Balanced Information Potentials for PDF-Distance Algorithms with Constant Modulus Error

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ABSTRACT

Blind equalization techniques have been widely used in wireless communication systems. In this paper, we propose to apply the balanced information potentials to the criterion of minimum Euclidian distance between two PDFs with constant modulus errors for adaptive blind equalizers. One of the two PDFs is constructed with constant modulus error samples and another does with Dirac delta functions. Two information potentials derived from the criterion are balanced in order to have better performance by putting a weighting factor to each information potentials. The proposed blind algorithm has shown in the MSE convergence performance that it can produce enhanced performance by over 3 dB of steady state MSE.

Keywords : Blind equalization, PDF, Constant modulus error, balanced, Information potential

1. Introduction

Multipoint communication networks and the wireless broadcasting networks require efficient blind equalizers to cope with intersymbol interference in the environment that training sequences are not available [1][2].

Most blind algorithms are developed based on mean-squared-error (MSE) criterion for minimization of constant modulus error (CME). One of the drawbacks of MSE criterion can be the truth that it utilizes only second order statistics such as error power. The constant modulus algorithm (CMA) popularly being used is to use the second order statistics for CME. In the recently developed information theoretic optimization criteria, higher order informations are utilized in that optimization method introduced by Princepe [3]. This approach is to adjust the adaptive system based on a combination of a nonparametric kernel density estimator for probability density function (PDF) and a procedure to compute entropy or information potential (IP) [3]. For approximating Shannon's entropy, Renyi's generalized entropy [4] is employed by way of the kernel density estimation method with a Gaussian kernel [5]. The Renyi entropy with the Kernel density method leads to an estimation of information potential defined as interactions among pairs of samples.

For CME, when the error PDF is forced to match the delta function, the CME samples gather together around zero by minimizing the distance between the error PDF and the delta function [6] which is a different approach from

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the error-entropy minimization in blind equalization. In error-entropy method [7] the difference of two error samples are used. This error distance can have valuable information about error distribution in supervised systems, but in CME-type blind equalization the error difference between two CMEs loses the important information of constant modulus value, so that error-entropy method with CME can not be employed in blind equalization.

In the process of minimizing the distance between the PDF of CME and the Dirac delta function, the interactions among CMEs are revealed to have two information potentials acting towards opposite directions in this research, so we propose in this paper to balance the two IPs in order to mitigate their conflicts in the IP field.

2. The Distance of CME-PDF and Dirac delta function

When L-ary PAM signaling systems with L levels being equally likely to be transmitted are employed and the transmitted levels A_l are $A_l = 2l-1-L$, $l = 1,2,...,L$, the constant modulus R_2 becomes

$$R_2 = E[|A_l|^4]/E[|A_l|^2] \qquad (1)$$

In this section we introduce briefly the distance between the PDF of CME and the Dirac-delta function [6]. With the definition of $e_{CME} = |y_k|^2 - R_2$, we can describe the distance $D[f_{CME}(e_{CME}), \delta(e_{CME})]$ between the two PDFs, the CME-PDF $f_{CME}(e_{CME})$ and Dirac-delta function $\delta(e_{CME})$ as

$$D[f_{CME}(e_{CME}), \delta(e_{CME})] = \int f_{CME}^2(\xi)d\xi + \int \delta^2(\xi)d\xi - 2\int f_{CME}(\xi)\delta(\xi)d\xi . \qquad (2)$$

From the research [6] we can notice the first term on the right side $\int f_{CME}^2(\xi)d\xi$ is the information potential for CMEs, so we write it as V_{CME} in this paper.

Then we obtain

$$D[f_{CME}(e_{CME}), \delta(e_{CME})] = V_{CME} + 1 - 2f_{CME}(0) \qquad (3)$$

Since we can also remove the constant term from (3) for it is not controllable, the minimization of $D[f_{CME}(e_{CME}), \delta(e_{CME})]$ remains as the new criterion for constant modulus errors.

$$\min_W \{V_{CME} - 2f_{CME}(0)\} \qquad (4)$$

For convenience sake, we refer to (4) as minimum ED for CME (MED-CME) criterion.

Using the kernel density estimation method [5], the PDF of CME $f_{CME}(e_{CME})$ with Gaussian kernel and a block of N CME samples can be described as

$$f_{CME}(e_{CME}) = \frac{1}{N}\sum_{i=k-N+1}^{k} G_\sigma(e_{CME} - e_{CME,i}) \qquad (5)$$

The kernel density estimation method can make us write the IP V_{CME} in (4) as follows:

$$V_{CME} = \int f_{CME}^2(\xi)d\xi = \frac{1}{N^2}\sum_{i=k-N+1}^{k}\sum_{j=k-N+1}^{k} G_{\sigma\sqrt{2}}(e_{CME,i} - e_{CME,j}) \qquad (6)$$

From (6) with $e_{CME} = 0$, the probability $f_{CME}(0)$ in (4) becomes

$$f_{CME}(0) = \frac{1}{N}\sum_{i=k-N+1}^{k} G_\sigma(-e_{CME,i}) \qquad (7)$$

Then the cost function $P_{CME-MED}$ of MED-CME criterion becomes

$$P_{CME-MED} = \frac{1}{N^2}\sum_{i=k-N+1}^{k}\sum_{j=k-N+1}^{k} G_{\sigma\sqrt{2}}(e_{CME,i} - e_{CME,j})$$
$$-2\frac{1}{N}\sum_{i=k-N+1}^{k} G_{\sigma}(-e_{CME,i}) \quad (8)$$

3. The proposed Algorithm with Balanced Information Potentials

By inserting $e_{CMA} = |y_k|^2 - R_2$ into the proposed cost function $P_{CME-MED}$ and using a block of past output samples $Y_k = \{y_k, y_{k-1}, ..., y_{k-N+1}\}$, the cost function $P_{CME-MED}$ [8] can be rewritten as

$$P_{CME-MED} = \frac{1}{N^2}\sum_{i=k-N+1}^{k}\sum_{j=k-N+1}^{k} G_{\sigma\sqrt{2}}(|y_i|^2 - |y_j|^2)$$
$$-\frac{2}{N}\sum_{i=k-N+1}^{k} G_{\sigma}(-[|y_i|^2 - R_2]) \quad (9)$$

The first term on the right side which was the information potential for CMEs now has become the information potential for output samples so that we can express it as $V_{CME-MED}(y,y)$. We also can treat the second term on that side as an interaction among instant output powers and constant modulus, therefore it can be defined here as another information potential, $V_{CME-MED}(R_2,y)$. Then the cost function $P_{CME-MED}$ can be rewritten as $P_{CME-MED} = V_{CME-MED}(y,y) - 2\cdot V_{CME-MED}(R_2,y)$. The two information potentials $V_{CME-MED}(y,y)$, $V_{CME-MED}(R_2,y)$ contribute to minimizing the cost function (8). An important aspect to be noticed is that when $V_{CME-MED}(y,y)$ decreases, it contributes positively to minimization of the cost function, while $V_{CME-MED}(R_2,y)$ has to decrease in order to contribute to the minimization process. This leads us to the truth that the two information potentials are pushing and pulling each other on minimization process of the cost function. If we place some balanced point on the line of influences from the two information potentials, we might be able to find an optimal balance of forces between the two information potentials.

From this motivation, we propose to put a weighting factor α to the information potentials $V_{CME-MED}(y,y)$ and $V_{CME-MED}(R_2,y)$ to balance their effect on the cost function as

$$P_{proposed} = V_{CME-MED}(y,y)\cdot\alpha$$
$$-2\cdot V_{CME-MED}(R_2,y)\cdot(1-\alpha) \quad (10)$$

Now we apply the gradient descent method for the minimization of the cost function (10). The gradient is evaluated from

$$\frac{\partial P_{proposed}}{\partial W_k} = \frac{\alpha}{\sigma^2 N^2}\sum_{i=k-N+1}^{k}\sum_{j=k-N+1}^{k}(|y_i|^2 - |y_j|^2)$$
$$\cdot G_{\sigma\sqrt{2}}(|y_i|^2 - |y_j|^2)(y_j X_j^* - y_i X_i^*)$$
$$-\frac{2(1-\alpha)}{\sigma^2 N}\sum_{i=k-N+1}^{k} G_{\sigma}(|y_i|^2 - R_2)$$
$$\cdot(R_2 - |y_i|^2)\cdot y_i \cdot X_i^*, \quad (11)$$

Using this gradient, we can write the proposed algorithm (we will refer to this in this paper as Balanced-MED-CME) as

$$W_{k+1} = W_k - \mu[\frac{\alpha}{\sigma^2 N^2}\sum_{i=k-N+1}^{k}\sum_{j=k-N+1}^{k}(|y_i|^2 - |y_j|^2)$$
$$\cdot G_{\sigma\sqrt{2}}(|y_i|^2 - |y_j|^2)(y_j X_j^* - y_i X_i^*)$$
$$-\frac{2(1-\alpha)}{\sigma^2 N}\sum_{i=k-N+1}^{k} G_{\sigma}(|y_i|^2 - R_2)$$
$$\cdot(R_2 - |y_i|^2)\cdot y_i \cdot X_i^*] \quad (12)$$

where μ is the step-size for convergence control of the proposed algorithm.

4. Simulation results and Discussion

For performance evaluation, in this section, we present and discuss simulation results that illustrate the comparative performance of the proposed algorithm versus MED-CME in blind equalization. The 4 level random signal $\{\pm 3, \pm 1\}$ is transmitted to the channel and the impulse response, h_i of the channel model in [9] is

$$h_i = \frac{1}{2}\{1 + \cos[2\pi(i-2)/BW]\}, \quad i = 1,2,3 \qquad (13)$$

where BW =3.3, the eigenvalue spread ratio ESR=21.71. The number of weights is set to 11. The step-size for convergence control is $\mu = 0.006$ The data-block size $N = 20$ and the fixed kernel size $\sigma = 0.5$. The information potential weighting factor < is set to 2. The MSE curve performance of the MED-CME algorithm and the proposed is illustrated in Fig. 1.

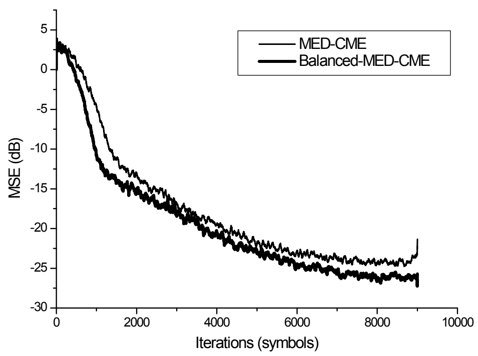

Fig. 1. MSE convergence performance

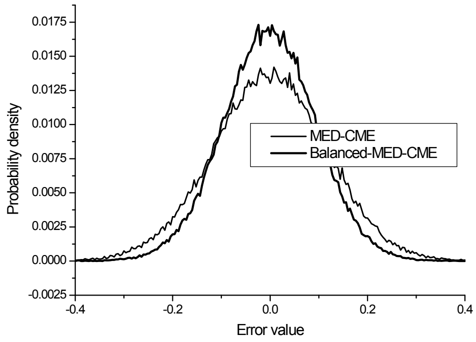

Fig. 2. Probability density for errors

In case of channel 1 with ESR=11.12, the MSE performance in Fig. 1 shows that the conventional MED-CME converges to -23 dB of steady state MSE, while the proposed algorithm reaches -27 dB. The proposed algorithm gives enhanced performance by above 3 dB of steady state MSE with faster convergence speed in comparison with MED-CME. In Fig. 2, the system error densities are shown and their performance difference is clearer showing the system error distribution of the proposed algorithm more concentrated around zero.

5. Conclusion

In this paper, a new algorithm balancing two information potentials derived from the criterion of minimizing PDF-distance between the PDF of constant modulus error and Dirac delta function is presented. By placing a appropriate weighting value on the proposed balancing equation composed of the two information potentials, the proposed method is proved to produce significantly enhanced performance in blind equalization.

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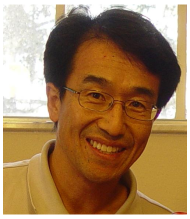

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