Average Rate Performance of Two-Way Amplify-and-Forward Relaying in Asymmetric Fading Channels

Jae Cheol Park, Iickho Song, Sung Ro Lee, and Yun Hee Kim

Abstract: **A two-way relaying (TWR) system is analyzed, where two source terminals with unequal numbers of antennas exchange data via an amplify-and-forward relay terminal with a single antenna. In the system considered herein, the link quality between the sources and relay can generally be asymmetric due to the non-identical antenna configuration, power allocation, and relay location. In such a general setup, accurate bounds on the average sum rate (ASR) are derived when beamforming or orthogonal space time block coding is employed at the sources. We show that the proposed bounds are almost indistinguishable from the exact ASR under various system configurations. It is also observed that the ASR performance of the TWR system with unequal numbers of source antennas is more sensitive to the relay location than to the power allocation.**

Index Terms: **Amplify-and-forward (AF), average sum rate (ASR), beamforming, Nakagami fading, two-way relaying (TWR).**

I. INTRODUCTION

Relays have found many applications in wireless networks for reliable communication and coverage enhancement by acting as distributed antennas [1], [2]. However, one-way relaying (OWR) suffers from a loss in the spectral efficiency due to the half-duplex constraint. This loss can be compensated by employing two-way relaying (TWR) protocols proposed recently [3]–[11], where two source terminals communicating each other transmit their symbols to a relay simultaneously in the first phase and then the relay broadcasts the received signal back to the two sources in the second phase. In broadcasting, the relay commonly adopts either a decode-and-forward (DF) approach in which the received signal from the two sources is decoded and re-encoded [3]–[5], or an amplify-and-forward (AF) approach in which the received signal is amplified [5]–[11].

In this paper, we concentrate our attention on simple AF relays equipped with a single antenna. We refer an AF relay considered herein *simple* in that the relay just scales the received signal without knowing the transmission strategies of the sources and the channel state information (CSI) between the sources and relay. More complicated multi-antenna relays can be found in the literature [9], [11], where the relays transform the received signal with the matrix derived from the estimated CSI. The source terminals are, on the other hand, equipped with multiple antennas for transmission via either beamforming (BF) [12] or orthogonal space time block coding (OSTBC) [13], [14] and for reception via maximal ratio combining (MRC). Taking the coexistence of heterogeneous terminals into consideration, we assume that the numbers of antennas at the sources may be unequal.

Under such a configuration called the asymmetric channels,[1] we derive in this paper accurate bounds on the average sum rate (ASR) of the TWR system. Clearly, upper- and lower-bounds are provided on the ASR of TWR in [7] when the two channels from the two sources to the relay have an identical diversity order of one or two. However, the bounds in [7] are unfortunately observed to be rather loose, deviating from the exact values considerably as the SNR decreases or the relay power increases. In this paper, we provide more accurate upper- and lower-bounds on the ASR of TWR, which are in addition applicable in more general configurations of the TWR system.

The remaining of this paper is organized as follows. Section II describes the system model of the TWR system in the asymmetric channels. Section III derives new bounds on the ASR, which are then verified with simulation results and also used for performance investigation in Section IV. Finally, conclusions are provided in Section V.

Notations: We use bold lowercase (uppercase) letters for the vectors (matrices) with $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ denoting the conjugate, transpose, and Hermitian of a vector (matrix), respectively. The space of $n \times m$ matrices with complex-valued elements is denoted by $\mathbb{C}^{n\times m}$. We also denote by $\mathbf{0}$, $\mathbf{I}_n$, and $||\cdot||$ the all-zero vector, $n \times n$ identity matrix, and Euclidean norm, respectively. We use $\mathcal{CN}(\mathbf{m}, \mathbf{\Sigma})$ to denote the distribution of a circularly symmetric complex Gaussian random vector with mean vector $\mathbf{m}$ and covariance matrix $\mathbf{\Sigma}$. The notation $\mathcal{G}(\alpha, \beta)$ signifies a gamma distribution of which the probability density function (pdf) is given by

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[1] Not only the values of the signal-to-noise ratio (SNR) but also the diversity orders of the two channels from the sources to the rely are unequal.

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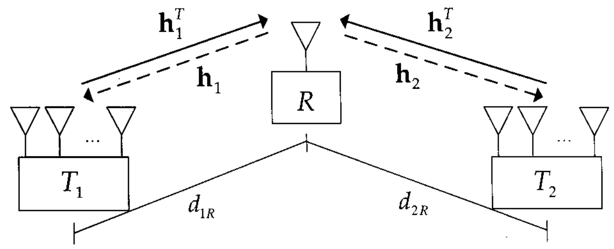

Fig. 1. System model of a TWR system.

$$p_{\mathcal{G}(\alpha,\beta)}(x) = \frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\beta^\alpha \Gamma(\alpha)}, \; x \geq 0 \tag{1}$$

where $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1}e^{-t}dt$ is the gamma function. We also use $\mathcal{E}\{\cdot\}$ for the expected value and $\sim$ for "distributed as."

II. SYSTEM MODEL

Consider a two-hop half-duplex TWR system as shown in Fig. 1, which consists of two source terminals T_1 and T_2 and a relay terminal R located at the distance d_{iR} from T_i for $i = 1, 2$. The sources T_1 and T_2 are equipped with M_1 and M_2 antennas, respectively, while the relay is equipped with a single antenna. The system utilizes two orthogonal equi-duration time slots for a two-phase signal transmission: In the first phase, both T_1 and T_2 transmit their symbols simultaneously to the relay R, and in the second phase, the relay broadcasts the received signal back to T_1 and T_2 after amplification. Multiple antennas at the sources are exploited for either BF or OSTBC transmission and for MRC reception.

Assuming that the channels are flat-fading, reciprocal, and time-invariant over the two time slots, the channels from T_i to R and from R to T_i can be denoted as $\mathbf{h}_i^T = [h_{i,1} \; h_{i,2} \; \cdots \; h_{i,M_i}] \in \mathbb{C}^{1\times M_i}$ and $\mathbf{h}_i$, respectively, for $i = 1$ and 2. The channels are assumed to be Nakagami fading such that $|h_{i,k}|^2 \sim \mathcal{G}\left(m_i, \frac{1}{m_i d_i^\nu}\right)$, where m_i, $d_i = \frac{d_{iR}}{d_{1R}+d_{2R}}$, and ν denote the Nakagami fading parameter, normalized distance, and path-loss exponent, respectively, of the link between T_i and R. Note that, for $m_i = 1$, Nakagami fading reduces to Rayleigh fading.

A. Beamforming

In this subsection, we assume that the CSI on $\mathbf{h}_i$ is available at T_i before transmission. Using the CSI, source T_i transmits its symbols with the BF vector $\mathbf{w}_i = \mathbf{h}_i^*/||\mathbf{h}_i||$ in the first time slot. The received signal at the relay is then expressed as

$$y_R(l) = ||\mathbf{h}_1||x_1(l) + ||\mathbf{h}_2||x_2(l) + w_R(l) \tag{2}$$

for $l = 1, 2, \cdots, L$, where $x_i(l) \sim \mathcal{CN}(0, P_i)$ is the information symbol of T_i with transmit power P_i at symbol index l, $w_R(l) \sim \mathcal{CN}(0, \sigma_R^2)$ is the additive white Gaussian noise (AWGN) at R at symbol index l, and L denotes the number of symbols in a time slot. The AF relay amplifies $y_R(l)$ by

$$g_{BF} = \sqrt{\frac{P_R}{P_1||\mathbf{h}_1||^2 + P_2||\mathbf{h}_2||^2 + \sigma_R^2}} \tag{3}$$

to make the average transmit power P_R, and broadcasts $g_{\text{BF}}y_R(l)$ to T_1 and T_2 in the second time slot.

The received signal vector $\mathbf{y}_j(l) \in \mathbb{C}^{M_j\times 1}$ at T_j is thus expressed as

$$\begin{aligned} y_j(l) = \; & g_{\text{BF}}\mathbf{h}_j||\mathbf{h}_1||x_1(l) + g_{\text{BF}}\mathbf{h}_j||\mathbf{h}_2||x_2(l) \\ & + g_{\text{BF}}\mathbf{h}_j w_R(l) + \mathbf{w}_j(l) \end{aligned} \tag{4}$$

for $j = 1, 2$, where $\mathbf{w}_j(l) \sim \mathcal{CN}(\mathbf{0}, \sigma_j^2\mathbf{I}_{M_j})$ is the AWGN vector at T_j. Over the vector $\mathbf{y}_j(l)$, T_j performs MRC and then cancels the self-interference in the MRC output as

$$\begin{aligned} \tilde{y}_j(l) &= \frac{\mathbf{h}_j^\dagger \mathbf{y}_j(l)}{||\mathbf{h}_j||} - g_{\text{BF}}||\mathbf{h}_j||^2 x_j(l) \\ &= g_{\text{BF}}||\mathbf{h}_j||||\mathbf{h}_i||x_i(l) + g_{\text{BF}}||\mathbf{h}_j||w_R(l) + \tilde{w}_j(l) \end{aligned} \tag{5}$$

where $\tilde{w}_j(l) = \frac{\mathbf{h}_j^\dagger \mathbf{w}_j(l)}{||\mathbf{h}_j||} \sim \mathcal{CN}(0, \sigma_j^2)$ and $(i,j) \in \mathcal{D} = \{(1,2),(2,1)\}$ denotes the pair of transmitting and receiving terminals.

From (5), the rate (mutual information) delivered from T_i to T_j is obtained as

$$\begin{aligned} \mathcal{R}_{\text{BF}_2}^{i\to j} &= \frac{1}{2}\log_2\left(1 + \frac{g_{\text{BF}}^2 P_i||\mathbf{h}_i||^2||\mathbf{h}_j||^2}{g_{\text{BF}}^2||\mathbf{h}_j||^2\sigma_R^2 + \sigma_j^2}\right) \\ &= \frac{1}{2}\log_2\left(1 + \frac{X_{iR}X_{Rj}}{X_{iR} + (1+\eta_j)X_{Rj} + 1}\right) \end{aligned} \tag{6}$$

for $(i,j) \in \mathcal{D}$, where $X_{iR} = \frac{P_i}{\sigma_R^2}||\mathbf{h}_i||^2$, $X_{Rj} = \frac{P_R}{\sigma_j^2}||\mathbf{h}_j||^2$, $\eta_j = \frac{P_j}{\sigma_R^2}\frac{\sigma_j^2}{P_R}$, and the factor $1/2$ stems from the use of two time slots in the TWR. Hence, the total sum rate of TWR employing the BF can be expressed as

$$\begin{aligned} \mathcal{R}_{\text{BF}_2} &= \mathcal{R}_{\text{BF}_2}^{1\to 2} + \mathcal{R}_{\text{BF}_2}^{2\to 1} \\ &= \frac{1}{2}\sum_{(i,j)\in\mathcal{D}} \log_2\left(1 + \frac{X_{iR}X_{Rj}}{X_{iR} + (1+\eta_j)X_{Rj} + 1}\right). \end{aligned} \tag{7}$$

In passing, we would like to note that the OWR, which requires four time slots for information exchange through separate source transmission and subsequent relaying, can provide the rate

$$\mathcal{R}_{\text{BF}_1} = \frac{1}{4}\sum_{(i,j)\in\mathcal{D}} \log_2\left(1 + \frac{X_{iR}X_{Rj}}{X_{iR} + X_{Rj} + 1}\right). \tag{8}$$

B. Orthogonal Space Time Block Coding

Let us now assume that the sources transmit information symbols via OSTBC with no CSI available. In such a case, source T_i maps an information symbol vector $\mathbf{x}_i = [x_i(1) \; x_i(2) \; \cdots \; x_i(K_i)]^T \in \mathbb{C}^{K_i\times 1}$ to the OSTBC symbol matrix $\mathcal{B}_i(\mathbf{x}_i) \in \mathbb{C}^{M_i\times N_i}$ such that $\mathcal{B}_i^H(\mathbf{x}_i)\mathcal{B}_i(\mathbf{x}_i) = ||\mathbf{x}_i||^2\mathbf{I}_{M_i}$. For instance, the OSTBC symbol matrix for $M_i = 2$ is given by [13]

$$\mathcal{B}_i(\mathbf{x}_i) = \begin{bmatrix} x_i(1) & -x_i^*(2) \\ x_i(2) & x_i^*(1) \end{bmatrix} \tag{9}$$

with $K_i = 2$ and $N_i = 2$: The OSTBC symbol matrices for $M_i > 2$ can be found in [14]. The OSTBC exhibits no rate loss with $\rho_i = 1$ when $M_i = 2$ but exhibits an unavoidable rate loss with $\rho_i = K_i/N_i < 1$ when $M_i > 2$. Then, the OSTBC symbol matrix $\mathcal{B}_i(\mathbf{x}_i)$ is transmitted column by column over the M_i antennas in order.

For simplicity in description, let us consider the first OSTBC symbol matrices transmitted by the sources. The received symbol at the relay in the first time slot can be represented as

$$y_R(l) = \sqrt{\frac{1}{M_1}}\mathbf{h}_1^T\mathbf{b}_1(l) + \sqrt{\frac{1}{M_2}}\mathbf{h}_2^T\mathbf{b}_2(l) + w_R(l) \quad (10)$$

where $\mathbf{b}_i(l)$ is the lth column of $\mathcal{B}_i(\mathbf{x}_i)$ transmitted at the lth OSTBC symbol time and $\sqrt{1/M_i}$ is the scaling factor to make the transmit power per information symbol the same as that without OSTBC. The relay subsequently scales $y_R(l)$ by

$$g_{\text{ST}} = \sqrt{\frac{P_R}{\frac{P_1}{M_1}||\mathbf{h}_1||^2 + \frac{P_2}{M_2}||\mathbf{h}_2||^2 + \sigma_R^2}} \quad (11)$$

and broadcasts $g_{\text{ST}}y_R(l)$ to T_1 and T_2 in the second time slot.

The received signal vector at T_j in the second time slot is then given by

$$\mathbf{y}_j(l) = \frac{g_{\text{ST}}}{\sqrt{M_1}}\mathbf{h}_j\mathbf{h}_1^T\mathbf{b}_1(l) + \frac{g_{\text{ST}}}{\sqrt{M_2}}\mathbf{h}_j\mathbf{h}_2^T\mathbf{b}_2(l) + g_{\text{ST}}\mathbf{h}_j w_R(l) + \mathbf{w}_j(l). \quad (12)$$

After MRC and self-interference cancellation, the signal at T_j can be expressed as

$$\begin{aligned}\tilde{y}_j(l) &= \frac{\mathbf{h}_j^\dagger\mathbf{y}_j(l)}{||\mathbf{h}_j||} - \frac{g_{\text{ST}}}{\sqrt{M_j}}||\mathbf{h}_j||\mathbf{h}_j^T\mathbf{b}_j(l)\\ &= \frac{g_{\text{ST}}}{\sqrt{M_i}}||\mathbf{h}_j||\mathbf{h}_i^T\mathbf{b}_i(l) + g_{\text{ST}}||\mathbf{h}_j||w_R(l) + \tilde{w}_j(l). \quad (13)\end{aligned}$$

By processing $\{\tilde{y}_j(l)\}_{l=1,2,\cdots,N_i}$ linearly with the CSI $\mathbf{h}_i$, we finally have the decision metrics [14]

$$\hat{\mathbf{y}}_j = \frac{g_{\text{ST}}}{\sqrt{M_i}}||\mathbf{h}_j||||\mathbf{h}_i||\mathbf{x}_i + g_{\text{ST}}||\mathbf{h}_j||\hat{\mathbf{w}}_R + \hat{\mathbf{w}}_j \quad (14)$$

for $\mathbf{x}_i$, where $\hat{\mathbf{w}}_R \sim \mathcal{CN}(\mathbf{0}, \sigma_R^2\mathbf{I}_{K_i})$ and $\hat{\mathbf{w}}_j \sim \mathcal{CN}(\mathbf{0}, \sigma_j^2\mathbf{I}_{K_i})$.

This process indicates that, for coherent demodulation, the source T_j should estimate $\mathbf{h}_j$ and $\mathbf{h}_i$ in the case of OSTBC: Note that T_j should estimate $\mathbf{h}_j$ and $||\mathbf{h}_i||$ for $i \neq j$ in the case of BF.

On the analogy of the case of BF, the rate delivered from T_i to T_j via TWR employing the OSTBC can be obtained as

$$\mathcal{R}_{\text{ST}_2}^{i\to j} = \frac{\rho_i}{2}\log_2\left(1 + \frac{\frac{1}{M_i}X_{iR}X_{Rj}}{\frac{1}{M_i}X_{iR} + (1+\frac{\eta_j}{M_j})X_{Rj} + 1}\right) \quad (15)$$

from (14), where the factor ρ_i incorporates the rate loss of OSTBC. Hence, the sum rate of TWR with the OSTBC is given by

$$\begin{aligned}\mathcal{R}_{\text{ST}_2} &= \mathcal{R}_{\text{ST}_2}^{1\to 2} + \mathcal{R}_{\text{ST}_2}^{2\to 1}\\ &= \frac{1}{2}\sum_{(i,j)\in\mathcal{D}}\rho_i\log_2\left(1 + \frac{\frac{1}{M_i}X_{iR}X_{Rj}}{\frac{1}{M_i}X_{iR} + (1+\frac{\eta_j}{M_j})X_{Rj} + 1}\right). \quad (16)\end{aligned}$$

Note that the sum rate of OWR employing OSTBC is given by

$$\mathcal{R}_{\text{ST}_1} = \frac{1}{4}\sum_{(i,j)\in\mathcal{D}}\rho_i\log_2\left(1 + \frac{\frac{1}{M_i}X_{iR}X_{Rj}}{\frac{1}{M_i}X_{iR} + X_{Rj} + 1}\right). \quad (17)$$

III. UNIFIED ANALYSIS ON THE ASR

A. Bounds on the Sum Rate

Noting the inequality

$$\begin{aligned}\frac{a_iX_{iR}X_{Rj}}{a_iX_{iR} + b_jX_{Rj} + b_j} &\leq \frac{a_iX_{iR}X_{Rj}}{a_iX_{iR} + b_jX_{Rj} + 1}\\ &\leq \frac{a_iX_{iR}X_{Rj} + b_j - 1}{a_iX_{iR} + b_jX_{Rj} + 1} \quad (18)\end{aligned}$$

for $a_i \geq 0$, $b_j \geq 1$, and $(i,j) \in \mathcal{D}$, where the equalities hold when $b_j = 1$, we define the lower-bound function

$$\begin{aligned}f^{\mathcal{L}}(X_{iR}, X_{Rj}, a_i, b_j) &\triangleq \log_2\left(1 + \frac{a_iX_{iR}X_{Rj}}{a_iX_{iR} + b_jX_{Rj} + b_j}\right)\\ &= \log_2(1 + b_j^{-1}a_iX_{iR}) + \log_2(1 + X_{Rj})\\ &\quad - \log_2(1 + b_j^{-1}a_iX_{iR} + X_{Rj}) \quad (19)\end{aligned}$$

and upper-bound function

$$\begin{aligned}f^{\mathcal{U}}(X_{iR}, X_{Rj}, a_i, b_j) &\triangleq \log_2\left(1 + \frac{a_iX_{iR}X_{Rj} + b_j - 1}{a_iX_{iR} + b_jX_{Rj} + 1}\right)\\ &= \log_2(b_j) + \log_2(1 + b_j^{-1}a_iX_{iR}) + \log_2(1 + X_{Rj})\\ &\quad - \log_2(1 + a_iX_{iR} + b_jX_{Rj}). \quad (20)\end{aligned}$$

With the lower- and upper-bound functions defined above, the bounds on $\mathcal{R}_{\text{BF}_2}$ and $\mathcal{R}_{\text{ST}_2}$ can be expressed as

$$\mathcal{R}_{\text{BF}_2}^{y} = \frac{1}{2}\sum_{(i,j)\in\mathcal{D}} f^{y}(X_{iR}, X_{Rj}, 1, 1+\eta_j) \quad (21)$$

and

$$\mathcal{R}_{\text{ST}_2}^{y} = \frac{1}{2}\sum_{(i,j)\in\mathcal{D}}\rho_i f^{y}\left(X_{iR}, X_{Rj}, \frac{1}{M_i}, 1+\frac{\eta_j}{M_j}\right), \quad (22)$$

from (7) and (16), respectively: Here, the lower- and upper-bounds are obtained with $y = \mathcal{L}$ and $y = \mathcal{U}$, respectively. It should be noted that the exact sum rates (8) and (17) of OWR are given by $\mathcal{R}_{\text{BF}_1} = \frac{1}{2}\mathcal{R}_{\text{BF}_2}^{\mathcal{U}}\big|_{\eta_1=\eta_2=0}$ and $\mathcal{R}_{\text{ST}_2} = \frac{1}{2}\mathcal{R}_{\text{ST}_2}^{\mathcal{U}}\big|_{\eta_1=\eta_2=0}$, respectively.

B. Average Sum Rate

To obtain the bounds on the ASR of TWR, we need to derive $\mathcal{E}\{f^{y}(X_{iR}, X_{Rj}, a_i, b_j)\}$ for $y = \mathcal{L}$ and $\mathcal{U}$, where $a_i > 0$ and $b_j \geq 1$, which in turn requires the derivation of $\mathcal{E}\{\log_2(1 + c_1X_{iR} + c_2X_{Rj})\}$ for nonnegative c_1 and c_2. The following observations are necessary in deriving $\mathcal{E}\{\log_2(1 + c_1X_{iR} + c_2X_{Rj})\}$.

Theorem 1: When $\{X_k \sim \mathcal{G}(\alpha_k, \beta)\}$ are independent, $\sum X_k \sim \mathcal{G}(\sum\alpha_k, \beta)$ and $cX_k \sim \mathcal{G}(\alpha_k, c\beta)$ for a constant $c > 0$ [15].

Theorem 2: When $X_1 \sim \mathcal{G}(\alpha_1, \beta_1)$ and $X_2 \sim \mathcal{G}(\alpha_2, \beta_2)$ are independent, the distribution $\mathcal{K}(\alpha_1, \alpha_2, \beta_1, \beta_2)$ of $X_1 + X_2$ has the pdf [16]

$$p_{\mathcal{K}(\alpha_1,\alpha_2,\beta_1,\beta_2)}(z) = \left(\frac{\beta_{o_1}}{\beta_{o_2}}\right)^{\alpha_{o_2}} \sum_{k=0}^{\infty} \delta_k \cdot \frac{z^{\alpha_1+\alpha_2+k-1} e^{-z/\beta_{o_1}}}{\beta_{o_1}^{\alpha_1+\alpha_2+k} \Gamma(\alpha_1+\alpha_2+k)}, \; z \geq 0 \quad (23)$$

for $\alpha_i, \beta_i > 0$, where $o_1 = \arg\min_i \beta_i$, $o_2 = \arg\max_i \beta_i$, and

$$\delta_k = \begin{cases} 1, & k = 0 \\ \frac{1}{k} \sum_{i=1}^{k} \alpha_{o_2} \left(1 - \frac{\beta_{o_1}}{\beta_{o_2}}\right)^i \delta_{k-i}, & k = 1, 2, \cdots. \end{cases} \quad (24)$$

When α_1 and α_2 are integers, the pdf (23) can be expressed in finite sums as [17]

$$p_{\mathcal{K}(\alpha_1,\alpha_2,\beta_1,\beta_2)}(z) = \sum_{j=1}^{2} \sum_{r=1}^{\alpha_j} \frac{(-1)^{\alpha_1+\alpha_2+r} b_{j,r}}{\beta_1^{\alpha_1} \beta_2^{\alpha_2}} \cdot \frac{z^{r-1} e^{-z/\beta_j}}{(r-1)!} \quad (25)$$

for $z \geq 0$, where

$$b_{j,r} = \lim_{t \to \frac{1}{\beta_j}} \left\{ \frac{1}{(\alpha_j - r)!} \frac{\partial^{\alpha_j - r}}{\partial t^{\alpha_j - r}} [(t - \frac{1}{\beta_j})^{\alpha_j} \prod_{i=1}^{2} (t - \frac{1}{\beta_i})^{\alpha_i}] \right\}. \quad (26)$$

Corollary 1: For $i = 1$ and 2, $X_{iR} \sim \mathcal{G}(\mu_i, \gamma_{iR})$ and $X_{Ri} \sim \mathcal{G}(\mu_i, \gamma_{Ri})$, where $\mu_i = m_i M_i$, $\gamma_{ir} = \frac{P_i}{\sigma_R^2 m_i d_i^\nu}$, and $\gamma_{Ri} = \frac{P_R}{\sigma_i^2 m_i d_i^\nu}$.

Proof: Note that $X_{iR} = \frac{P_i}{\sigma_R^2} \sum_{l=1}^{M_i} |h_{i,l}|^2$ and $X_{Ri} = \frac{P_R}{\sigma_i^2} \sum_{l=1}^{M_i} |h_{i,l}|^2$, where $|h_{i,l}|^2 \sim \mathcal{G}\left(m_i, \frac{1}{m_i d_i^\nu}\right)$. From Theorem 1, we have $S = \sum_{l=1}^{M_i} |h_{i,l}|^2 \sim \mathcal{G}\left(m_i M_i, \frac{1}{m_i d_i^\nu}\right)$, and consequently, $X_{iR} = \frac{P_i}{\sigma_R^2} S \sim \mathcal{G}\left(m_i M_i, \frac{P_i}{\sigma_R^2 m_i d_i^\nu}\right)$ and $X_{Ri} = \frac{P_R}{\sigma_i^2} S \sim \mathcal{G}\left(m_i M_i, \frac{P_R}{\sigma_i^2 m_i d_i^\nu}\right)$. □

Corollary 2: When $c_1 > 0$ and $c_2 > 0$, we have $c_1 X_{iR} + c_2 X_{Rj} \sim \mathcal{K}(\mu_i, \mu_j, c_1 \gamma_{ir}, c_2 \gamma_{Rj})$.

Proof: Since $X_{iR} \sim \mathcal{G}(\mu_i, \gamma_{iR})$ and $X_{Rj} \sim \mathcal{G}(\mu_j, \gamma_{Rj})$, the result is straightforward from Theorem 2. □

Let us now define the average capacity function over $\mathcal{G}(\alpha, \beta)$ and $\mathcal{K}(\alpha_1, \alpha_2, \beta_1, \beta_2)$ as

$$\bar{C}_{\mathcal{G}(\alpha,\beta)} \triangleq \int_0^\infty \log_2(1+x) p_{\mathcal{G}(\alpha,\beta)}(x) dx \quad (27)$$

and

$$\bar{C}_{\mathcal{K}(\alpha_1,\alpha_2,\beta_1,\beta_2)} \triangleq \int_0^\infty \log_2(1+z) p_{\mathcal{K}(\alpha_1,\alpha_2,\beta_1,\beta_2)}(z) dz, \quad (28)$$

respectively. Using (23), we can rewrite $\bar{C}_{\mathcal{K}(\alpha_1,\alpha_2,\beta_1,\beta_2)}$ as

$$\bar{C}_{\mathcal{K}(\alpha_1,\alpha_2,\beta_1,\beta_2)} = \left(\frac{\beta_{o_1}}{\beta_{o_2}}\right)^{\alpha_{o_2}} \sum_{k=0}^{\infty} \delta_k \bar{C}_{\mathcal{G}(\alpha_1+\alpha_2+k, \beta_{o_1})}. \quad (29)$$

When α_1 and α_2 are integers, the result (29) can be expressed as

$$\bar{C}_{\mathcal{K}(\alpha_1,\alpha_2,\beta_1,\beta_2)} = \sum_{j=1}^{2} \sum_{r=1}^{\alpha_j} \frac{\beta_j^r (-1)^{\alpha_1+\alpha_2+r}}{\beta_1^{\alpha_1} \beta_2^{\alpha_2}} b_{j,r} \bar{C}_{\mathcal{G}(r,\beta_j)}, \quad (30)$$

using (25). Similarly, when α is an integer, $\bar{C}_{\mathcal{G}(\alpha,\beta)}$ is given in a closed form as [18]

$$\bar{C}_{\mathcal{G}(\alpha,\beta)} = \frac{e^{1/\beta}}{\ln 2} \sum_{l=1}^{\alpha} \beta^{-\alpha+l} \Gamma(-\alpha+l, 1/\beta) \quad (31)$$

where $\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$ is the incomplete Gamma function. In passing, let us note that (30) and (31) reduce to

$$\bar{C}_{\mathcal{K}(1,1,\beta_1,\beta_2)} = \begin{cases} \frac{\beta_1 \bar{C}_{\mathcal{G}(1,\beta_1)} - \beta_2 \bar{C}_{\mathcal{G}(1,\beta_2)}}{\beta_1 - \beta_2}, & \beta_1 \neq \beta_2, \\ \bar{C}_{\mathcal{G}(2,\beta_1)}, & \beta_1 = \beta_2, \end{cases} \quad (32)$$

and

$$\bar{C}_{\mathcal{G}(1,\beta)} = -\frac{e^{1/\beta}}{\ln 2} \mathrm{Ei}\left(-\frac{1}{\beta}\right), \quad (33)$$

respectively, when $\alpha_1 = \alpha_2 = \alpha = 1$ (which represents the scenario of single source antenna with Rayleigh fading), where $\mathrm{Ei}(x) = -\int_{-x}^\infty \frac{e^{-t}}{t} dt$.

From (19), (20), (27), and (28) with Corollaries 1 and 2, we have

$$\mathcal{E}\{f^{\mathcal{L}}(X_{iR}, X_{Rj}, a_i, b_j)\} = \bar{C}_{\mathcal{G}(\mu_i, b_j^{-1} a_i \gamma_{iR})} + \bar{C}_{\mathcal{G}(\mu_j, \gamma_{Rj})} - \bar{C}_{\mathcal{K}(\mu_i, \mu_j, b_j^{-1} a_i \gamma_{iR}, \gamma_{Rj})} \quad (34)$$

and

$$\mathcal{E}\{f^{\mathcal{U}}(X_{iR}, X_{Rj}, a_i, b_j)\} = \log_2 b_j + \bar{C}_{\mathcal{G}(\mu_i, b_j^{-1} a_i \gamma_{iR})} + \bar{C}_{\mathcal{G}(\mu_j, \gamma_{Rj})} - \bar{C}_{\mathcal{K}(\mu_i, \mu_j, a_i \gamma_{iR}, b_j \gamma_{Rj})}. \quad (35)$$

Hence, the bounds on the ASR of TWR are obtained with $a_i = 1$ and $b_j = 1 + \eta_j$ in (34) and (35) for the BF case as

$$\mathcal{E}\{\mathcal{R}_{\mathrm{BF}_2}^{\mathcal{L}}\} = \frac{1}{2} \sum_{i=1}^{2} \bar{C}_{\mathcal{G}(\mu_i, \gamma_{Ri})} + \frac{1}{2} \sum_{(i,j) \in \mathcal{D}} \left\{ \bar{C}_{\mathcal{G}(\mu_i, (1+\eta_j)^{-1} \gamma_{iR})} - \bar{C}_{\mathcal{K}(\mu_i, \mu_j, (1+\eta_j)^{-1} \gamma_{iR}, \gamma_{Rj})} \right\} \quad (36)$$

and

$$\mathcal{E}\{\mathcal{R}_{\mathrm{BF}_2}^{\mathcal{U}}\} = \frac{1}{2} \sum_{i=1}^{2} \left\{ \bar{C}_{\mathcal{G}(\mu_i, \gamma_{Ri})} + \log_2(1+\eta_i) \right\} + \frac{1}{2} \sum_{(i,j) \in \mathcal{D}} \left\{ \bar{C}_{\mathcal{G}(\mu_i, (1+\eta_j)^{-1} \gamma_{iR})} - \bar{C}_{\mathcal{K}(\mu_i, \mu_j, \gamma_{iR}, (1+\eta_j) \gamma_{Rj})} \right\}. \quad (37)$$

Similarly, the ASR of TWR employing the OSTBC is obtained with $a_i = \frac{1}{M_i}$ and $b_j = 1 + \frac{\eta_j}{M_j}$ in (34) and (35) as

$$\mathcal{E}\{\mathcal{R}_{\mathrm{ST}_2}^{\mathcal{L}}\} = \frac{1}{2} \sum_{i=1}^{2} \rho_i \bar{C}_{\mathcal{G}(\mu_i, \gamma_{Ri})} + \frac{1}{2} \sum_{(i,j) \in \mathcal{D}} \rho_i \left\{ \bar{C}_{\mathcal{G}(\mu_i, (M_i + M_i \eta_j / M_j)^{-1} \gamma_{iR})} - \bar{C}_{\mathcal{K}(\mu_i, \mu_j, (M_i + M_i \eta_j / M_j)^{-1} \gamma_{iR}, \gamma_{Rj})} \right\} \quad (38)$$

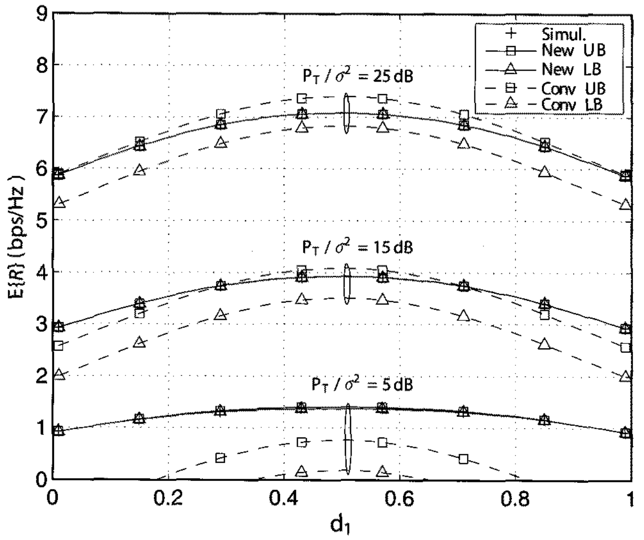

Fig. 2. The ASR of TWR as a function of the normalized relay location d_1 when $M_1 = 1$, $M_2 = 1$, $\zeta = 0.618$, and $\chi = 0.5$.

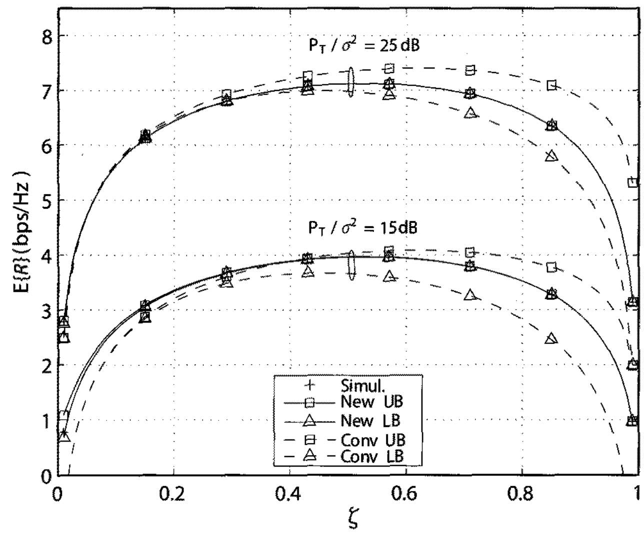

Fig. 3. The ASR of TWR as a function of the relay power allocation ζ when $M_1 = 1$, $M_2 = 1$, $d_1 = 0.5$, and $\chi = 0.5$.

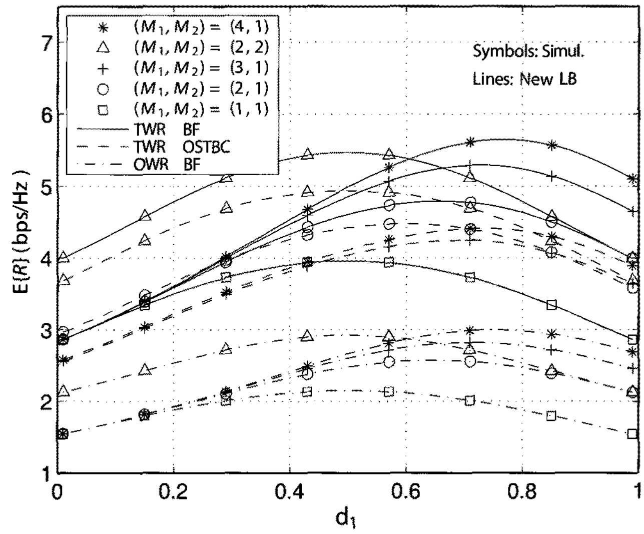

Fig. 4. The ASR of TWR employing the BF and OSTBC as a function of the normalized relay location d_1 when $\zeta = 0.5$, $\chi = 0.5$, and $P_T/\sigma^2 = 15$ dB.

and

$$\begin{aligned}\mathcal{E}\{\mathcal{R}_{\mathrm{ST}_2}^{\mathcal{U}}\} = &\frac{1}{2}\sum_{i=1}^{2}\rho_i\left\{\bar{C}_{\mathcal{G}(\mu_i,\gamma_{Ri})} + \log_2\left(1+\frac{\eta_i}{M_i}\right)\right\}\\ &+\frac{1}{2}\sum_{(i,j)\in\mathcal{D}}\rho_i\left\{\bar{C}_{\mathcal{G}(\mu_i,(M_i+M_i\eta_j/M_j)^{-1}\gamma_{iR})}\right.\\ &\left.-\bar{C}_{\mathcal{K}(\mu_i,\mu_j,M_i^{-1}\gamma_{iR},(1+\eta_j/M_j)\gamma_{Rj})}\right\}. \end{aligned} \quad (39)$$

In passing, we would like to add that the exact ASR of OWR can easily be obtained by $\mathcal{E}\{\mathcal{R}_{\mathrm{BF}_1}\} = \frac{1}{2}\,\mathcal{E}\{\mathcal{R}_{\mathrm{BF}_2}^{\mathcal{U}}\}\big|_{\eta_1=\eta_2=0}$, and $\mathcal{E}\{\mathcal{R}_{\mathrm{ST}_1}\} = \frac{1}{2}\,\mathcal{E}\{\mathcal{R}_{\mathrm{ST}_2}^{\mathcal{U}}\}\big|_{\eta_1=\eta_2=0}$.

IV. NUMERICAL AND SIMULATION RESULTS

In this section, we verify the validity of the bounds obtained in this paper and investigate the performance of TWR in various configurations. We assume a relay power constraint $P_R = \zeta P_T$ and source power constraints $P_1 = \chi(1-\zeta)P_T$ and $P_2 = (1-\chi)(1-\zeta)P_T$, where P_T is the total transmit power of the system, $\zeta \in (0,1)$ denotes the fraction of the total transmit power allocated to the relay R, and $\chi \in (0,1)$ denotes the fraction of the source transmit power $(1-\zeta)P_T$ allocated to T_1. It is also assumed that the path loss exponent is $\nu = 3$ and the Nakagami fading parameters are $m_1 = m_2 = 1$: The bounds with the shape parameter $\mu_i = m_i M_i$ of gamma distribution will be verified by varying M_i with m_i fixed.

Figs. 2 and 3 compare the bounds derived in this paper on the ASR of TWR with the conventional bounds [7] and simulation results when $M_1 = M_2 = 1$ and $\chi = 1/2$. The ASR is shown as a function of the normalized relay location d_1 in Fig. 2 when the relay power allocation is $\zeta = 0.618$ (asserted to be optimal in [7]) and as a function of the relay power allocation ζ in Fig. 3 when $d_1 = 1/2$. It is observed that the new upper-bound (UB) and lower-bound (LB) are barely distinguishable from the simulation results for various values of d_1 and ζ. On the other hand, the conventional bounds deviate considerably from the simulation results: in addition, the upper-bound becomes invalid especially when the relay approaches T_1 or T_2 and P_T/σ^2 is small. It is also observed in Fig. 3 that the optimal relay power allocation achieved is not at $\zeta = 0.618$ but at around $\zeta = 0.5$.

Fig. 4 provides the ASR of TWR with the BF and OSTBC as a function of the normalized relay location d_1 when $\zeta = 0.5$, $\chi = 0.5$, and $P_T/\sigma^2 = 15$ dB. We provide both the new lower-bound and simulation results for several antenna configurations (M_1, M_2), where the codes of rate 1 [13] and rate 3/4 [14] are employed for the OSTBC when M_1 or M_2 is 2 and 4, respectively. For comparison, we have also included the ASR of OWR employing the BF. As we could anticipate easily, the BF exploiting the CSI provides a rate gain over the OSTBC without CSI. It is observed that the maximum ASR is achieved when the relay is located closer to the source T_2 when $M_1 > M_2$: For exam-

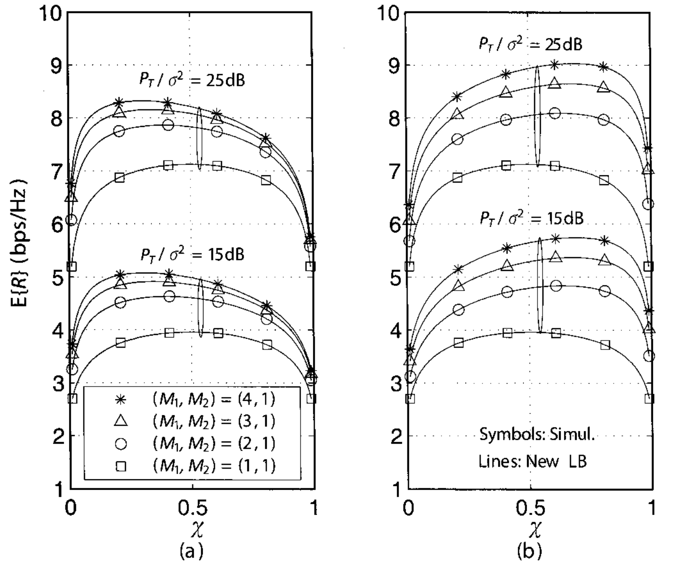

Fig. 5. The ASR of TWR employing the BF as a function of source power allocation χ in the asymmetric channels: (a) At $d_1 = 0.5$, (b) at the optimal $d_1 = d_1^*$.

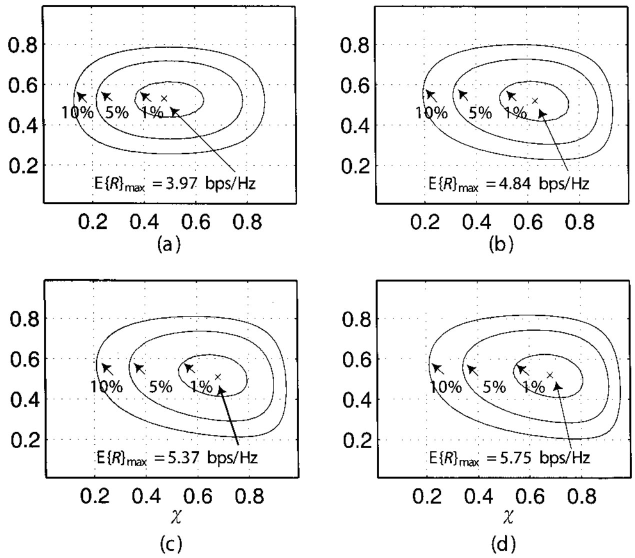

Fig. 6. The region of (χ, ζ) providing the ASR within 1%, 5%, and 10% loss of the maximum ASR: (a) $(M_1, M_2) = (1,1)$, (b) $(M_1, M_2) = (2,1)$, (c) $(M_1, M_2) = (3,1)$, (d) $(M_1, M_2) = (4,1)$.

ple, with $d_1 = 0.76$ when $(M_1, M_2) = (4,1)$. As M_1 increases with M_2 fixed, the ASR of BF increases for all d_1 while the ASR of OSTBC does not always increase due to the rate loss when $M_1 = 3$ or 4. It is also observed that the ASR of TWR is about twice that of OWR.

Fig. 5 provides the ASR of TWR employing the BF with several degrees of channel asymmetry as a function of the source power allocation χ when $\zeta = 0.5$ and $M_2 = 1$. The relay is assumed to be located at the center $d_1 = 0.5$ in Fig. 5(a) and, in Fig. 5(b), at the location $d_1 = d_1^*$ maximizing the ASR in Fig. 4. As M_1 increases from 1 to 4, the optimal source allocation χ^* shifts from 0.5 to 0.26 allocating more power to T_2 when $d_1 = 0.5$ in Fig. 5(a) while it shifts from 0.5 to 0.75 to T_1 when $d_1 = d_1^*$ in Fig. 5(b). Nonetheless, the difference in the ASR values at $\chi = 0.5$ and $\chi = \chi^*$ is not significant.

The effect of power allocation on the ASR of TWR is investigated further in Fig. 6 by providing the contours of (χ, ζ) at which the ASR loss is 1%, 5%, and 10% of the maximum ASR achievable when the relay location is $d_1 = d_1^*$, $P_T/\sigma^2 = 15$ dB, and the BF is employed. In the figure, the mark 'x' represents the optimal (χ^*, ζ^*) leading to the maximum ASR $\mathcal{E}\{\mathcal{R}\}_{\max}$. It is observed in Fig. 6 that the ASR of TWR is rather robust to the source and relay power allocation at various degrees of the channel asymmetry. For several source antenna configurations up to four antennas, we can guarantee 95% of the maximum ASR with $\zeta \approx 0.5$ and $\chi \approx 0.5$.

V. CONCLUSION

In this paper, we have derived more accurate bounds on the ASR when the TWR system consists of one AF relay and two source terminals employing either the BF or OSTBC. The new bounds are obtained by applying simple lower- and upper-bounds on the instantaneous sum rate and applying some properties of gamma distribution. From the results shown in this paper, it is observed that the proposed bounds are almost indistinguishable from the exact values and are much more accurate than the conventional bounds in various conditions. It is also observed that the relay location is more important than the power allocation to improve the ASR of TWR in asymmetric channels caused by the heterogeneous antenna configuration of the source terminals.

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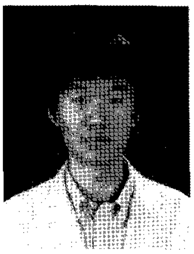

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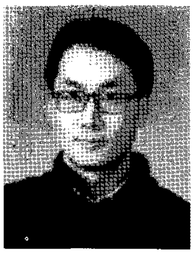

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