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다변수 슬라이딩 모드 제어에 의한 부정합조건 불확실성을 갖는 다입출력 비선형 시스템의 강인 그로벌 지수 안정화

A Robust Global Exponential Stabilization of Uncertain Affine MIMO Nonlinear Systems with Mismatched Uncertainties by Multivariable Sliding Mode Control

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Abstract - In this paper, a systematic design of a robust nonlinear multivariable variable structure controller based on state dependent nonlinear form is presented for the control of MIMO uncertain affine nonlinear systems with mismatched uncertainties and matched disturbance. After a MIMO uncertain affine nonlinear system is represented in the form of state dependent nonlinear system, a systematic design of a robust nonlinear variable structure controller is presented. To be linear in the closed loop resultant dynamics, the linear sliding surface is applied. A corresponding diagonalized control input is proposed to satisfy the closed loop global exponential stability and the existence condition of the sliding mode on the linear sliding surface, which will be investigated in Theorem 1. Through a design example and simulation study, the usefulness of the proposed controller is verified.

Key Words : MIMO uncertain nonlinear system, Variable structure system, Sliding mode control, Mismatched uncertainties

1. Introduction

Stability analysis and controller design for uncertain nonlinear systems is open problems now[1]. So far numerous design methodologies exist for the controller design of nonlinear systems[2]. These include any of a huge number of linear design techniques[3][4] used in conjuction with gain scheduling[5]; nonlinear design methodologies such as Lyapunov function approach[1][2][6][7][10][11], feedback linearization method[8][9][10], dynamics inversion [10], backstepping[11], adaptive technique which encompass both linear adaptive[13] and nonlinear adaptive control[14], and sliding mode control[15]-[24] etc[25]-[27].

The sliding mode control(SMC) can provide the effective means to the problem of controlling uncertain nonlinear systems under parameter variations and external disturbances[15][16]. One of its essential advantages is the robustness of the controlled system to variations of parameters and external disturbances in the sliding mode on the predetermined sliding surface, $s=0$[17]. In [18], one order increased sliding surface is proposed, the corresponding control input suggested, and the chattering problems are improved by menas of the boundary layer method. But, the internal stability of the proposed sliding mode controller is not proved. In [19] the convergence to a sliding manifold which can be attained relying on a control strategy still based on a simplex of control vectors are identified for multi input uncertain nonlinear systems. A continuous finite reaching time sliding mode controller with enhanced robustness is designed for MIMO nonlinear control systems in [20]. A MIMO second order sliding mode control is presented for multivariable nonlinear uncertain systems[21]. The SISO second order sliding mode technique is extended to the MIMO nonlinear systems. In [22] the finite reaching time to the origin is analyzed when using the continuous input. Using a high order integral sliding surface, Defoort et al designed the finite time controller for a class of MIMO nonlinear systems. While the closed loop stability is proved by the Lyapunov theory, the existence condition of the sliding mode is not investigated. For 2nd order uncertain nonlinear systems with mismatched uncertainties, a switching control law between a first order sliding mode control and a second order sliding mode control is proposed to obtain the globally or locally asymptotic stability[23]. The design of a continuous sliding mode controller for the tracking control of a class

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of MIMO nonlinear uncertain systems is presented. The on-line estimation of the uncertainties is performed using proportional-integral action of the sliding function for continuous implementation of the SMC[24]

For MIMO uncertain affine nonlinear systems with mismatched uncertainties and matched disturbance, the systematic design of the sliding mode control is not reported until now.

In this technical paper, a systematic design of a new multivariable nonlinear variable structure controller based on state dependent nonlinear form is presented for the control of uncertain MIMO affine nonlinear systems with mismatched uncertainties and matched disturbance. This is a extension from the SISO nonlinear system research [33]. After an uncertain MIMO affine nonlinear system is represented in the form of state dependent nonlinear systems[33][36], a systematic design of a new nonlinear variable structure controller is presented. To be linear in the closed loop resultant dynamics, the linear sliding surface is applied to[33]. A corresponding control input is proposed in order to satisfy the closed loop global exponential stability and the existence condition of the sliding mode on the linear sliding surface, which will be investigated in Theorem 1 based on one of the two diagonalization methods in [15], [16], [39], and [43]. Through a design example and simulation studies, the usefulness of the proposed controller is verified.

2. A MIMO Nonlinear Variable Structure Systems

2.1 Description of plants

Consider an affine uncertain multivariable nonlinear system

$$\begin{aligned}\dot{x} &= f'(x,t)+g(x,t)u+d'(x,t), \qquad x(0) \\ &= f'(x,t)+\sum_{i=1}^{m} g_i(x,t)u_i+d'(x,t)\end{aligned} \tag{1}$$

where $x\in R^n$ is the state, $x(0)$ is its initial state, $u\in R^m$ is the control to be designed, $f'(x,t)\in C^k$ and $g(x,t)\in C^k$, $k\geq 1$, $g(x,t)\neq 0$, for $all\ x\in R^n$ and for $all\ t\geq 0$ are of suitable dimensions, and $d'(x,t)$ implies bounded matched external disturbances.

Assumption[27]

A1: $f'(x,t)$ is continuously differentiable and $f'(0,t)=0$ for all $t\geq 0$.

Then, the uncertain multivariable nonlinear system (1) can be represented in more affine nonlinear system of state dependent coefficient form[27][28][33][36]

$$\begin{aligned}\dot{x} &= f_0(x,t)x+\triangle f(x,t)+[g_0(x,t)+\triangle g(x,t)]u+d'(x,t) \\ &= f_0(x,t)x+g_0(x,t)u+d(x,t)\end{aligned} \tag{2}$$

$$d(x,t)=\triangle f(x,t)+\triangle g(x,t)u+d'(x,t) \tag{3}$$

where $f_0(x,t): R^n\times[0,+\infty)\rightarrow R^n$ and $g_0(x,t): R^n\times[0,+\infty)\rightarrow R^{n\times m}$ is each nominal value such that $f'(x.t)=[f_0(x,t)x+\triangle f(x,t)]: R^n\times[0,+\infty)\rightarrow R^n$ and $g(x.t)=[g_0(x,t)+\triangle g(x,t)]: R^n\times[0,+\infty)\rightarrow R^{n\times m}$, respectively, $\triangle f(x,t)$ and $\triangle g(x,t)$ are matched or mismatched uncertainties, and $d(x,t): R^n\times[0,+\infty)\rightarrow R^n$ is the mismatched lumped uncertainties.

Assumption:

A2: The pair $[f_0(x,t), g_0(x,t)]$ is involutive[10] for $all\ x\in R^n$ and for $all\ t\geq 0$

A3: The mismatched lumped uncertainties $d(x,t)$ is bounded

A4: $\ddot{x}$ is bounded if $\dot{u}$ is bounded.

The goal of a MIMO sliding mode controller design is to control the more affine uncertain multivariable nonlinear system (2) to the origin in state space with global exponential stability. This design of the MIMO sliding mode controller will be done through the two steps, i.e. sliding surface choice and control input design. The sliding surface will be designed as first stage in the next.

2.2 Sliding Surface

To control the uncertain multivariable nonlinear system (1) or (2) with a linear closed loop dynamics, the linear sliding surface used in this design is as follows:

$$s=\begin{bmatrix} s_1 \\ \vdots \\ s_m\end{bmatrix}=\begin{bmatrix} C_1^T x \\ \vdots \\ C_m^T x\end{bmatrix}=C^T x(=0) \tag{4}$$

where s_i, $i=1,...m$ is the sub sliding surface. The vector relative degree $r=[r_1\cdots r_m]$ of the sub sliding surface with respect to the system (1) with (4) is assumed to be constant and known[20]. It means that the $m\times m$ matrix:

$$\begin{bmatrix} L_{g_1}L_f^{r_1-1}C_1^T x & \cdots & L_{g_m}L_f^{r_1-1}C_m^T x \\ \vdots & & \vdots \\ L_{g_1}L_f^{r_m-1}C_1^T x & \cdots & L_{g_m}L_f^{r_m-1}C_m^T x\end{bmatrix} \tag{5}$$

must be nonsingular and

$$L_{g_j}L_f^k C_i^T x=0, \qquad . \tag{6}$$
$$\text{for}\quad 1\leqq i\leqq m,\quad 1\leqq j\leqq m,\quad \text{and}\quad 1\leqq k\leqq r_i-1$$

which plays a role of the sufficient condition for the design of the sliding surface (4). In this paper, the vector relative degree $r=[r_1\cdots r_m]$ is assumed to be all first order $r=[1\cdots 1]$, which means that each sub-sliding surface in the MIMO SMCs has the state which derivative is a direct function of each input at least. In (4), C is a non-zero element as the design parameter such that the following assumptions are satisfied.

Assumption

A5: $C^T g(x,t)$ and $C^T g_0(x,t)$ have the full rank, i,e they are invertible.

A6: $C^T \triangle g(x,t)\left[C^T g_0(x,t)\right]^{-1} = \triangle I$, $\triangle I$ is diagonal, and $|\triangle I_i| < \delta_i < 1,\ 0<\delta_i<1,\ i=1,...,m.$

Using $\dot{s}=0$[15], the equivalent control input of the linear sliding surface is obtained as

$$u_{eq} = -\left[C^T g(x,t)\right]^{-1} C^T f_0(x,t)x - \left[C^T g(x,t)\right]^{-1} C^T \triangle f(x,t) - \left[C^T g(x,t)\right]^{-1} d'(x,t) \tag{7}$$

This control input can not be implemented because of the uncertainties and external disturbance, but which is used to obtaining the ideal sliding mode dynamics. The ideal sliding mode dynamics of the sliding surface (4) can be derived by the equivalent control approach[16][33] as

$$\dot{x}_s = \left[f_0(x_s,t) - g_0(x_s,t)(C^T g_0(x_s,t))^{-1} C^T f_0(x_s,t)\right]x_s, \quad \text{for} \quad x_s(t_s) = x(t_s) \tag{8}$$

$$\dot{x}_s = \left[f_0(x_s,t) - g_0(x_s,t)K(x_s)\right]x_s, \tag{9}$$

$$K(x_s) = \left[C^T g_0(x_s,t)\right]^{-1} C^T f_0(x_s,t) \tag{10}$$

where t_s the reaching time. The solution of the ideal sliding mode dynamics (8) identically defines the sliding surface after reaching. Hence to design the sliding surface as stable, this ideal sliding mode dynamics is designed to be stable. To choose the stable gain based on the Lyapunov stability theory, the ideal sliding mode dynamics (8) or (9) is represented by the nominal plant of (2) as

$$\dot{x} = f_0(x,t)x + g_0(x,t)u, \quad u = -K(x)x$$
$$= f_c(x,t)x, \quad f_c(x,t) = f_0(x,t) - g_0(x.t)K(x) \tag{11}$$

To select the stable gain in the ideal sliding mode dynamics, take a Lyapunov function candidate as

$$V(x) = x^T P x, \qquad P>0 \tag{12}$$

The time derivative of (12) becomes

$$\dot{V}(x) = x^T\left[f_0(x,t)^T P + P f_0(x,t)\right]x + u^T g_0^T(x,t) P x + x^T P g_0(x,t) u \tag{13}$$

If take the control input as

$$u = -g_0^T(x,t) P x \tag{14}$$

and $Q(x,t)>0$ for $all\, x \in R^n$ and for $all\, t \ge 0$ is

$$f_0(x,t)^T P + P f_0(x,t) = -Q(x,t) \tag{15}$$

then

$$\dot{V}(x) = -x^T Q(x,t)x - 2x^T P g_0(x,t) g_0^T(x,t) P x$$
$$= -x^T\left[Q(x,t) + 2P g_0(x,t) g_0^T(x,t) P\right]x$$
$$= -x^T\left[f_c^T(x,t)P + P f_c(x,t)\right]x$$
$$= -x^T Q_c(x,t) x, \quad Q_c(x,t) = f_c^T(x,t)P + P f_c(x,t) \tag{16}$$
$$\le -\lambda_{\min}\{Q_c(x,t)\}x^2$$
$$\le 0$$

Therefore the stable gain is chosen as

$$K(x) = g_0^T(x,t)P \text{ or} = \left[C^T g_0(x_s,t)\right]^{-1} C^T f_0(x_s,t) \tag{17}$$

Now as the second design stage of the nonlinear MIMO sliding mode controller, the control input will be suggested.

2.3 Control Input

A corresponding transformed control input is proposed as follows:

$$u = -[C^T g_0(x.t)]^{-1}[K(x)x + \triangle K x + K_1 s + K_2 sign(s)] \tag{18}$$

where $K(x)$ is a continuous nonlinear feedback gain, $\triangle K$ is a discontinuous switching gain, K_1 is a continuous feedback gain of the sliding surface, and K_2 is a discontinuous switching gain, respectively as

$$K(x) = C^T f_0(x,t) \text{ or} = \left[C^T g_0(x,t)\right] g_0^T(x,t)P \tag{19}$$

$$\triangle K = [\triangle k_{ij}] \tag{20}$$

$$\triangle k_{ij} = \begin{cases} \ge \dfrac{\max\left\{\triangle I C^T f_0(x,t)\right\}_{ij}}{\min\{I+\triangle I\}_i} & sign(s_i x_j) > 0 \\ \le \dfrac{\min\left\{\triangle I C^T f_0(x,t)\right\}_{ij}}{\min\{I+\triangle I\}_i} & sign(s_i x_j) < 0 \end{cases}$$
$$i=1,...,m,\ j=1,...,n \tag{21}$$

$$K_1 = [k_{1_{ii}}] \qquad k_{1_{ii}} > 0,\ i=1,...,m \tag{22}$$

$$K_2 = [k_{2_{ii}}] \tag{23}$$

$$\triangle k_{2_{ii}} = \begin{cases} \ge \dfrac{\max\left\{C^T \triangle f(x,t) + C^T d'(x,t)\right\}_i}{\min\{I+\triangle I\}_{ii}} & sign(s_i) > 0 \\ \le \dfrac{\min\left\{C^T \triangle f(x,t) + C^T d'(x,t)\right\}_i}{\min\{I+\triangle I\}_{ii}} & sign(s_i) < 0 \end{cases} \tag{24}$$

The control input is transformed by means of the diagonalization method[15][16][39][43], which means that the term of $[C^T g_0(x,t)]^{-1}$ is multiplied to each gain of the control input for easy stabilization and easy proof of the existence condition of the sliding mode. The real sliding dynamics by the proposed control with the linear sliding surface is obtained as follows:

$$\dot{s} = C^T \dot{x}$$
$$= C^T\left[f_0(x,t)x + \triangle f(x,t) + g(x,t)u + d'(x,t)\right]$$
$$= C^T\left[f_0(x,t)x + \triangle f(x,t)\right]$$
$$+ C^T\left[g(x,t)[C^T g_0(x,t)]^{-1}\left\{-K(x)x - \triangle K x - K_1 s - K_2 sign(s)\right\}\right]$$
$$+ C^T\left[d'(x,t)\right]$$

$$\dot{s} = C^T f_0(x,t)x - K(x)x + C^T \triangle f(x,t) - C^T \triangle g(x,t)K(x)x$$
$$- C^T g(x,t)[C^T g_0(x,t)]^{-1}\triangle K x - C^T g(x,t)[C^T g_0(x,t)]^{-1}K_1 s$$
$$+ C^T d'(x,t) - C^T g(x,t)[C^T g_0(x,t)]^{-1}K_2 sign(s)$$
$$= C^T \triangle f(x,t) - C^T \triangle g(x,t)K(x)x - [I+\triangle I]\triangle K x$$
$$- [I+\triangle I]K_1 s + C^T d'(x,t) - [I+\triangle I]K_2 sign(s) \tag{25}$$

The closed loop stability by the proposed control input with linear sliding surface together with the existence condition of the sliding mode will be investigated in next Theorem 1.

Theorem 1: *If the linear sliding surface is designed in the stable, i.e. stable design of $K(x)$, the proposed input with Assumption A1-A6 satisfies the existence condition of the sliding mode on the sliding surface and the global exponential stability.*

Proof; Take a Lyapunov function candidate as

$$V(x)=\frac{1}{2}s^T s \tag{26}$$

Differentiating (26) with respect to time leads to and substituting (25) into (27)

$$\begin{aligned}\dot{V}(x) &= s^T\dot{s} \\ &= s^T C^T \triangle f(x,t) - s^T C^T \triangle g(x,t)K(x)x \\ &\quad - s^T[I+\triangle I]\triangle Kx - s^T[I+\triangle I]K_1 s \\ &\quad + s^T C^T d'(x,t) - s^T[I+\triangle I]K_2 sign(s)\end{aligned} \tag{27}$$

$$\begin{aligned}\dot{V}(x) &\le -\epsilon K_1\|s\|^2, \qquad \epsilon=\min[\|(I+\triangle I)\|] \\ &= -\epsilon K_1 s^T s \\ &= -2\epsilon K_1 V(x)\end{aligned}$$

From the above equation, the following equations are obtained as

$$\dot{V}(x)+2\epsilon K_1 V(x) \le 0 \tag{28}$$

$$V(t) \le V(0)e^{-2\epsilon K_1 t} \tag{29}$$

And for the each sub sliding surface

$$s_i \cdot \dot{s}_i \leftarrow 2\epsilon K_{1_i} s_i \cdot s_i < 0, \quad i=1,\cdots,m \tag{30}$$

is obtained from (27) by (18) with (19)-(24) which means that the MIMO version of the existence condition of the sliding mode on the linear sliding surface is satisfied. The second order derivative of $V(x)$ become

$$\ddot{V}(x)=\dot{s}\dot{s}+s\ddot{s}=(\dot{s})^2+sC^T\ddot{x}<\infty \tag{31}$$

and by Assumption A4 $\ddot{V}(x)$ is bounded which completes the proof of Theorem 1.

3. Design Example and Simulation Studies

Consider a fourth order affine uncertain multivariable nonlinear system with mismatched uncertainties and matched disturbance

$$\begin{aligned}\dot{x}_1 &= x_2+1.2\sin(x_1)-0.9\sin(x_2)+0.1u_1 \\ \dot{x}_2 &= -x_1+(1-x_1^2)x_2+[2.5+0.5\sin(3t)+\frac{1}{1+(x_1x_3)^2}]u_1+d'_1(x,t) \\ \dot{x}_3 &= x_4+0.7\sin(x_1)+0.8\sin(x_2) \\ \dot{x}_4 &= -2x_3+(1-x_3^2)x_4+[2+0.5\sin(5t) \\ &\quad +\frac{1}{1+(x_2x_4)^2}]u_2+d'_2(x,t)\end{aligned} \tag{32}$$

$$\begin{aligned}d'_1(x,t) &= 0.7\sin(x_1)-0.8\sin(x_2)+0.2(x_1^2+x_3^2)+2\sin(2t) \\ d'_2(x,t) &= 0.5\sin(x_1)+0.9\sin(x_2)+1.5\sin(10t)+0.02(x_1^2+x_2^2)\end{aligned} \tag{33}$$

Since (32) and (33) satisfy the Assumption A1, (32) with (33) can be represented in state dependent coefficient form as

$$\begin{aligned}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} &= \begin{bmatrix}0 & 1 & 0 & 0\\ 1 & (1-x_1^2) & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 2 & (1-x_3^2)\end{bmatrix}x \\ &+ \begin{bmatrix}0.1 & 0\\ 2.5+0.5\sin(3t)+\frac{1}{1+(x_1x_3)^2} & 0\\ 0 & 0\\ 0 & 2.0+0.5\sin(5t)+\frac{1}{1+(x_2x_4)^2}\end{bmatrix}u \\ &+ \begin{bmatrix}1.2\sin(x_1)-0.9\sin(x_2)\\ d'_1(x,t)\\ 0.7\sin(x_1)+0.8\sin(x_2)\\ d'_2(x,t)\end{bmatrix}\end{aligned} \tag{34}$$

where the nominal parameter $f_0(x,t)$ and $g_0(x,t)$ and mismatched uncertainties $\triangle f(x,t)$ and $\triangle g(x,t)$ are

$$\begin{aligned}f_0(x,t) &= \begin{bmatrix}0 & 1 & 0 & 0\\ 1 & (1-x_1^2) & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 2 & (1-x_3^2)\end{bmatrix}, \\ g_0(x.t) &= \begin{bmatrix}0 & 0\\ 2.5+\frac{1}{1+(x_1x_3)^2} & 0\\ 0 & 0\\ 0 & 2.0+\frac{1}{1+(x_2x_4)^2}\end{bmatrix} \\ \triangle f(x,t) &= \begin{bmatrix}1.2\sin(x_1)-0.9\sin(x_2)\\ 0\\ 0.7\sin(x_1)+0.8\sin(x_2)\\ 0\end{bmatrix}, \\ \triangle g(x.t) &= \begin{bmatrix}0.1 & 0\\ 0.5\sin(3t) & 0\\ 0 & 0\\ 0 & 0.5\sin(5t)\end{bmatrix}\end{aligned} \tag{35}$$

To be linear in the sliding mode, the coefficient of the linear sliding surface is chosen as

$$C^T=\begin{bmatrix}5 & 1 & 1 & 0\\ 0 & 0 & 7 & 1\end{bmatrix} \tag{36}$$

with considering the sliding surface design idea in [29] for MIMO uncertain linear VSS, which vector relative degree of the two sub sliding surface is first order $r=[1 \quad 1]$.

$$C^T g_0(x,t)=\begin{bmatrix}2.5+\frac{1}{1+(x_1x_3)^2} & 0\\ 0 & 2.0+\frac{1}{1+(x_2x_4)^2}\end{bmatrix} \tag{37}$$

$$\begin{aligned}C^T\triangle g(x,t)\left[C^T g_0(x,t)\right]^{-1} &= \triangle I \\ &= \begin{bmatrix}\frac{[0.5+0.5\sin(3t)][1+(x_1x_3)^2]}{3.5+2.5(x_1x_3)^2} & 0\\ 0 & \frac{[0.5\sin(5t)][1+(x_2x_4)^2]}{3+2(x_2x_4)^2}\end{bmatrix}\end{aligned} \tag{38}$$

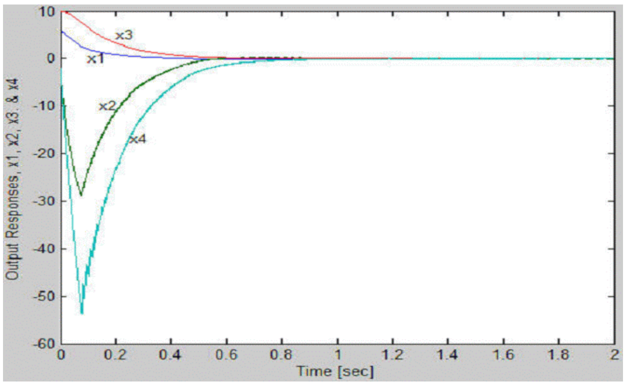

Fig. 1 x_1, x_2, x_3, and x_4 time trajectories

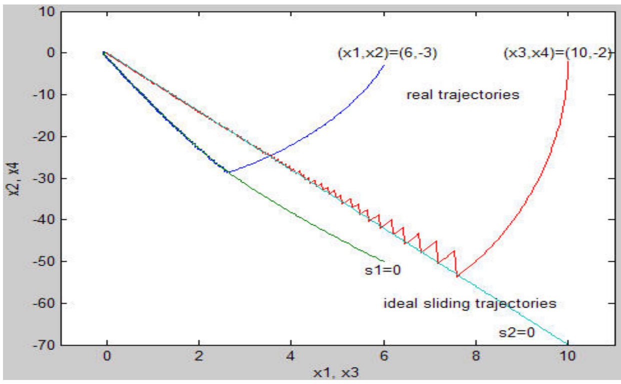

Fig. 2 Real phase trajectories and ideal sliding trajectories

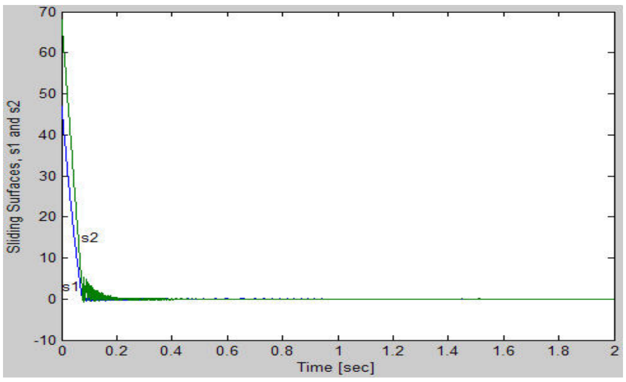

Fig. 3 Sliding surfaces

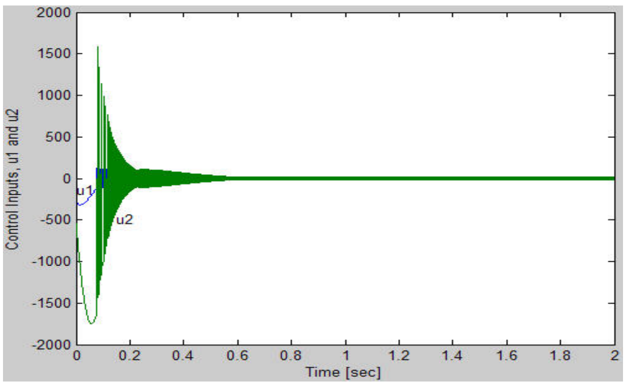

Fig. 4 Control inputs

The assumption A5 and A6 are satisfied. The selected gains in the control input (18) are as follows:

$$K(x)=C^T f_0(x,t)=\begin{bmatrix}1 & (6-x_1^2) & 0 & 1\\ 0 & 0 & 2 & (8-x_3^2)\end{bmatrix},$$

$$\triangle K=\begin{bmatrix}\triangle k_{11} & \triangle k_{12} & \triangle k_{13} & \triangle k_{14}\\ \triangle k_{21} & \triangle k_{22} & \triangle k_{23} & \triangle k_{24}\end{bmatrix}, \triangle k_{11}=\begin{cases}4.5 & \text{if } s_1x_1>0\\ -4.5 & \text{if } s_1x_1<0\end{cases},$$

$$\triangle k_{12}=\begin{cases}|(6-x_1^2)| & \text{if } s_1x_2>0\\ -|(6-x_1^2)| & \text{if } s_1x_2<0\end{cases}, \triangle k_{13}=\begin{cases}3.5 & \text{if } s_1x_3>0\\ -3.5 & \text{if } s_1x_3<0\end{cases}$$

$$\triangle k_{14}=\begin{cases}3.3 & \text{if } s_1x_4>0\\ -3.3 & \text{if } s_1x_4<0\end{cases}, \quad \triangle k_{21}=\begin{cases}7.5 & \text{if } s_2x_1>0\\ -7.5 & \text{if } s_2x_1<0\end{cases},$$

$$\triangle k_{22}=\begin{cases}13.4 & \text{if } s_2x_2>0\\ -13.4 & \text{if } s_2x_2<0\end{cases} \quad \triangle k_{23}=\begin{cases}6.5 & \text{if } s_2x_3>0\\ -6.5 & \text{if } s_2x_3<0\end{cases},$$

$$\triangle k_{24}=\begin{cases}|(8-x_3^2)| & \text{if } s_2x_4>0\\ -|(8-x_3^2)| & \text{if } s_2x_4<0\end{cases},$$

$$K_1=\begin{bmatrix}K_{1_{11}} & 0\\ 0 & K_{1_{22}}\end{bmatrix}=\begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix}$$

$$K_2=\begin{bmatrix}K_{2_{11}} & 0\\ 0 & K_{2_{22}}\end{bmatrix}=\begin{bmatrix}50.5+0.2(x_1^2+x_3^2) & 0\\ 0 & 50.5+0.2(x_1^2+x_3^2)\end{bmatrix} \quad (39)$$

which satisfy the equation (19) and inequalities (20)-(24).

The simulation is carried out under 1[msec] sampling time and with $x(0)=[6\ -3\quad 10\ -2]^T$ initial state. Fig. 1 shows x_1, x_2, x_3 and x_4 time trajectories. The phase trajectories and ideal sliding trajectories are depicted in Fig. 2. The sliding surfaces s_1 and s_2 are shown in Fig. 3. The control inputs u_1 and u_2 are depicted in Fig. 4. From the simulation studies, the effectiveness of the proposed SMC is proven.

4. Conclusions

In this paper, a global exponential stabilization by using a proposed multivariable robust nonlinear variable structure controller based on state dependent nonlinear form and diagonalization method is presented for the control of MIMO uncertain affine nonlinear systems with mismatched uncertainties and matched disturbance. After a MIMO affine uncertain nonlinear system is represented in the form of state dependent nonlinear system, a systematic design of a new robust nonlinear variable structure controller is suggested. To be linear in the sliding mode of the closed loop controlled system, the first order linear sliding surface is applied to the designed algorithm. A corresponding control input is proposed, which is composed of the continuous terms and discontinuous switching terms and is transformed by means of the diagonalization method for easy stabilization and easy proof of the existence condition of the sliding mode. The closed loop global exponential stability by the proposed sliding mode control input with the first order linear sliding surface together with the existence condition of the sliding mode on the selected sliding surface is investigated in Theorem 1 for all mismatched uncertainties and matched disturbance. Through an illustrative design example and simulation study, the effectiveness of the proposed MIMO sliding mode controller is verified.

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