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온실가스배출 규제를 고려한 경제적 선박 운항속도 및 운행대수 결정

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Determining Economic Ship Speeds and Fleet Sizes Considering Greenhouse Gas Emissions

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지속적으로 강화되고 있는 국제 환경규제로, 해운사들은 선박의 온실가스 배출량을 줄이기 위한 노력을 해야 하는 상황에 직면해 있다. 본 연구에서는 총 온실가스 배출량에 대한 제약이 있는 상황에서 복수개의 항로를 운영하는 해운사에서 일 평균 선박 운영비용의 총합을 최소화하는 항로별 최적 선박대수와 운항속도를 결정하는 문제를 다룬다. 이 문제를 풀기위해 라그랑지안 휴리스틱 알고리즘을 개발하고 라그랑지안 쌍대문제를 풀어 최적해에 대한 하한값을 구한다. 제시한 알고리즘의 성능을 평가하기 위해 비용인자들의 값을 달리하면서 랜덤하게 테스트 문제들을 생성하였으며, 실험결과 제시한 알고리즘이 짧은 시간에 최적해에 매우 근접한 좋은 해를 찾음을 알 수 있었다.

Keyword : Maritime Transport, Greenhouse Gas Emissions, Ship Speed, Fleet Size

1. Introduction

Following the Kyoto Treaty and UN's sequential actions, the International Maritime Organization (IMO) and Intergovernmental Panel on Climate Change are discussing legal measures to reduce emissions of greenhouse gas (GHG) such as emission taxation and emission trading schemes. Due to the recent developments in GHG regulations, maritime transport can no longer enjoy its image as an environmentally friendly mode. In addition, since maritime transport has expanded in line with the rapid growth of international trade over the past half century, maritime transport is now regarded as a major contributor of GHG emissions and to environmental degradation. To reduce the GHG emissions, many shipping companies are using technological and operational approaches such as power and propulsion systems, ship hull design, voyage optimization by speed reduction, and renewable energy use (IMO [4]). Ship speed reduction has become popular owing to its simplicity and effectiveness in reducing GHG emissions.

Bulk and container ships are usually operated on closed routes and follow a published schedule of sailings. A route

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is a specified sequence of calling ports that each ship assigned to that route visits cyclically during its voyage. Commonly, a ship takes a few weeks/months to complete its voyage on a route and multiple ships are assigned to the route to provide cyclic services (usually weekly) to each port on the route (Ronen [7]). Many shipping companies run multiple routes and are required to obey the international regulations on total GHG emissions caused by ship operation. This paper focuses on determining economic ship speeds and fleet sizes on multiple routes according to GHG emissions.

In recent research in this field, few studies have used the speed reduction of ships as an option for reducing fuel consumption and thereby alleviating the environmental harm. For convenience of the literature review, we classify previous research articles with respect to their problem objectives : fuel consumption saving and reduction of GHG emissions. The former has been considered by Ronen [6, 7], Brown et al. [1], and Fagerholt et al. [3]. Ronen [7] performed pioneering research related to ship speed determination using the relation that fuel consumption is a cubic function of speed. He suggested cost models for determining an optimal ship speed for different types of legs (arcs in a network). Later, Ronen [7] extended his previous models by considering the weekly shipping service prevalent in the maritime industry and suggested a procedure to determine the optimal speed and fleet size (the number of deployed ships) on a single route. Brown et al. [1] suggested a linear programming model to optimize the fuel consumption for a given ship transit time. Fagerholt et al. [3] considered the problem with calling time (ship arrival time) window constraints at ports on a single route. They formulated the problem as a non-linear programming model and proposed a heuristic algorithm by discretizing the time window and formulating the problem as a shortest path model. A few research papers have recently examined the problem of reducing GHG emissions. Matsukura et al. [5] considered a ship route design problem and suggested a mixed integer programming model with the objective of minimizing CO_2 emissions while satisfying customer demand and time window constraints. Using the model and data on the container transportation network in Japan, they obtained the routes of ships with different capacities, speeds, and main engine horsepowers. Corbett et al. [2] and Wang et al. [8] developed a profit-maximization equation to estimate route-specific, economically-efficient speeds. They explored the policy impacts of a fuel tax and speed reduction mandate on CO_2 emissions.

Our paper focuses on the problem of determining the ship speeds and fleet sizes in routes in order to satisfy the maximally allowed CO_2 emission requirement. The emission requirement may be set by a company in order to reduce its GHG emissions. The objective of the problem is to minimize the bunker fuel consumption and ship time costs. In fact, our research extends the work of Ronen [7], who optimized ship speed and fleet size for a single route with weekly service cycle, by considering multiple routes with different service cycles and total CO_2 emission restriction. To solve this problem, we suggest a Lagrangian heuristic algorithm with an efficient feasible solution generation method and an optimal algorithm for the relaxed problem resulting from the Lagrangian decomposition.

The rest of this paper is organized as follows. The next section defines the problem along with a non-linear program in detail. In Section 3, we present the Lagrangian heuristic algorithm by deriving an optimal property for the Lagrangian-relaxed problem. Section 4 illustrates the execution of the algorithm using an example problem. Section 5 summarizes the computational test results on randomly generated problem instances. The final section describes the summary of research results and suggestions for future research.

2. Problem Description

The problem considered in this paper is the determination of the ship speeds and fleet sizes operating on multiple shipping routes while satisfying the maximally allowed CO_2 emission requirement. The fleet size in a route is the number of ships operating on the route. The maximally allowed CO_2 emissions may be set by a company in order to reduce the total CO_2 amount emitted from all of the ships to observe environmental regulations. We assume that ships on the routes have their minimum and maximum allowable speeds. We further assume that all ships deployed on a route have similar physical and economic characteristics and, hence, consume the same amount of fuel at the same navigation speed. In this paper, a *service cycle time* for a route is defined as the time interval between two consecutive shipping services at each port on the route.

The notations used throughout the paper are as follows :

Parameters

r	index for a route
n	number of routes
c_f	fuel price [USD/ton]
d_r	nautical distance of route r [nautical mile]
e	CO_2 emissions per ton of fuel consumption [ton]
$f(v)$	daily fuel consumption of a ship navigating at speed v [ton]
k_r	factor of converting the speed to the fuel consumption of a ship operating on route r
t_r	service cycle time of route r [hour]
pt_r	total port time during the cycle of route r [hour]
c_r	daily cost of the ship operating on route r [USD/day]
MAX_{CO_2}	maximally allowed CO_2 emission in all routes
$v_r^{\min}$	minimum allowable speed of the ship operating on route r [knot]
$v_r^{\max}$	maximum allowable speed of the ship operating on route r [knot]

Decision variables

V_r	speed of ships operating on route r [knot]
X_r	number of ships operating on route r

The objective of the problem is to minimize the average daily bunker fuel consumption and daily ship costs of all routes. To provide a shipping service to each port on route r at every service cycle time t_r, the total distance sailed by all ships on route r during the service cycle time should be equal to d_r. Therefore, the bunker fuel consumption on route r per service cycle time is equal to $f(V_r)\cdot d_r/V_r$, and the average daily bunker fuel consumption on route r is obtained as:

$$\frac{f(V_r)\cdot d_r/V_r}{t_r}$$

Obviously, the average daily cost of bunker fuel for all the ships operating on route r is :

$$c_f\cdot\frac{f(V_r)\cdot d_r}{t_r\cdot V_r}$$

Since fuel consumption can be approximated by the following well-known cubic function of speed (Ronen [6, 7]), $f(V_r)=k_r\cdot V_r^3$, the average daily cost of bunker fuel for all the ships operating on route r can be expressed as:

$$\frac{c_f\cdot k_r\cdot d_r}{t_r}\cdot V_r^2.$$

On the other hand, the daily cost of all the ships on route r is c_rX_r. The daily cost of a ship includes crew, repair and maintenance, insurance, stores and lubes, fuel for auxiliary power, administration, and (possibly) capital costs that are all the costs incurred when the ship is not sailing. For a time-chartered ship, it is the daily charter hire.

Due to the cyclic service for all ports on a route, the number of ships operating on the route should be an integer greater than or equal to the time needed to complete services for the route by a vessel divided by the service cycle time of the route, i.e.,

$$X_r=\left\lceil\frac{d_r/V_r+pt_r}{t_r}\right\rceil$$

where $\lceil\bullet\rceil$ is the ceiling function giving the smallest integer greater than or equal to •.

Based on the various cost factors and the equation for X_r defined above, the problem considered in this research can be formulated as the following non-linear integer program.

$$\textbf{[P]}\ \text{Minimize}\ \sum_{r=1}^{n}\frac{c_f\cdot k_r\cdot d_r}{t_r}\cdot V_r^2+\sum_{r=1}^{n}c_r\cdot X_r$$

subject to

$$\frac{d_r}{t_r\cdot V_r}+\frac{pt_r}{t_r}\le X_r \qquad \text{for } r=1,2,\cdots,n \qquad (1)$$

$$\sum_{r=1}^{n}\frac{e\cdot k_r\cdot d_r}{t_r}\cdot V_r^2\le MAX_{CO_2} \qquad (2)$$

$$v_r^{\min}\le V_r\le v_r^{\max} \qquad \text{for } r=1,2,\cdots,n \qquad (3)$$

$$X_r\ge 0 \text{ and integers} \qquad \text{for } r=1,2,\cdots,n \qquad (4)$$

The objective to be minimized is the sum of average daily bunker fuel consumption and daily ship costs. Constraint (1) is the non-equalities for calculating the number of ships on each route. Constraint (2) represents that the CO_2 amount emitted by all vessels should not exceed the maximally allowed CO_2 emissions. Constraint (3) represents that the ship speed on a route is bounded by its minimum and maximum speeds. Constraint (4) is the integer restriction on the number of ships operating on a route.

3. Solution Algorithm

This section presents the Lagrangian relaxation method used to solve the problem. We show that the relaxed problem can be solved optimally owing to the property proposed in this research. Then, we suggest a Lagrangian heuristic along with a method of finding feasible solutions while considering the trade-offs among the various cost factors.

The Lagrangian relaxation approach is based on the dualization of constraint (2) with a Lagrangian multiplier λ. The relaxed problem resulted from the dualization is :

[LR] Minimize

$$\sum_{r=1}^{n} \frac{(c_f+\lambda \cdot e) \cdot k_r \cdot d_r}{t_r} \cdot V_r^2 + \sum_{r=1}^{n} c_r \cdot X_r - \lambda \cdot MAX_{CO2}$$

subject to (1), (3), (4), and $\lambda \geq 0$

In the above objective function, the last term is a constant that can be eliminated from further consideration. Therefore, the problem can be decomposed into n mutually independent problems, i.e., problem [LR$_r$] for r = 1, 2, $\cdots$, n. The following property characterizes the optimal solutions of the problem [LR$_r$].

Property 1 : *For given* λ *and* X_r, *in the problem* [LR$_r$] *there is an optimal speed with*

$$V_r = \begin{cases} \dfrac{d_r}{t_r \cdot X_r - pt_r} & \text{if } v_r^{\min} \leq \dfrac{d_r}{t_r \cdot X_r - pt_r} \leq v_r^{\max} \\ v_r^{\min} & \text{if } \dfrac{d_r}{t_r \cdot X_r - pt_r} < v_r^{\min} \end{cases}$$

Proof. Since X_r is given, the problem [LR$_r$] can be rewritten in the following form:

[LRr'] Minimize $\dfrac{(c_f+\lambda \cdot e) \cdot k_r \cdot d_r}{t_r} \cdot V_r^2 + c_r \cdot X_r$

subject to

$$V_r \geq \frac{d_r}{t_r \cdot X_r - pt_r} \text{ and } v_r^{\min} \leq V_r \leq v_r^{\max}$$

In the above model, the last term in the objective function is a constant eliminated from consideration. Since the objective function is to minimize the ship speed V_r, the optimal speed is the minimum V_r in its feasible region, which leads to this property. The case $d_r/(t_r \cdot X_r - pt_r) > v_{\max}$ clearly implies that given X_r renders the problem infeasible.

Property 1 ensures that the optimal solution of the problem [LR$_r$] can be obtained if X_r is determined optimally. Using Property 1, we suggest a procedure for finding an optimal solution of the problem [LR$_r$] by extending the algorithm of Ronen [7] as follows.

Procedure 1 : (Obtaining an optimal solution of the problem [LR$_r$])

Step 1-1 : Let the best total cost TC_r^* be an arbitrary large number and set $V_r = v_r^{\max}$. Calculate the number of ships using:

$$X_r = \left\lceil \frac{d_r}{t_r \cdot V_r} + \frac{pt_r}{t_r} \right\rceil$$

Step 1-2 : Determine the ship speed by Property 1 as :

$$V_r = v_r^{\min} \text{ if } \frac{d_r}{t_r \cdot X_r - pt_r} < v_r^{\min} \text{ and}$$

$$V_r = \frac{d_r}{t_r \cdot X_r - pt_r}, \text{ otherwise.}$$

Step 1-3 : Calculate the resulting total cost TC_r as :

$$TC_r = \frac{(c_f+\lambda \cdot e) \cdot k_r \cdot d_r}{t_r} \cdot V_r^2 + c_r \cdot X_r$$

If $TC_r < TC_r^*$, set $TC_r^* = TC_r$, $V_r^* = V_r$ and $X_r^* = X_r$. If $V_r = v_r^{\min}$, stop.

Step 1-4 : Set X_r = X_r + 1 and go to Step 1-2.

Then, we can compute a lower bound on the optimal solution value of the original problem [P]. For a given λ, $L(\lambda)$, which is an optimal solution value of [LR] and also a lower bound for [P], can be obtained as follows :

$$L(\lambda)=\sum_{r=1}^{n}\frac{(c_f+\lambda\cdot e)\cdot k_r\cdot d_r}{t_r}\cdot V_r^{*2}+\sum_{r=1}^{n}c_r\cdot X_r^{*}-\lambda\cdot MAX_{CO2}$$

where V_r^* and X_r^* are optimal solutions of the relaxed problem [LR].

To find a better lower bound, we need to find better values for the Lagrangian multiplier. In the following, we present a solution method for the following Lagrangian dual problem to find the best multiplier.

$$L(\lambda^*)=\max_{\lambda\geq 0} L(\lambda)$$

By using a subgradient optimization algorithm to solve the Lagrangian dual problem, we solve a relaxed problem optimally to compute subgradients for the Lagrangian multiplier and change the multiplier. Given the multiplier at iteration w, the multiplier for the next iteration is generated by :

$$\lambda_w=\max\left[0,\ \lambda_w+\sigma_w\cdot\left(\sum_{r=1}^{n}\frac{e\cdot k_r\cdot d_r}{t_r}\cdot V_r^{*2}-MAX_{CO2}\right)\right]$$

where V_r^* is the optimal solution for the problem [LR] obtained at iteration i and σ_w is a positive scalar step size at iteration w. A commonly used step size at iteration w is :

$$\sigma_w=\phi_w\cdot\frac{Z^*-L(\lambda_w)}{(\sum_{r=1}^{n}e\cdot k_r\cdot d_r/t_r\cdot V_r^{*2}-MAX_{CO2})^2}$$

where $\phi_w\leq 2$ is a positive scalar, and Z^* the best feasible solution value of the problem [P1]. Initially, the value for ϕ_w is set to be equal to 2 and halved when the solution of the relaxed problem [LR] has not increased for a given number of iterations.

Now, we explain a Lagrangian heuristic to obtain a feasible solution to the problem [P] by perturbing the solution of the relaxed problem [LR]. The solution obtained from the relaxed problem is infeasible to the problem [P] and is therefore modified iteratively in such a way that constraint (2) is satisfied. The solution is modified by decreasing the excess speed by increasing the number of ships while considering the trade-offs between the various cost factors.

Consider the speed reduction of ships on route r with $V_r>v_r^{\min}$. Let the solutions before (after) the speed reduction be $V_r(V_r')$ and $X_r(X_r')$. To reduce their speeds, we increase the number of ships X_r by one using a similar idea to that presented in Procedure 1. Then, the ship speed is determined as :

$$V_r'=\begin{cases}\dfrac{d_r}{t_r\cdot X_r'-pt_r} & \text{if } v_r^{\min}\leq\dfrac{d_r}{t_r\cdot X_r'-pt_r}\\ v_{\min} & \text{if } \dfrac{d_r}{t_r\cdot X_r'-pt_r}<v_r^{\min}\end{cases}$$

The resulting cost increase can be calculated as :

$$\Delta TC_r=\frac{(c_f+\lambda\cdot e_r)\cdot k_r\cdot d_r}{t_r}\cdot(V_r^2-V_r'^2)+c_r$$

Then, among the routes, we select the best route r^* that minimizes the cost increase :

$$r^*=\arg\min[\Delta TC_r\mid V_r>v_r^{\min}]\quad r=1,2,\cdots,n$$

This routine continues until the over emission of GHG is eliminated, i.e., constraint (2) is satisfied.

An overall procedure for obtaining a feasible solution is summarized below.

Procedure 2 : (Obtaining a feasible solution)

Step 2-1 : Calculate the over emission of GHG of the solution of the relaxed problem [LR] using :

$$OE=\sum_{r=1}^{n}\frac{e\cdot k_r\cdot d_r}{t_r}\cdot V_r^2-MAX_{CO2}$$

If $OE\leq 0$, stop. Otherwise, go to Step 2-2.

Step 2-2 : From route r = 1 to n with $V_r > v_r^{\min}$, set $X_r = X_r + 1$ and calculate the modified solution and the increased cost using the equations described above. Update the solution of the best route r^* minimizing the cost increase and the over emission of GHG using :

$$OE = OE - \frac{e_{r*} \cdot k_{r*} \cdot d_{r*}}{t_{r*}} \cdot (V_{r*}^2 - V_{r*}'^2)$$

where V_{r*}' is the updated ship speed of the best route r^*.

Step 2-3 : If $OE \leq 0$, stop and otherwise, go to Step 2-2.

The Lagrangian heuristic proposed in this paper can now be summarized as follows. The algorithm is terminated when the iteration count (w) reaches a predetermined limit (W).

Procedure 3 : (The Lagrangian heuristic algorithm)

Step 3-1 : Set w = 1 and λ = 0. Let the feasible solution value and lower bound be an arbitrary large number and 0, respectively.

Step 3-2 : Solve the problem [LP$_r$] for r = 1, 2, ⋯, n, using the algorithm in Procedure 1.

Step 3-3 : Obtain a lower bound by computing the objective function value using the solution of [LR] using the method described earlier. Update the lower bound solution once the lower bound is improved. Then, find a feasible solution using Procedure 2 and update the best upper bound solution Z^* once it is improved. Stop if the best feasible solution value equals the lower bound.

Step 3-4 : Set $w = w+1$, If $w > W$, stop. Otherwise, update the Lagrangian multiplier with the step size using the methods described earlier and go to Step 3-2.

4. Example Problem

In this section, an example problem is solved to show the execution of the algorithm proposed in this paper. Consider five example routes with fuel price c_f = 194.229, CO_2 emissions e = 3.170, the maximally allowed CO_2 emission MAX_{CO2} = 3801.240, minimum allowable speed of the ship $v_r^{\min}$ = 13, and maximum allowable speed of the ship $v_r^{\max}$ = 26. The nautical distance d_r, converting factor k_r, service cycle time t_r, total port time pt_r, and daily cost of the ship c_r are summarized in <Table 1>.

<Table 1> Example Data

Route (r)	1	2	3	4	5
d_r	14852.901	8189.184	14342.326	6786.248	13575.701
k_r	0.014	0.015	0.013	0.010	0.015
t_r	84	168	168	84	336
pt_r	153.706	82.594	233.315	100.836	172.817
c_r	21203.803	6859.340	11756.035	14000.671	11382.183

The following calculation summarizes the Lagrangian heuristic algorithm's run on the data of the example problem.

Initialization

At first, we set w = 1 and λ = 0. The iteration limit W was set to 500, and ϕ_w was set to 2 initially and halved if the lower bound had not been improved in 10 iterations. Also, the feasible solution value and lower bound were set to an arbitrary large number and 0, respectively.

Lower bound generation

We solve the subproblem [LP$_r$] for all routes using

Procedure 1 : As an example, the run of Procedure 1 for route 1 is given below.

Step 1-1.

$TC_1^* = M$ (an arbitrary large number)

$V_1 = v_1^{\max} = 26$

$$X_r = \left\lceil \frac{d_1}{t_r \cdot V_1} + \frac{pt_r}{t_1} \right\rceil = \left\lceil \frac{14852.901}{84.26} + \frac{153.706}{84} \right\rceil$$

Step 1-2.

$$V_1 = \frac{d_1}{t_1 \cdot X_1 - pt_1} = \frac{14852.901}{84.9 - 153.706} = 24.661$$

which is more than $v_1^{\min}$ (13 knot).

Step 1-3.

$$TC_1 = \frac{(c_f + \lambda \cdot e) \cdot k_1 \cdot d_1}{t_1} \cdot V_1^2 + c_1 \cdot X_1$$

$$= 482509.853$$

Since $TC_1 < TC_1^*$, solutions are updated as follows :
$TC_1^* = 482509.853,\ V_1^* = 24.661,\ X_1^* = 9.$

Step 1-4.

Increase the number of ships operating on route 1 by one, i.e.,

$$X_1 = X_1 + 1 = 9 + 1 = 10$$

and go to Step 1-2.

This routine continues until $V_1 = v_1^{\min} = 13$. After all the routines, the obtained optimal solution for route 1 is $TC_1^* = 395831.472$, $V_1^* = 15.830$, and $X_1^* = 13$. The optimal solution of the subproblem [LP_r], V_r and X_r, is given in <Table 2>.

<Table 2> A Lower-Bound Solution of the Example Problem

Route (r)	1	2	3	4	5
V_r	15.830	13.894	13	13.930	13
X_r	13	4	8	7	4

Then, by using the optimal solutions of the subproblem [LP_r], we obtained a lower bound $L(\lambda)$ on the optimal solution value of the original problem [P] as

$$L(\lambda) = \sum_{r=1}^{5} \frac{(c_f + \lambda \cdot e) \cdot k_r \cdot d_r}{t_r} \cdot V_r^{*2} + \sum_{r=1}^{5} c_r \cdot X_r^* - \lambda \cdot MAX_{CO2} = 776753.541$$

Upper bound generation

The next step is to obtain an upper bound, which is feasible with respect to the original problem. The following calculation summarizes the run of Procedure 2.

Step 2-1.

The over emission of GHG of the solution of the relaxed problem is:

$$OE = \sum_{r=1}^{5} \frac{e \cdot k_r \cdot d_r}{t_r} \cdot V_r^2 - MAX_{CO2} = 51.890$$

which is not less than zero. Therefore, the over emission of GHG should be eliminated.

Step 2-2.

The candidate routes for the speed reduction are routes 1, 2, and 4 since routes 3 and 5 have the minimum speed $v_r^{\min} = 13$.

Route 1 :

$$X_1' = X_1 + 1 + 14$$

$$V_1' = \frac{d_1}{t_1 \cdot X_1' - pt_1} = 14.529$$

$$\Delta TC_1 = \frac{(c_f + \lambda \cdot e_1) \cdot k_1 \cdot d_1}{t_1} \cdot (V_1^2 - V_1'^2) + c_1 = 40142.643$$

Route 2 :

$$X_2' = X_2 + 1 = 5$$

$$V_2' = \frac{d_2}{t_2 \cdot X_2' - pt_2} = 10.812 < v_2^{\min}$$

Hence, $V_2' = v_2^{\min} = 13$

$$\Delta TC_2 = \frac{(c_f + \lambda \cdot e_2) \cdot k_2 \cdot d_2}{t_2} \cdot (V_2^2 - V_2'^2) + c_2 = 10215.372$$

Route 4 :

$$X_4' = X_4 + 1 = 8$$

$$V_4' = \frac{d_4}{t_4 \cdot X_4' - pt_4} = 11.881 < v_4^{\min}$$

Hence, $V_4' = v_4^{\min} = 13$

$$\Delta TC_4 = \frac{(c_f + \lambda \cdot e_4) \cdot k_4 \cdot d_4}{t_4} \cdot (V_4^2 - V_4'^2) + c_4 = 10215.372$$

The best route is selected from

$r^* = \arg\min \Delta TC_r = 2$ or 4

$r = 1, 2, 4$

Then, if selecting route 2 among routes 2 and 4 with the same cost increase, the over emission of GHG is updated by

$$OE = OE - \frac{e_2 \cdot k_2 \cdot d_2}{t_2} \cdot (V_2^2 - V_2'^2) = -2.883$$

Step 2-3.

Since the updated over emission OE is less than zero, we stopped the procedure. The feasible solution of the example problem is that $X_2 = 5$, $V_2 = 13$, and the other solutions are the same as the solution in <Table 2>.

Then, the new and best upper bound is

$$Z^* = \sum_{r=1}^{5} \frac{c_f \cdot k_r \cdot d_r}{t_r} \cdot V_r^2 + \sum_{r=1}^{5} c_r \cdot X_r = 780256.849$$

Lagrangian multiplier update

The next step is to update the Lagrangian multiplier with the current lower and best bounds and the lower bound solution obtained above.

$$\sigma_2 = \phi_1 \cdot \frac{Z^* - L(\lambda_1)}{(\sum_{r=1}^{5} e \cdot k_r \cdot d_r/t_r \cdot V_r^{*2} - MAX_{CO2})^2}$$

$$= 2 \cdot \frac{780256.849 - 776753.541}{51.890^2} = 2.602$$

$$\lambda_2 = \max[0,\ \lambda_1 + \sigma_2 \cdot (\sum_{r=1}^{5} \frac{e \cdot k_r \cdot d_r}{t_r} \cdot V_r^{*2} - MAX_{CO2})]$$

$$= \max[0,\ 0 + 2.602 \cdot 51.890] = 135.018$$

This routine repeated 500 iterations and we obtained a final solution in <Table 3>.

<Table 3> A Final Solution of the Example Problem

Route (r)	1	2	3	4	5
V_r	14.529	13.894	13	13.93	13
X_r	14	4	8	7	4

Its solution value was 779018.503 and the lower bound was 777133.773.

5. Computational Results

To demonstrate the performance of the heuristic algorithm proposed in this paper, computational tests were performed on various test instances with different problem sizes and different values of the problem parameters, i.e., maximally allowed CO_2 emission, fuel price, and daily ship cost. The Lagrangian heuristic requires specific values for several parameters. After a preliminary experiment, the parameters in these experiments were set as follows: the iteration limit W was set to 500, and ϕ_w was set to 2 initially and halved if the lower bound had not been improved in 10 iterations. To demonstrate the effectiveness of the Lagrangian heuristic algorithm, two performance measures were used in the test: percentage deviation from the lower bound obtained by solving the Lagrangian dual problem, and percentage deviation from the optimal solution value or lower bound obtained using LINGO 12.0, commercial linear and non-linear programming software. Here, we set the time limit for the run of the LINGO to one hour due to the excessive computational burden to obtain the optimal solutions. However, it is important to note that lower bounds can be obtained if the LINGO is allowed to run for longer than one hour. The algorithms tested in this paper were coded in C and run on a PC with a Pentium processor operating at 1.73GHz.

For the test on various sized problems, we generated 45 test instances, i.e., 5 for each of the following number of routes: 20, 40, 60, 80, 100, 200, 300, 400, and 500. Data were generated based on the data setting in Ronen [7]. The fuel price c_f was generated from $U(100, 500)$ and the nautical distance of a route d_r was generated as $U(5000, 15000)$, respectively, where $U(a, b)$ is the uniform distribution with a range of $[a, b]$. The CO_2 emission factor e was set to 3.17 according to Corbett et al. [2]. The factor for converting the speed to the fuel consumption, k_r, was generated from $U(0.01, 0.016)$. The cycle duration (hours) T_r was set to 84 (i.e., 0.5 week), 168 (i.e., 1 week), and 336 (i.e., 2 week) with probabilities 0.3, 0.6, and 0.1, respectively. The total port time (hour) pt_r was generated from $d_r \cdot U(0.01, 0.02)$ for each route. The daily cost of a ship c_r was generated from $U(5000, 30000)$. The maximally allowed CO_2 emissions was set to $(1-\alpha) \cdot L + \alpha \cdot T$, where α was generated from $U(0, 1)$, $L = \sum_{r=1}^{n} e \cdot k_r \cdot d_r/t_r \cdot (V_r^*)^2$ and $T = \sum_{r=1}^{n} e \cdot k_r \cdot d_r/t_r \cdot (v_r^{\min})^2$. Here, V_r^* is the ship speed of route r obtained from Procedure 1 by setting $\lambda = 0$. The restriction of the maximally allowed CO_2 emissions becomes tighter as the factor α increases. The maximum and minimum allowable ship speeds were set to 26 and 13 knots, respectively.

The test results summarized in <Table 4> show the mean values and standard deviations of the percentage deviations from the lower bounds and optimal solution values or lower bounds (if the LINGO runs for more than one hour). The mean and standard deviations in the below tables are those of the percentage deviations for five test instances. The Lagrangian heuristic proposed in this paper provided near optimal solutions for all instances. That is, the overall mean percentage deviation from the lower bound was 0.03% and that from the optimal solution or lower bound by LINGO was 1.27%, which implied that the lower bound by the Lagrangian dual problem was tighter than that by LINGO. The percentage deviation from the lower bound by Lagrangian dual problem improved with increasing number of routes, which implied that the Lagrangian heuristic algorithm can provide better solutions with more routes. This result may be due to the smoothing effect of solution values which may occur more often with increasing problem size. For example, the percentage deviation of a large- sized problem is lower than that of a small-sized problem if the large-sized problem's difference between the lower bound (or optimal solution value) and the solution value of the Lagrangian heuristic is the same as that of small-sized problems, since the large-sized problem may have bigger solution value than the small-sized problem. On the contrary, the percentage deviation from the optimal solution value or the lower bound by LINGO was degraded with increasing number of routes since the lower bound by LINGO became looser as it increased. The computational times of the Lagrangian heuristic were significantly shorter than those of LINGO, that is, less than 2 seconds were required while a time more than the time limit of one hour was required by LINGO for many of the problems. This implies that the Lagrangian heuristic suggested in this paper can be used to solve practical- sized problems within a reasonable computational time. As anticipated, the computational time required increased with increasing number of routes.

The effects of the maximally allowed CO_2 emission were tested with one test instance for each number of routes with factor α ranging from 0.1 to 0.9 at intervals of 0.2 and with the other data generated using the same method in the test of different problem sizes. The maximally allowed CO_2 emission restriction became as α increases from 0.1 to 0.9. <Table 5> summarizes the test results, which showed that the percentage deviation was degraded as the factor became bigger, as expected. However, no trend for the effect of the different factors on the computational time was evident.

The effects of different fuel prices were analyzed using one test instance for each number of routes with fuel prices

<Table 4> Test Results on Different Problem Sizes

No. Route	LB DEV[a]		LINGO DEV[b]		CPU Seconds			
					Lagrangian Heuristic		LINGO	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
20	0.13	0.05	0.03	0.04	0.03	0.01	243.09	147.55
40	0.05	0.04	0.16	0.66	0.08	0.02	2823.53	1397.51
60	0.02	0.02	1.35	0.58	0.13	0.05	3601.08	0.53
80	0.02	0.02	1.52	0.72	0.16	0.02	3600.62	0.42
100	0.03	0.03	1.46	0.67	0.19	0.05	3601.02	0.76
200	0.01	0.01	1.61	0.81	0.49	0.23	3600.79	0.57
300	0.01	0.00	1.66	0.82	1.16	0.55	3600.59	0.71
400	0.01	0.01	1.61	0.81	1.26	0.56	3600.45	0.36
500	0.01	0.00	1.65	0.84	1.60	0.72	3601.25	0.71
Overall	0.03	0.04	0.87	0.81	0.57	0.67	2567.08	1554.61

[a] percentage deviation from the lower bound obtained by solving the Lagrangian dual problem.

[b] percentage deviation from the optimal solution or lower bound obtained from LINGO.

ranging from 100 to 500 USD at intervals of 100 USD and with the other data generated using the same method in the test of different problem sizes. The test results summarized in <Table 6> do not show any trend for the effect of fuel price on the percentage deviations and computational time.

The effects of different daily ship costs on the algorithm performance were also analyzed using one test instance for each number of routes with the fuel price ranging from 5000 to 25000 USD at intervals of 5000 USD and with the other data generated using the same method in the test of different problem sizes. As can be seen from <Table 7>, the percentage deviations and the computation time increased with increasing daily ship cost, which can be explained by the effect of the greater daily ship costs in increasing the ship speed which in turn increased the search space of the algorithm.

<Table 5> Effect of Maximally Allowed CO_2 Emissions

CO_2 factor (α)	LB DEV		LINGO DEV		CPU Seconds			
					Lagrangian Heuristic		LINGO	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
0.1	0.02	0.04	0.86	0.32	0.41	0.38	3275.14	977.34
0.3	0.03	0.06	0.87	0.46	0.49	0.48	3120.67	959.07
0.5	0.03	0.06	1.09	0.45	0.58	0.60	3428.07	516.98
0.7	0.03	0.03	1.33	0.76	0.42	0.42	2994.99	1202.28
0.9	0.07	0.08	1.98	1.09	0.44	0.51	3350.47	750.90

See footnotes in <Table 4>.

<Table 6> Effect of Fuel Price

Fuel Price(USD)	LB DEV		LINGO DEV		CPU Seconds			
					Lagrangian Heuristic		LINGO	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
100	0.04	0.03	0.53	0.22	0.98	1.28	3252.18	1046.79
200	0.05	0.07	0.85	0.32	1.48	2.58	3221.49	1138.03
300	0.03	0.03	1.04	0.57	0.55	0.69	3023.42	1208.77
400	0.02	0.01	1.33	0.56	0.42	0.43	3329.17	814.06
500	0.03	0.03	1.64	0.63	0.43	0.47	3226.19	1122.91

See footnotes in <Table 4>.

<Table 7> Effect of Daily Ship Cost

Daily Ship Cost(USD)	LB DEV		LINGO DEV		CPU Seconds			
					Lagrangian Heuristic		LINGO	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
5000	0.02	0.02	1.97	0.75	0.40	0.47	3288.62	937.41
10000	0.02	0.02	1.15	0.48	0.57	0.55	3214.21	1159.96
15000	0.03	0.05	0.57	0.24	1.20	1.52	3213.75	1160.59
20000	0.03	0.03	0.38	0.16	1.37	2.08	3216.01	1153.21
25000	0.07	0.10	0.40	0.18	1.59	2.50	3297.71	909.53

See footnotes in <Table 4>.

6. Concluding Remarks

We considered the problem of determining economic ship speeds and fleet sizes on multiple routes while considering the total GHG emission restrictions with the aim of minimizing the sum of average daily bunker fuel consumption and ship time costs. We used a Lagrangian relaxation approach in which the relaxed problem is solved by an extended algorithm of an existing algorithm and a feasible solution is generated considering the cost trade-offs. The test results on a number of randomly generated problems showed that the Lagrangian heuristic provided near optimal solutions in a very short computation time, which demonstrated the practical viability of this tool. The sensitivity analyses conducted with the three parameters (maximally allowed CO_2 amount emitted, fuel price, and daily ship cost) demonstrated the sensitivity of the heuristic performance to the maximally allowed CO_2 amount emitted and the daily ship cost, but not to the fuel price.

This research may be extended in the following ways. First, ship speed variability in different legs of the route needs to be considered as a future research topic. Second, the ship schedule may become less flexible as the time gets closer to the departure day, so a time window constraint of the ship calling time at ports should be considered.

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