2(x))$ , which implies  $\delta_M^S(x, \sigma) = \delta_{M'}(x, \sigma)$  by the definition of  $\delta_M^S$ . Thus,  $\delta_M^S(x, \sigma) = \delta_{M'}(x, \sigma)$ . (2)  $\sigma \in \Sigma_1^c$ : Since  $\Sigma_1^c \cap \Sigma_2^c = \emptyset$ , it holds that  $\sigma \in \Sigma_2^{uc}$ . Also since  $\sigma \in S_1(\Phi_1(x))$  from the above definition of  $S_i$ , it holds that  $\sigma \in S_1(\Phi_1(x)) \cap S_2(\Phi_2(x))$ , which implies  $\delta_M^S(x, \sigma) = \delta_{M'}(x, \sigma)$  by the definition of  $\delta_M^S$ . Thus,  $\delta_M^S(x, \sigma) = \delta_{M'}(x, \sigma)$ . (3)  $\sigma \in \Sigma_2^c$ : Analogous to the case (2).

- Let  $\delta_M^S(x, \sigma)$  be defined. Then since  $S_1 \wedge S_2 / M$  is a sub-automaton, it holds that  $\delta_M^S(x, \sigma) = \delta_{M'}(x, \sigma)$ , and  $\sigma \in S_1(\Phi_1(x)) \cap S_2(\Phi_2(x))$  according to the definition of  $\delta_M^S$ . Then we can consider the following three cases.

(1)  $\sigma \in \Sigma^{uc}$ : Since  $M'$  is controllable, it holds that  $\Sigma_{M'}(x) \cap \Sigma^{uc} = \Sigma_M(x) \cap \Sigma^{uc}$ , and hence  $\delta_{M'}(x, \sigma)$  is defined. Thus it holds that  $\delta_{M'}(x, \sigma) = \delta_M(x, \sigma) = \delta_M^S(x, \sigma)$  since  $S_1 \wedge S_2 / M$  and  $M'$  are the sub-automata of  $M$ . (2)  $\sigma \in \Sigma_1^c$ : Since  $\delta_M^S(x, \sigma)$  is defined, it holds that  $\sigma \in S_1(\Phi_1(x)) \cap S_2(\Phi_2(x))$ , which implies  $\sigma \in S_1(\Phi_1(x))$ . According to the definition of  $S_i$ , there exists some  $x' \in X_{M'}$  such that  $\delta_{M'}(x', \sigma)$  is defined and  $\Phi_i(x) = \Phi_i(x')$ . Then, by the observability of  $M'$ , it follows that  $\delta_{M'}(x, \sigma)$  is also defined. Thus it holds that  $\delta_{M'}(x, \sigma) = \delta_M(x, \sigma) = \delta_M^S(x, \sigma)$ . (3)  $\sigma \in \Sigma_2^c$ : Analogous to the case (2).

This completes the proof of the induction step.

**(ONLY IF)** Let  $S_1$  and  $S_2$  satisfy  $S_1 \wedge S_2 / M = M'$ . Then for any  $x \in X_{M'}$ , it holds that  $\Sigma_M(x) \cap \Sigma^{uc} \subseteq \Sigma_{S_1 \wedge S_2 / M}(x)$  since  $\Sigma^{uc} \subseteq S_1(\Phi_1(x)) \cap S_2(\Phi_2(x))$ . Then, because  $S_1 \wedge S_2 / M = M'$ , it follows that  $\Sigma_M(x) \cap \Sigma^{uc} \subseteq \Sigma_{M'}(x) \cap \Sigma^{uc}$ . Also, since  $M'$  is a sub-automaton of  $M$ , it is true that  $\Sigma_{M'}(x) \subseteq \Sigma_M(x)$  and then  $\Sigma_{M'}(x) \cap \Sigma^{uc} \subseteq \Sigma_M(x) \cap \Sigma^{uc}$ . Thus we have shown that  $\Sigma_{M'}(x) \cap \Sigma^{uc} = \Sigma_M(x) \cap \Sigma^{uc}$  which is the controllability condition.

Let some  $x_1, x_2 \in X_{M'}$  satisfy  $\Phi_i(x_1) = \Phi_i(x_2)$  for some  $i \in \{1, 2\}$ , and assume that  $\delta_{M'}(x_1, \sigma)!$ ,  $\delta_{M'}(x_2, \sigma)!$ , but  $\delta_{M'}(x_2, \sigma)$  is not defined for some  $\sigma \in \Sigma_i^c$ . That is,  $M'$  is

not observable. Then, it follows from  $S_1 \wedge S_2/M = M'$  that  $\sigma \in S_1(\Phi_1(x_1)) \cap S_2(\Phi_2(x_1))$ . Since  $\Phi_1(x_1) = \Phi_1(x_2)$  and  $x_2 \in X_{M'}$ , it holds that  $\sigma \in S_1(\Phi_1(x_2)) \cap S_2(\Phi_2(x_2))$ , which implies that  $\delta_{M'}^S(x_2, \sigma)$  is defined. Then it follows from  $S_1 \wedge S_2/M = M'$  that  $\delta_{M'}(x_2, \sigma)$  is also defined, which is a contradiction. ■

The main result of this paper is presented as follows.

**Theorem 1:** For a sub-automaton  $M' = (X_{M'}, \Sigma_{M'}, \delta_{M'}, x_{M'}^0, X_{M'}^m)$  of an oligopolistic market  $M = (X_M, \Sigma_M, \delta_M, x_M^0, X_M^m)$ , suppose that  $M'$  is convergent and  $X_{M'}^m \neq \emptyset$ . Then, there exist nonblocking oligopoly supervisors  $S_1$  and  $S_2$  such that  $S_1 \wedge S_2/M = M'$  if and only if  $M'$  is controllable, observable, and nonblocking.

**Proof: (IF)** According to Proposition 1, it follows from the controllability and observability of  $M'$  that the oligopoly supervisors  $S_i(\Phi_i(x)) = \{\sigma \in \Sigma_i^c \mid \delta_{M'}(x', \sigma) \text{ is defined for some } x' \in X_{M'} \text{ such that } \Phi_i(x) = \Phi_i(x')\} \cup \Sigma_i^{nc}$  for  $i=1,2$  achieve  $S_1 \wedge S_2/M = M'$ . Since  $M'$  is convergent, the supervisors can be designed as follows: for any  $x \in X_{M'}$ ,

$$S_i(\Phi_i(x)) := \begin{cases} \{de\_out\_i\} \cup \Sigma_i^{nc} & \text{if } \delta_{M'}(x, de\_out\_i) \text{ and } \\ & MC_i(x) > MR_i(x); \\ \{in\_out\_i\} \cup \Sigma_i^{nc} & \text{if } \delta_{M'}(x, in\_out\_i) \text{ and } \\ & MC_i(x) < MR_i(x); \\ \Sigma_i^{nc} & \text{if } MC_i(x) = MR_i(x). \end{cases}$$

Since  $M'$  is nonblocking, it is true that  $S_1 \wedge S_2/M$  is also nonblocking. Thus,  $S_1$  and  $S_2$  are also nonblocking.

**(ONLY IF)** Straightforward. ■

The supervisor  $S_i$  (oligopolistic firm  $i$ ) constructed in the proof means that if marginal cost is larger than marginal revenue at a state, it decreases its output, and otherwise it increases its output. If the two values equal, firm  $i$  does nothing since it maximizes its profit at the state. Through the repetition of these processes by the two firms, the oligopolistic market in disequilibrium eventually reaches equilibrium states (i.e., two firms' profit-maximizing states) under the controllability, observability, and nonblocking property. We note that when the market is in the state  $x$ , each firm  $S_i$  observes only the partial information  $\Phi_i(x)$ , i.e., its present output, marginal cost, and a market price at the state. It does not know how much its rival produces in the state. Based on the partial observation, each firm decides only whether it increases its output or decreases. In summary, Theorem 1 states that under the partial observation, the controllability, observability, and nonblocking property of the convergent sub-automaton  $M'$  are necessary and sufficient conditions for two oligopolistic firms in disequilibrium to eventually reach an equilibrium.

We note that the controllability of a sub-automaton presented in this paper corresponds to those in state feedback control based upon predicates of [2,5]. The observability of a

sub-automaton presented also corresponds to  $n$ -observability in decentralized state feedback control based upon predicates of [12]. However, the nonblocking control problem has not been addressed in [12]. The nonblocking property defined in this paper is similar to the stability in [7] where given a set of desired states, a system is stable if all paths from any state go through the set of desired states in a finite number of transitions and then visit the set infinitely often. The difference between them is that while the stability of [7] does not allow a cycle which does not go through a given set of desired states, the nonblocking property defined in this paper allows such a cycle. For example, consider a state  $x \in X_{M'}$  with  $MR_i(x) < MC_i(x)$  and  $\delta_{M'}(x, in\_pc\_i \ de\_pc\_i) = x$  in a sub-automaton  $M'$  of an oligopolistic market  $M$ . Since  $MR_i(x) \neq MC_i(x)$ , the state  $x$  is not marked, and there exists a cycle at the state  $x$  by the uncontrollable events  $in\_pc\_i$  and  $de\_pc\_i$ . However, it is not realistic that the increase and decrease of production cost infinitely occur in a certain period without the occurrences of other events. It seems to be more realistic that without the infinite occurrences of  $in\_pc\_i$  and  $de\_pc\_i$  at the state  $x$ , firm  $i$  can decrease output (i.e., enable the event  $de\_out\_i$ ) in order to reduce the difference between marginal cost and marginal revenue, and through the finite repetitive decreasing of output, the equilibrium of them can be achieved. In this respect, the nonblocking property defined in this paper guarantees that an oligopolistic market eventually reaches marked states.

#### IV. AN ILLUSTRATIVE EXAMPLE

Let us consider the illustrative oligopolistic market data shown in Table 3. Using the supervisory control theory presented in this paper, let us explain how this market's behavior can occur.

The marginal revenues in Table 3 are computed by the formula presented in the previous section, i.e.,  $MR_i(x) = P(x) + P(x)(1/E_i(x))$  for firm  $i$  where  $E_i(x)$  is the price

표 3. 한 과점시장의 시장 데이터.

Table 3. Market data of an oligopolistic market.

Time	MC1	Q1	MR1	MC2	Q2	MR2	P	State
Feb. 2008	50	22	50	32	6	42	54	x0
Jun. 2008	50	22	44	36	8	34	50	x1
Oct. 2008	48	15	48	36	8	36	52	x2
Feb. 2009	48	15	48	8	8	36	52	x3
Jun. 2009	48	15	42	10	10	28	48	x4
Oct. 2009	46	10	46	10	10	25	50	x5
Feb. 2010	46	10	41	15	12	15	45	x6
Jun. 2010	45	5	45	15	12	27	47	x7
Oct. 2010	45	5	40	17	15	17	42	x8

elasticity of demand of firm  $i$ . From this data, we construct a FSA  $M$  of the market as shown in Fig. 4 where  $x_0$  is an initial state and  $x_2$  is a marked state. We have omitted the component FSAs,  $F_1$ ,  $F_2$ , and  $C$ . However, they can be easily derived from the FSA  $M$ .

First, let us consider a sub-automaton  $M_1$  denoted by dotted lines in Fig. 4. It is convergent. For example, at the initial state  $x_0$ , it holds that  $MC_2(x_0) = 32 < MR_2(x_0) = 42$  for firm 2. Then,  $\delta_{M_1}(x_0, in\_out\_2) = x_1$ ,  $MR_2(x_1) - MC_2(x_1) = -2$  and  $MR_2(x_0) - MC_2(x_0) = 10$ . In addition,  $M_1$  is nonblocking since for the states  $x_0, x_1 \in X_{M_1}$ , there exist paths of transitions from these states to the marked state  $x_2$ . The observability is trivially true since all the states of  $M_1$  can be obviously discriminated by the firms. However,  $M_1$  is not controllable since, at the state  $x_2 \in X_{M_1}$ , it holds that  $\Sigma_{M_1}(x_2) \cap \Sigma^{uc} = \emptyset$  but  $\Sigma_M(x_2) \cap \Sigma^{uc} = \{de\_pc\_2\}$ . Hence, according to Theorem 1, there do not exist two nonblocking oligopoly firms to meet  $S_1 \wedge S_2 / M = M_1$ . This is the reason for that the market at the state  $x_2$  on Oct. 2008 has eventually

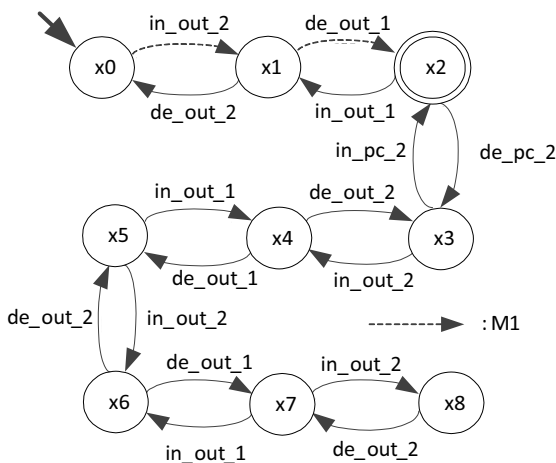


그림 4. 시장  $M$  과 서브오토마톤  $M_1$ .  
Fig. 4. Market  $M$  and a sub-automaton  $M_1$ .

gone to the state  $x_3$  on Feb. 2009 as shown in Table 3.

Let us the second sub-automaton  $M_2$  denoted by dotted lines in Fig. 5. Since the uncontrollable event  $de\_pc\_2$  is defined at the state  $x_2$  in  $M_2$ , it is then controllable. It is also observable. For example, for the two states  $x_2, x_3 \in X_{M_2}$ , it holds that  $\Phi_1(x_2) = \Phi_1(x_3) = (48, 15, 52)$ . However, there is no event  $\sigma \in \Sigma_1^c$  such that  $\delta_{M_2}(x_2, \sigma)!$  or  $\delta_{M_2}(x_3, \sigma)!$  since  $MR_1(x_2) = MC_1(x_2) = 48$  and  $MR_1(x_3) = MC_1(x_3) = 48$ . In addition,  $M_2$  is convergent and nonblocking. Therefore, there exist two nonblocking oligopoly firms  $S_1$  and  $S_2$  such that  $S_1 \wedge S_2 / M = M_2$  according to Theorem 1. As presented in the sufficiency part of the proof, the control actions at the state  $x_3$  by  $S_1$  and  $S_2$  are  $S_i(\Phi_i(x_3)) = \Sigma_i^{uc}$  for  $i=1, 2$ , i.e., there are no controllable events to be enabled at the state  $x_3$  to achieve  $M_2$ . This is because there is no controllable event defined at the state  $x_3$  in  $M_2$ . We note that even though  $MR_2(x_3) = 36 > MC_2(x_3) = 8$ , firm2 does not increase output to achieve  $M_2$  which aims to maximize the profits of both firms. However, it is only possible when firm 2 accepts and follows a cooperative agreement with firm 1.

Finally, let us explain how the market could reach the state  $x_8$  eventually. At  $x_3$ ,  $MC_2(x_3) = 8 < MR_2(x_3) = 36$  for firm 2. It implies that firm 2 has an incentive to increase output to maximize its profit. If the two firms are noncooperative, firm 1 has no mean to prevent firm 2 from increasing its output. Thus, firm 2 increases output, and then the market reaches the state  $x_4$ . The sub-automaton  $M_3$  denoted by dotted lines in Fig. 6 models the behaviors of two firms to maximize their profits in a noncooperative manner. It finally evolves into the state  $x_8$  at which firm 1 has an incentive to decrease its output since  $MR_1(x_8) = 40 < MC_1(x_8) = 45$ . However, it cannot reduce its output at the state since the production below the quantity does not return any profit, i.e., it may lead to the firm 1's shutdown. We can explain it using Theorem 1 as follows. The sub-automaton  $M_3$  is convergent, controllable, and observable. However, it is not nonblocking since at the state  $x_8$ , there is no path of transitions from the state to the market state  $x_2$ .

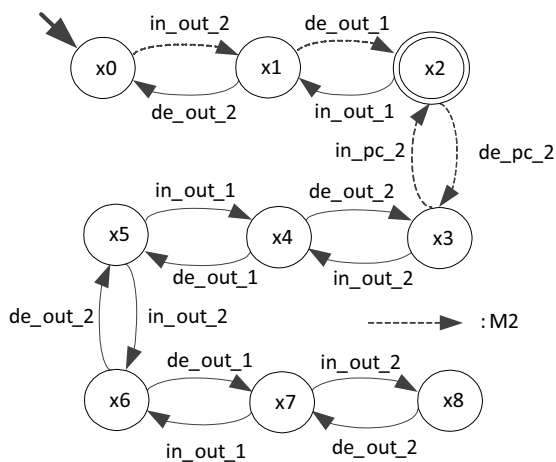


그림 5. 서브오토마톤  $M_2$ .  
Fig. 5. Sub-automaton  $M_2$ .

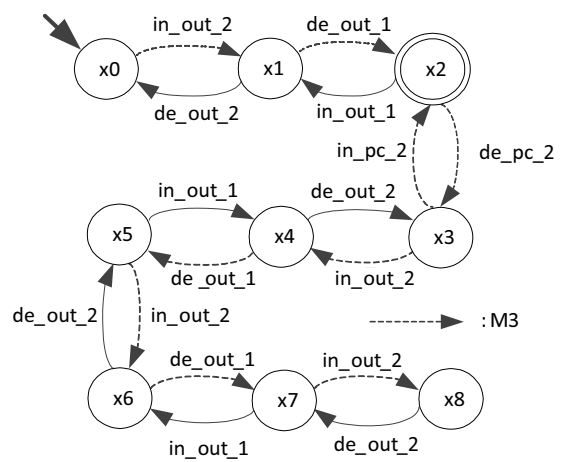


그림 6. 서브오토마톤  $M_3$ .  
Fig. 6. Sub-automaton  $M_3$ .



Therefore, there do not exist two nonblocking oligopoly firms such that  $S_1 \wedge S_2/M = M3$ .

### V. CONCLUSIONS

In this paper, we have studied a dynamic oligopolistic market in which two firms compete in quantities in the absence of information about their rivals' outputs and marginal costs. To maximize their profits, each firm adopts the conventional strategy that if marginal cost is larger than marginal revenue, it decreases output, and otherwise it increases output. For this market, we have shown that the controllability, observability, and nonblocking property are the necessary and sufficient conditions for two firms following the conventional strategy in disequilibrium to eventually reach an equilibrium of maximizing the profits of both firms.

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