

미지의 파라미터를 갖는 이동 로봇들을 위한 선도-추종 방법 기반 적응 군집 제어

논 문

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Leader-following Approach Based Adaptive Formation Control for Mobile Robots with Unknown Parameters

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Abstract - In this paper, a formation control method based on the leader-following approach for nonholonomic mobile robots is proposed. In the previous works, it is assumed that the followers know the leader's velocity by means of communication. However, it is difficult that the followers correctly know the leader's velocity due to the contamination or delay of information. Thus, in this paper, an adaptive approach based on the parameter projection algorithm is proposed to estimate the leader's velocity. Moreover, the adaptive backstepping technique is used to compensate the effects of a dynamic model with the unknown time-invariant and time-varying parameters. From the Lyapunov stability theory, it is proved that the errors of the closed-loop system are uniformly ultimately bounded. Simulation results illustrate the effectiveness of the proposed control method.

Key Words : Adaptive formation control, Leader-following approach, Time-varying parameters, Projection algorithm, Backstepping technique.

1. INTRODUCTION

Over the past decades, the formation control of mobile robots has been focused by many research communities [1]-[2], because the control of a group of mobile robots has the advantages of efficiency and simplicity in many fields such as the explosives detection, the freight transportation, and so on. There have been three approaches for the formation control: virtual structure [3]-[5], behavior based [6]-[7] and leader-following approaches [8]-[10]. Especially, the leader-following approach has been widely used because it is easy to add new robots to the group.

Because of this advantage, several control techniques have been developed based on the leader-following approach. Shao et al. [8] designed the control inputs using the velocity of each robot based on the perfect velocity assumption [12]. However, it is not easy to release the perfect velocity assumption due to the effects of the dynamic model which contains the inertia, the centripetal force and so on. Thus, to deal with the dynamic model of the mobile robot, the neural network

and the sliding mode control were used in [9]-[10]. However, these control schemes require the leader's velocity, which is difficult to know correctly due to the contamination or the delay of information, to design the control inputs. Moreover, the unknown velocity information leads to the uncertainties in the dynamic model. In our previous paper [11], we do not require the leader's velocity, however, the perfect velocity assumption is still required because it does not consider the dynamic model of the mobile robot.

In this paper, an adaptive formation control based on the leader-following approach is proposed, which does not require the leader robot's velocity. The velocity information, which is considered as the time-varying parameters, is estimated by the projection algorithm [13]. The adaptive backstepping technique is used to compensate the effects of the uncertain dynamic model. Then, from the Lyapunov stability theorem, it is proved that all signals in the designed control system are uniformly ultimately bounded.

This paper is organized as follows: In Section 2, some preliminaries about the kinematics and dynamics of the single mobile robot and the group of the mobile robots are introduced. In Section 3, the control input based on the adaptive backstepping technique for the follower robots is derived. It is shown that the stability of the formation system is guaranteed by the Lyapunov method. Simulation results are presented in Section 4, and some concluding remarks are provided in Section 5.

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2. PRELIMINARIES

2.1 Kinematic and Dynamic Models of a Mobile Robot

In this paper, we consider a nonholonomic mobile robot having two actuated wheels. The kinematic and dynamic models of each mobile robot can be described by [4], [14]

$$\begin{aligned} \dot{q} &= S(q)z, \\ \overline{M}\dot{\eta} + \overline{C}(\dot{q})\eta + \overline{D}\eta &= \tau, \end{aligned} \quad (1)$$

where

$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}, S(q) = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix}, z = \begin{bmatrix} v \\ \omega \end{bmatrix}, \overline{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix},$$

$$\overline{C} = \begin{bmatrix} 0 & \dot{\theta} \\ -\dot{\theta} & 0 \end{bmatrix}, \overline{D} = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}, m_{11} = m_{22} = \frac{r^2}{4R^2}(mR^2 + I) + I_w,$$

$$m_{12} = m_{21} = \frac{r^2}{4R^2}(mR^2 - I), m = m_c + 2m_w, c = \frac{r^2}{2R^2}m_c d,$$

$$I = m_c d^2 + 2m_w R^2 + I_c + 2I_m, \eta = [v_r \ v_l]^T.$$

In these expressions, x and y are the coordinates of the center of the mass of the mobile robot, θ is the heading angle of the mobile robot. v and ω are the linear and angular velocities of the mobile robot, respectively. v_r and v_l are the angular velocities of the right and left wheels, respectively. τ is the torque of the mobile robot. R is the half of the width of the mobile robot, and r is the radius of the wheel. d is the distance between the center of the mass and the axis of the wheels of the mobile robot. m_c and m_w are the mass of the body and wheels of the mobile robot, respectively. I_c , I_w and I_m are the moment of inertia of the body about the vertical axis through the point of the center of the mass, the wheel about the wheel diameter and the wheel about the wheel axis, respectively. d_{11} and d_{22} are the positive damping coefficients.

The relationship between η and z are as follows:

$$\eta = \begin{bmatrix} a_1 & a_2 \\ a_1 & -a_2 \end{bmatrix} z, \quad (2)$$

where $a_1 = r^{-1}$ and $a_2 = r^{-1}R$.

2.2 Leader-following Based Formation Model of Mobile Robots

The formation model is designed based on the leader-following approach. To design the control input, the $l-\Psi$ model in [9] is used. The objective of the $l-\Psi$ model is to maintain the desired relative distance and the angle between the leader and the followers.

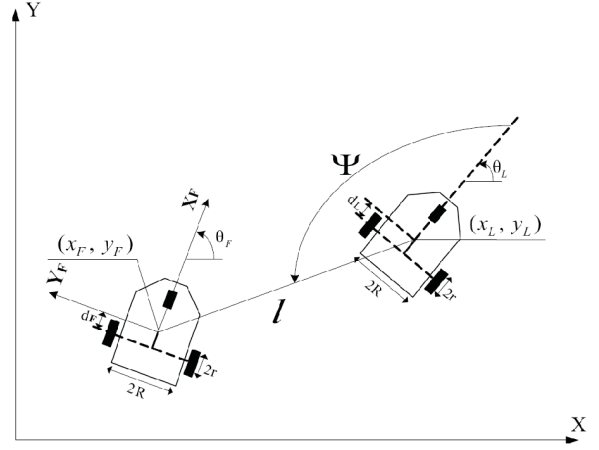


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Fig. 1 Formation model of mobile robots

Fig. 1 shows the formation model of mobile robots, where l is the relative distance between the center of the mass of the leader and the center of the mass of the follower, and Ψ is the relative angle of the center of the follower from the heading angle of the leader. In addition, subscripts “L” and “F” are used to identify the leader robot and the follower robots, respectively.

The relative distance l can be divided into x component and y component of Cartesian coordinates as follows:

$$l(t) = \sqrt{l_x^2(t) + l_y^2(t)}, \quad (3)$$

where

$$\begin{aligned} l_x(t) &= -l(t)\cos(\Psi(t) + \theta_L(t)) = x_L(t) - d_L\cos\theta_L(t) - x_F(t), \\ l_y(t) &= -l(t)\sin(\Psi(t) + \theta_L(t)) = y_L(t) - d_L\sin\theta_L(t) - y_F(t). \end{aligned} \quad (4)$$

Using (1), the time derivative of (3) can be written as

$$\dot{l} = v_F\cos\phi - v_L\cos\Psi + d_F\omega_F\sin\phi, \quad (5)$$

where $\phi(t) = \Psi(t) + \theta_L(t) - \theta_F(t)$, v_L and ω_L are the linear and angular velocities of the leader robot, v_F and ω_F are the linear and angular velocities of the follower robot, respectively. The relative angle Ψ is described as follows:

$$\Psi(t) = \arctan\frac{l_y(t)}{l_x(t)} - \theta_L(t) + \pi. \quad (6)$$

Using (1), the time derivative of (6) can be written as

$$\dot{\Psi} = \frac{1}{l}(v_L\sin\Psi - v_F\sin\phi + d_F\omega_F\cos\phi - l\omega_L). \quad (7)$$

Assumption 1: The velocities of the leader robot satisfy the following conditions:

$v_L(t), \omega_L(t) > 0$, $|v_L(t)| \leq \Omega_v$, and $|\omega_L(t)| \leq \Omega_\omega$ for $0 \leq t < t_f$ where Ω_v and Ω_ω are positive constants.

Assumption 2: The time derivative of velocities of the leader robot are smooth and satisfy the following conditions:

$|\dot{v}_L(t)| \leq \beta_v$, $|\dot{\omega}_L(t)| \leq \beta_\omega$ for $0 \leq t < t_f$ where β_v and β_ω are positive constants.

Assumption 3: The initial error between the heading angles of the leader and the follower is less than π .

2.3 Projection Algorithm

In this paper, the projection algorithm is used to estimate the information of the leader robot's velocity. The definition and the properties of the projection algorithm are as follows [13]:

Definition 1: $p(t)$ is the unknown time-varying parameter vector and exists in a closed ball of the arbitrary known radius $\delta > 0$. Ω is a set which consists of $p(t)$. Assume that $\hat{p}(0) \in \Omega$ and $p(t)$ is estimated as $\hat{p}(t)$, using the projection rule which is defined as follows:

$$\text{Proj}(\xi, \hat{p}) = \begin{cases} \xi, & \text{if } \hat{p}^2 \leq \delta^2 \\ \xi, & \text{if } \hat{p}^2 \geq \delta^2 \text{ and } \hat{p}\xi \leq 0 \\ \xi - \frac{(\hat{p}^2 - \delta^2)\hat{p}\xi}{\epsilon \hat{p}^2}, & \text{otherwise} \end{cases}$$

where ϵ is an arbitrary known positive constant.

Property 1: $\|\hat{p}\| \leq \delta + \epsilon, \forall t > 0$.

Property 2: $\tilde{p}(t)^T \|\text{Proj}(\xi, \hat{p}(t))\| \geq \tilde{p}(t)^T \xi$ where $\tilde{p} = p - \hat{p}$.

3. CONTROLLER DESIGN

Using (4), the errors between the actual position and the desired relative position are defined as follows:

$$\begin{bmatrix} \bar{l}_x \\ \bar{l}_y \\ \bar{\theta} \end{bmatrix} = \begin{bmatrix} l_x - l_{dx} \\ l_y - l_{dy} \\ \theta_L - \theta_F \end{bmatrix}, \quad (8)$$

where

$$l_{dx} = -l_d \cos(\Psi_d + \theta_L), \quad l_{dy} = -l_d \sin(\Psi_d + \theta_L), \quad (9)$$

l_d and Ψ_d denote the desired relative distance and angle, respectively. By coordinate transform, the error system can be redefined as follows:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\theta_F & \sin\theta_F & 0 \\ -\sin\theta_F & \cos\theta_F & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{l}_x \\ \bar{l}_y \\ \bar{\theta} \end{bmatrix}. \quad (10)$$

Using (4)-(7) and (9), the time derivative of (10) can be written as follows:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} -v_F + v_L \cos e_3 + \omega_F e_2 - \omega_L l_d \sin(\Psi_d + e_3) \\ -\omega_F e_1 + v_L \sin e_3 - d_F \omega_F + \omega_L l_d \cos(\Psi_d + e_3) \\ \omega_L - \omega_F \end{bmatrix}. \quad (11)$$

Now, we design the actual control input τ to stabilize (11) by the following two steps.

[Step 1] To design the virtual control inputs of v_F and ω_F , we take the Lyapunov function candidate as follows:

$$V_1 = \frac{1}{2} \{e_1^2 + e_2^2 + \gamma_1^{-1} \tilde{v}_L^2 + \gamma_2^{-1} \tilde{\omega}_L^2\}. \quad (12)$$

where $\tilde{v}_L = v_L - \hat{v}_L$, $\tilde{\omega}_L = \omega_L - \hat{\omega}_L$, γ_1 and γ_2 are positive

constants, \hat{v}_L and $\hat{\omega}_L$ are the estimates of v_L and ω_L , respectively. Using (11), the time derivative of (12) can be obtained as follows:

$$\begin{aligned} \dot{V}_1 = & e_1(-v_F + v_L \cos e_3 + \omega_F e_2 - \omega_L l_d \sin(\Psi_d + e_3)) \\ & + e_2(-\omega_F + v_L \sin e_3 - d_F \omega_F + \omega_L l_d \cos(\Psi_d + e_3)) \\ & + \gamma_1^{-1} \tilde{v}_L (\dot{v}_L - \dot{\hat{v}}_L) + \gamma_2^{-1} \tilde{\omega}_L (\dot{\omega}_L - \dot{\hat{\omega}}_L), \end{aligned} \quad (13)$$

Using (2), (13) can be rewritten as

$$\begin{aligned} \dot{V}_1 = & e_1 \left(\frac{1}{2a_1} (e_4 + e_5) - \bar{v}_F + v_L \cos e_3 + \bar{\omega}_F e_2 - \omega_L l_d \sin(\Psi_d + e_3) \right) \\ & + e_2 \left(\frac{d_F}{2a_2} (e_4 - e_5) - \bar{\omega}_F e_1 + v_L \sin e_3 - d_F \bar{\omega}_F + \omega_L l_d \cos(\Psi_d + e_3) \right) \\ & + \gamma_1^{-1} \tilde{v}_L (\dot{v}_L - \dot{\hat{v}}_L) + \gamma_2^{-1} \tilde{\omega}_L (\dot{\omega}_L - \dot{\hat{\omega}}_L), \end{aligned} \quad (14)$$

where \bar{v}_F and $\bar{\omega}_F$ are the virtual linear and angular velocities of the follower, respectively, $e_4 = \bar{v}_r - v_r$, $e_5 = \bar{v}_l - v_l$, $\bar{v}_r = a_1 v_F + a_2 \bar{\omega}_F$, and $\bar{v}_l = a_1 \bar{v}_F - a_2 \bar{\omega}_F$.

Then, the virtual control inputs \bar{v}_F and $\bar{\omega}_F$ are chosen as follows:

$$\begin{bmatrix} \bar{v}_F \\ \bar{\omega}_F \end{bmatrix} = \begin{bmatrix} k_1 e_1 + \hat{v}_L \cos e_3 - \hat{\omega}_L l_d \sin(\Psi_d + e_3) \\ \frac{1}{d_F} (k_2 e_2 + \hat{v}_L \sin e_3 + \hat{\omega}_L l_d \cos(\Psi_d + e_3)) \end{bmatrix}, \quad (15)$$

where k_1 and k_2 are positive constants. \hat{v}_L and $\hat{\omega}_L$ are updated as follows:

$$\begin{aligned} \dot{\hat{v}}_L &= \gamma_1 \text{Proj}(\xi_v, \hat{v}_L), \\ \dot{\hat{\omega}}_L &= \gamma_2 \text{Proj}(\xi_\omega, \hat{\omega}_L), \end{aligned} \quad (16)$$

where

$$\begin{aligned} \xi_v &= e_1 \cos e_3 + e_2 \sin e_3, \\ \xi_\omega &= -e_1 l_d \sin(\Psi_d + e_3) + e_2 l_d \cos(\Psi_d + e_3). \end{aligned}$$

Here, $\text{Proj}(\cdot)$ denotes the projection operator defined in Definition 1.

Substituting (15) and (16) into (14) yields

$$\dot{V}_1 = -k_1 e_1^2 - k_2 e_2^2 + \rho_1 + \gamma_1^{-1} \tilde{v}_L \dot{v}_L + \gamma_2^{-1} \tilde{\omega}_L \dot{\omega}_L, \quad (17)$$

where $\rho_1 = e_4 \left(\frac{e_1}{2a_1} + \frac{d_F e_2}{2a_2} \right) + e_5 \left(\frac{e_1}{2a_1} - \frac{d_F e_2}{2a_2} \right)$.

By Assumption 1 and Properties 1-2, \tilde{v}_L and $\tilde{\omega}_L$ satisfy the following inequalities:

$$\begin{aligned} |\tilde{v}_L| &\leq 2\Omega_v + \epsilon_v, \\ |\tilde{\omega}_L| &\leq 2\Omega_\omega + \epsilon_\omega, \end{aligned} \quad (18)$$

where ϵ_v and ϵ_ω are arbitrary positive constants. By Assumption 2 and (18), (17) can be rewritten as follows:

$$\dot{V}_1 \leq -k_1 e_1^2 - k_2 e_2^2 - \frac{1}{2} \gamma_1^{-1} \tilde{v}_L^2 - \frac{1}{2} \gamma_2^{-1} \tilde{\omega}_L^2 + \varpi_1 + \rho_1 \quad (19)$$

where $\varpi_1 = \frac{1}{2\gamma_1} \beta_v^2 + \frac{1}{2\gamma_2} \beta_\omega^2 + \frac{1}{\gamma_1} (2\Omega_v + \epsilon_v)^2 + \frac{1}{\gamma_2} (2\Omega_\omega + \epsilon_\omega)^2$.

[Step 2] In order to design the actual torque input τ , we rewrite the dynamic model (1) as follows:

$$\Phi W_1 + \Lambda W_2 - \bar{M} \dot{e}_d - \bar{C} e_d - \bar{D} e_d = \tau, \quad (20)$$

where

$$\begin{aligned} \Phi &= \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 & \phi_5 & 0 \\ \phi_2 & \phi_1 & \phi_4 & 0 & \phi_6 \end{bmatrix}, W_1 = [m_{11} \ m_{12} \ c \ d_{11} \ d_{22}]^T, \\ A &= \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ \lambda_2 & \lambda_1 & \lambda_4 & \lambda_3 \end{bmatrix}, W_2 = [\mu_1 \ \mu_2 \ \mu_3 \ \mu_4]^T, \\ \phi_1 &= a_1 \left\{ \frac{\partial v_F}{\partial e_1} (-v_F + \omega_F e_2) - \frac{\partial v_F}{\partial e_3} \omega_F + \frac{\partial v_F}{\partial v_L} \hat{v}_L + \frac{\partial v_F}{\partial \omega_L} \hat{\omega}_L \right\} \\ &\quad + a_2 \left\{ \frac{\partial \omega_F}{\partial e_2} (-\omega_F e_1 - d_F \omega_F) - \frac{\partial \omega_F}{\partial e_3} \omega_F + \frac{\partial \omega_F}{\partial v_L} \hat{v}_L + \frac{\partial \omega_F}{\partial \omega_L} \hat{\omega}_L \right\}, \\ \phi_2 &= a_1 \left\{ \frac{\partial v_F}{\partial e_1} (-v_F + \omega_F e_2) - \frac{\partial v_F}{\partial e_3} \omega_F + \frac{\partial v_F}{\partial v_L} \hat{v}_L + \frac{\partial v_F}{\partial \omega_L} \hat{\omega}_L \right\} \\ &\quad - a_2 \left\{ \frac{\partial \omega_F}{\partial e_2} (-\omega_F e_1 - d_F \omega_F) - \frac{\partial \omega_F}{\partial e_3} \omega_F + \frac{\partial \omega_F}{\partial v_L} \hat{v}_L + \frac{\partial \omega_F}{\partial \omega_L} \hat{\omega}_L \right\}, \end{aligned}$$

$$\begin{aligned} \phi_3 &= a_1 \theta v_F - a_2 \theta \omega_F, \phi_4 = -a_1 \theta v_F - a_2 \theta \omega_F, \\ \phi_5 &= a_1 v_F + a_2 \omega_F, \phi_6 = a_1 v_F - a_2 \omega_F, \end{aligned}$$

$$\lambda_1 = a_1 \left(\frac{\partial v_F}{e_1} \cos e_3 \right) + a_2 \left(\frac{\partial \omega_F}{e_2} \sin e_3 \right),$$

$$\lambda_2 = a_1 \left(\frac{\partial v_F}{e_1} \cos e_3 \right) - a_2 \left(\frac{\partial \omega_F}{e_2} \sin e_3 \right),$$

$$\begin{aligned} \lambda_3 &= a_1 \left(\frac{\partial v_F}{\partial e_1} (-l_d \sin(\Psi_d + e_3)) + \frac{\partial v_F}{\partial e_3} \right) \\ &\quad + a_2 \left(\frac{\partial \omega_F}{\partial e_2} (l_d \cos(\Psi_d + e_3)) + \frac{\partial \omega_F}{\partial e_3} \right), \end{aligned}$$

$$\begin{aligned} \lambda_4 &= a_1 \left(\frac{\partial v_F}{\partial e_1} (-l_d \sin(\Psi_d + e_3)) + \frac{\partial v_F}{\partial e_3} \right) \\ &\quad - a_2 \left(\frac{\partial \omega_F}{\partial e_2} (l_d \cos(\Psi_d + e_3)) + \frac{\partial \omega_F}{\partial e_3} \right), \end{aligned}$$

$$\mu_1 = m_{11} v_L, \mu_2 = m_{12} v_L, \mu_3 = m_{11} \omega_L, \mu_4 = m_{12} \omega_L, e_d = [e_4 \ e_5]^T.$$

Assumption 4: There exist some positive numbers Ω_W such that $0 < \|W_1\| \leq \Omega_W$.

To design the actual control input, we take the Lyapunov function as follows:

$$V_2 = V_1 + \frac{1}{2} (e_d^T \bar{M} e_d + \gamma_3^{-1} \hat{a}_\omega^2 + \bar{W}_2^T \Gamma^{-1} \bar{W}_2), \quad (21)$$

where $\gamma_3 > 0$, $\hat{a}_\omega = a_\omega - \bar{a}_\omega$, $a_\omega = \|W_1\|^2$, \bar{a}_ω is the estimate of a_ω . $\Gamma = \text{diag}(\gamma_4, \gamma_5, \gamma_6, \gamma_7)$ with $\gamma_4, \gamma_5, \gamma_6, \gamma_7 > 0$, $\bar{W}_2 = W_2 - \hat{W}_2$, \hat{W}_2 is the estimate vector of W_2 .

Using (14) and (20), the time derivative of (21) is obtained as

$$\dot{V}_2 \leq \dot{V}_1 + e_d^T (\Phi W_1 + A W_2 - \tau) - e_d^T \bar{D} e_d - \gamma_3^{-1} \hat{a}_\omega \dot{\hat{a}}_\omega + (\dot{W}_2^T - \hat{W}_2^T) \Gamma^{-1} \bar{W}_2. \quad (22)$$

Then, the actual torque input is chosen as follows:

$$\tau = K e_d + A \hat{W}_2 + \frac{\hat{a}_\omega \Phi \Phi^T e_d}{2\delta_1^2} + \chi, \quad (23)$$

where $\chi = \left[\frac{e_1}{2a_1} + \frac{d_F e_2}{2a_2} \ \frac{e_1}{2a_1} - \frac{d_F e_2}{2a_2} \right]^T$, $K = \text{diag}(k_3, k_4)$ with $k_3, k_4 > 0$, and δ_1 is a positive constant. \hat{a}_ω and \hat{W}_2 are updated by

$$\begin{aligned} \dot{\hat{a}}_\omega &= \gamma_3 \frac{e_d^T \Phi \Phi^T e_d}{2\delta_1^2} - \sigma \gamma_3 \hat{a}_\omega, \\ \dot{\hat{W}}_2 &= \Gamma \text{Proj}(\xi_\mu, \bar{W}_2), \end{aligned} \quad (24)$$

where $\xi_\mu = [\xi_{\mu_1}, \xi_{\mu_2}, \xi_{\mu_3}, \xi_{\mu_4}]^T$, $\xi_{\mu_1} = e_4 \lambda_1 + e_5 \lambda_2$, $\xi_{\mu_2} = e_4 \lambda_2 + e_5 \lambda_1$, $\xi_{\mu_3} = e_4 \lambda_3 + e_5 \lambda_4$, $\xi_{\mu_4} = e_4 \lambda_4 + e_5 \lambda_3$ and σ is a positive constant.

Substituting (19), (23) and (24) into (22) yields

$$\begin{aligned} \dot{V}_2 &\leq -k_1 e_1^2 - k_2 e_2^2 - k_3 e_4^2 - k_4 e_5^2 - \frac{1}{2} \gamma_1^{-1} \tilde{v}_L^2 - \frac{1}{2} \gamma_2^{-1} \tilde{\omega}_L^2 + \varpi_1 \\ &\quad + e_d^T \Phi W_1 - \hat{a}_\omega \frac{e_d^T \Phi \Phi^T e_d}{2\delta_1^2} - \bar{a}_\omega \frac{e_d^T \Phi \Phi^T e_d}{2\delta_1^2} + \sigma \hat{a}_\omega \dot{\hat{a}}_\omega + \bar{W}_2^T \Gamma^{-1} \bar{W}_2. \end{aligned} \quad (25)$$

where $\dot{W}_2 = [m_{11} \dot{v}_L \ m_{12} \dot{v}_L \ m_{11} \dot{\omega}_L \ m_{12} \dot{\omega}_L]^T$.

From the Young's inequality, $e_d^T \Phi W_1$ satisfies the following inequalities:

$$e_d^T \Phi W_1 \leq \frac{\delta_1^2}{2} + a_\omega \frac{e_d^T \Phi \Phi^T e_d}{2\delta_1^2}. \quad (26)$$

Using (26), (25) is rewritten as

$$\begin{aligned} \dot{V}_2 &\leq -k_1 e_1^2 - k_2 e_2^2 - k_3 e_4^2 - k_4 e_5^2 - \frac{1}{2} \gamma_1^{-1} \tilde{v}_L^2 - \frac{1}{2} \gamma_2^{-1} \tilde{\omega}_L^2 \\ &\quad + \varpi_1 + \frac{\delta_1^2}{2} - \frac{\sigma \hat{a}_\omega^2}{2} + \frac{\sigma a_\omega^2}{2} + \bar{W}_2^T \Gamma^{-1} \bar{W}_2. \end{aligned} \quad (27)$$

By Assumption 1 and Properties 1-2, \bar{W}_2 satisfies the following inequality:

$$\|\bar{W}_2\| \leq 2\Omega_\mu + \epsilon_\mu, \quad (28)$$

where $\Omega_\mu = \max(m_{11} \Omega_{v_L}, m_{12} \Omega_{v_L}, m_{11} \Omega_{\omega_L}, m_{12} \Omega_{\omega_L})$ and ϵ_μ is an arbitrary positive constant. By Assumption 2 and (28), (27) can be rewritten as follows:

$$\dot{V}_2 \leq -k_f V_2 + \varpi_1 + \varpi_2 + \frac{\delta_1^2}{2} + \frac{\sigma a_\omega^2}{2}. \quad (29)$$

where

$$k_f = \min(k_1, k_2, \frac{k_3}{M_M}, \frac{k_4}{M_M}, \gamma_3 \sigma),$$

$$\begin{aligned} \varpi_2 &= (\gamma_4^{-1} m_{11} \beta_v + \gamma_5^{-1} m_{12} \beta_v + \gamma_6^{-1} m_{11} \beta_\omega + \gamma_7^{-1} m_{12} \beta_\omega) (2\Omega_\mu + \epsilon_\mu), \\ &\quad + \frac{1}{2} (\gamma_4^{-1} + \gamma_5^{-1} + \gamma_6^{-1} + \gamma_7^{-1}) (2\Omega_\mu + \epsilon_\mu)^2 \end{aligned}$$

\bar{M}_M is the maximum eigenvalue of \bar{M} .

Theorem 1: Consider the nonholonomic mobile robot (1) that satisfies Assumptions 1-4. If the adaptive control law (23), and the adaptation laws (16) and (24) are applied to the mobile robot (1), then all error signals in the closed-loop system are uniformly ultimately bounded. Moreover, the formation errors can be reduced by increasing the value of gains.

Proof: Multiplying (29) by $e^{-k_f t}$, and then integrating with respect to time, the following inequality can be obtained:

$$V_2(t) e^{-k_f t} - V_2(0) \leq \frac{\rho_2}{k_f} (e^{-k_f t} - 1), \quad (30)$$

where $\rho_2 = \varpi_1 + \varpi_2 + \frac{1}{2} (\delta_1^2 + \sigma a_\omega^2)$. Multiplying (30) by $e^{-k_f t}$,

(30) is rewritten as follows:

$$V_2(t) \leq (V_2(0) - \frac{\rho_2}{k_f}) e^{-k_f t} + \frac{\rho_2}{k_f}. \quad (31)$$

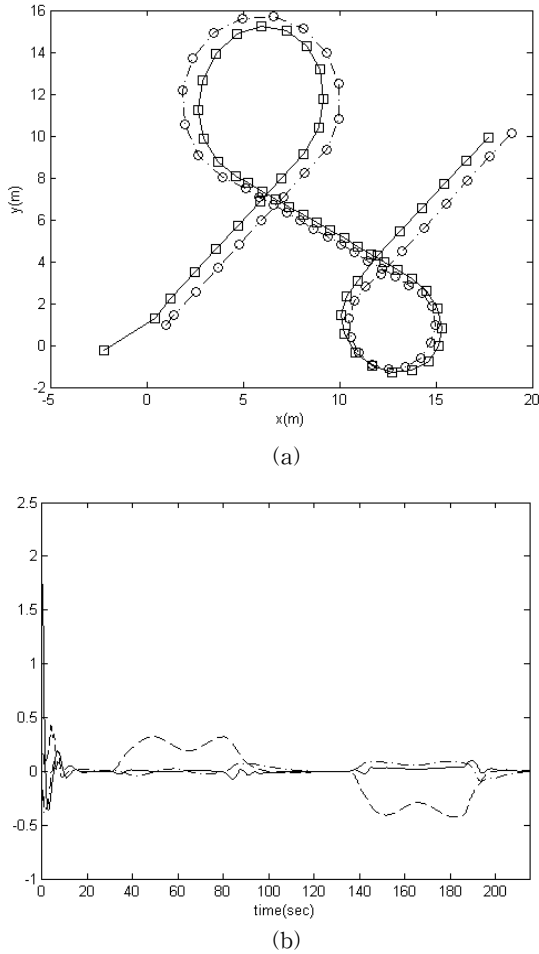


그림 2 모의 실험 결과 (a) 이동 로봇들의 궤적 (b) 추종 오차

Fig. 2 Simulation results (a) trajectories of the robots (solid : follower robot, dash-dot : leader robot) (b) tracking errors (solid : e_1 , dash-dot : e_2 , dashed : e_3). Since ρ_2 is bounded, it is proved that e_1, e_2, e_4 and e_5 are uniformly ultimately bounded.

To guarantee the stability of (10), it is necessary that e_1, e_2 and e_3 are bounded. Thus, it should be proved that e_3 is bounded by the control input. From (11), the dynamics of e_3 is obtained as follows:

$$\dot{e}_3 = \omega_L - \omega_F \quad (32)$$

Substituting (15) yields

$$\dot{e}_3 = -\frac{v_L}{d_F} \sin e_3 + \rho_3,$$

where

$$\rho_3 = \omega_L - \frac{1}{d_F} (k_2 e_2 - \tilde{v}_L \sin(e_3) + \hat{\omega}_L l_d \cos(\Psi_d + e_3)) + \frac{1}{2a_2} (e_4 - e_5)$$

Since ρ_3 is bounded, the nominal system of (32) is given by

$$\dot{e}_3 = -\frac{v_L}{d_F} \sin e_3 \quad (33)$$

By Assumptions 1 and 3, $v_L > 0$ and $|e_3(0)| < \pi$, respectively. Thus, (33) is stable, and it is guaranteed that e_3 is bounded by the stability theory of perturbed systems [15]. ■

4. SIMULATION RESULTS

In this section, some computer simulations are carried out to verify the effectiveness of the proposed controller. There exist a leader robot and a follower robot. The physical parameters are chosen as $r = 0.15$ [m], $R = 0.75$ [m], $d = 0.8$ [m], $m_c = 30$ [kg], $m_w = 1$ [kg], $I_c = 15.625$ [kg · m²], $I_w = 0.005$ [kg · m²], $I_m = 0.0025$ [kg · m²] and $d_{11} = d_{22} = 1$ [m]. The desired relative distance and relative angle are $l_d = 1$ [m] and $\Psi_d = 3\pi/4$ [rad]. The linear and angular velocities of the leader robot are used to generate the trajectory of the leader robot and are considered as the time-varying parameters of the follower robot. They are chosen as follows:

$$0 \leq t < 5 : v_L = -0.2 \cos\left(\frac{\pi t}{5}\right) + 0.2, \omega_L = 0$$

$$5 \leq t < 30 : v_L = 0.4, \omega_L = 0$$

$$30 \leq t < 35 : v_L = 0.4, \omega_L = -0.05 \cos\left(\frac{\pi t}{5}\right) + 0.05$$

$$35 \leq t < 80 : v_L = 0.4, \omega_L = 0.1$$

$$80 \leq t < 85 : v_L = 0.1 \cos\left(\frac{\pi t}{5}\right) + 0.3, \omega_L = 0.05 \cos\left(\frac{\pi t}{5}\right) + 0.05$$

$$85 \leq t < 135 : v_L = 0.2, \omega_L = 0$$

$$135 \leq t < 140 : v_L = 0.2, \omega_L = -0.05 \cos\left(\frac{\pi t}{5}\right) - 0.05$$

$$140 \leq t < 185 : v_L = 0.2, \omega_L = -0.1$$

$$185 \leq t < 190 : v_L = 0.1 \cos\left(\frac{\pi t}{5}\right) + 0.3, \omega_L = 0.05 \cos\left(\frac{\pi t}{5}\right) - 0.05$$

$$190 \leq t < 215 : v_L = 0.4, \omega_L = 0$$

The controller parameters and the gains of the adaptive laws are chosen as $k_1 = k_2 = 1$, $k_3 = k_4 = 0.8$, $\gamma_1 = 0.7$, $\gamma_2 = 0.4$, $\gamma_3 = 0.004$, $\Gamma = \text{diag}(0.04, 0.04, 0.04, 0.04)$, $\delta_1 = 15$, $\sigma = 0.00001$. The initial position and heading angle of the leader robot and the follower robot are chosen as $(x_L(0), y_L(0), \theta_L(0)) = (1, 1, \frac{\pi}{4})$ and $(x_F(0), y_F(0), \theta_F(0)) = (-2, 0, \frac{\pi}{4})$ respectively. Fig. 2(a) shows the trajectories of the leader and the follower robot. The position and heading angle errors are shown in Fig. 2(b). From the results of Figs. 2(a) and 2(b), it is confirmed that the follower keeps the formation with the leader by the designed torque input, while the errors are bounded in small neighborhood of zero. Fig. 3 shows the control inputs and the estimates of the parameters in the robot model. From these figures, it is shown that all signals are bounded. Therefore, we can conclude that the proposed method has good performance in keeping the desired formation without knowing the information of the leader robot's velocity.

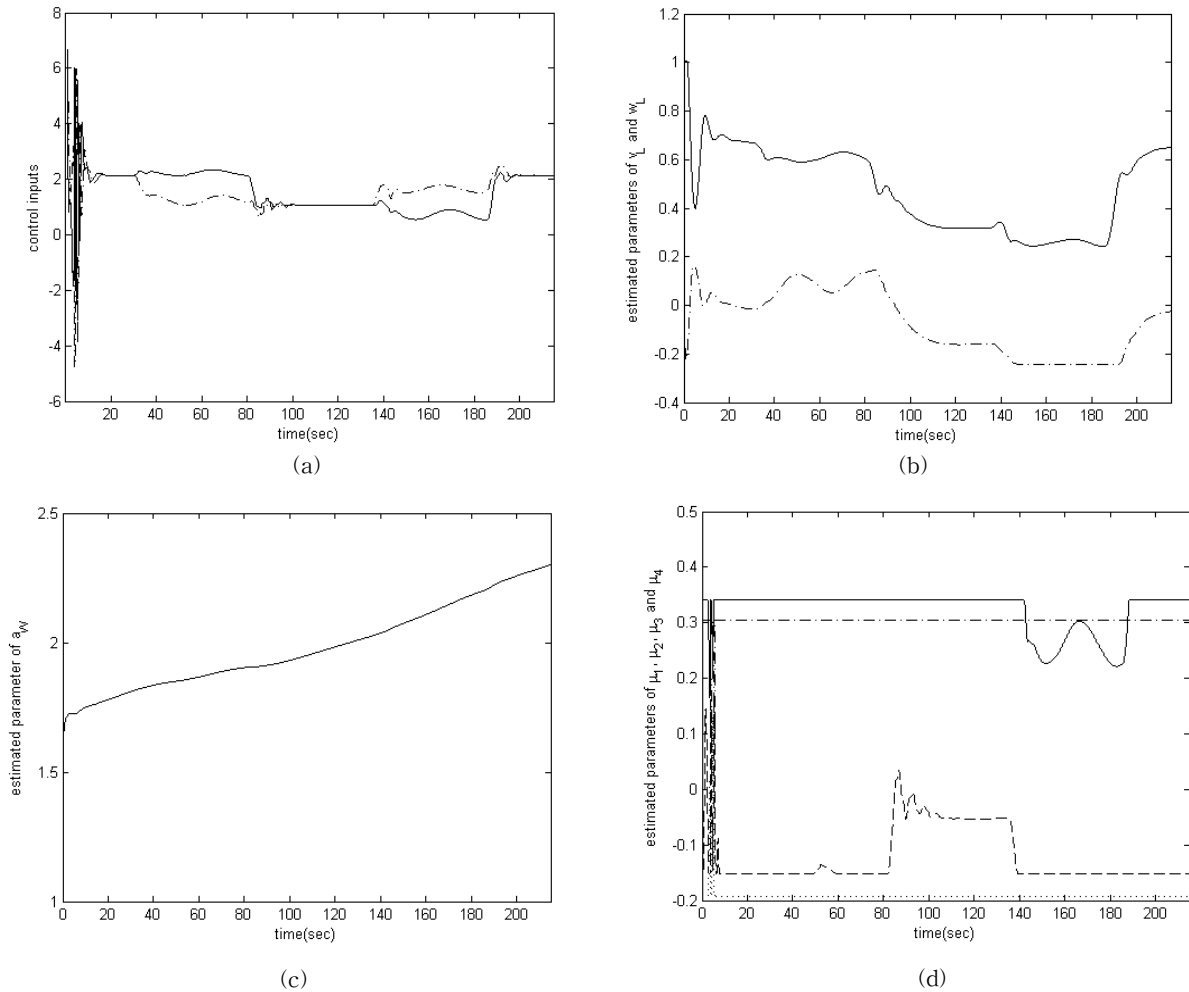


그림 3 모의 실험 결과 (a) 토크 입력 (b) v_L 과 ω_L 의 추정 매개 변수 (c) a_w 의 추정 매개 변수 (d) W_2 의 추정 매개 변수
Fig. 3 Simulation results (a) torque inputs (solid : τ_r , dash-dot : τ_l) (b) estimated parameters of v_L and ω_L (solid : \hat{v}_L , dash-dot : $\hat{\omega}_L$) (c) estimated parameter of a_w (d) estimated parameters of W_2 (solid : $\hat{\mu}_1$, dot : $\hat{\mu}_2$, dash-dot : $\hat{\mu}_3$, dash : $\hat{\mu}_4$).

5. CONCLUSION

In this paper, an adaptive formation control method based on the leader-following approach for nonholonomic mobile robots has been proposed. In the proposed method, the projection algorithm has been used to estimate the velocity information of the leader robot. The adaptive backstepping technique has been used to design the actual torque input which compensates the effects of an uncertain dynamic model. From the Lyapunov stability theorem, it has been proved that the system errors are uniformly ultimately bounded. Since all these signals can be reduced by increasing the value of gains, the mobile robots can keep the desired formation successfully. The effectiveness of the proposed controller has been verified by the simulation.

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