The Throughput Order of Multicast Traffics with Physical-Layer Network Coding in Random Wireless Ad Hoc Networks

Chen Chen, Lin Bai, Jianhua He, Haige Xiang, and Jinho Choi

Abstract: **This paper attempts to address the effectiveness of physical-layer network coding (PNC) on the throughput improvement for multi-hop multicast in random wireless ad hoc networks (WAHNs). We prove that the per session throughput order with PNC is tightly bounded as $\Theta((n\sqrt{m}R(n))^{-1})$ if $m = O(R^{-2}(n))$, where n is the total number of nodes, $R(n)$ is the communication range, and m is the number of destinations for each multicast session. We also show that per-session throughput order with PNC is tight bounded as $\Theta(n^{-1})$, when $m = \Omega(R^{-2}(n))$. The results of this paper imply that PNC cannot improve the throughput order of multicast in random WAHNs, which is different from the intuition that PNC may improve the throughput order as it allows simultaneous signal access and combination.**

Index Terms: **Physical-layer network coding (PNC), throughput capacity, wireless ad hoc networks (WAHNs).**

I. INTRODUCTION

The seminal work of Gupta and Kumar [1] has demonstrated that the per-node throughput of random wireless ad hoc networks (WAHNs) scales as $\lambda = \Theta\left(1/\sqrt{n \log n}\right)$,[1] where λ denotes the per-node throughput in WAHNs and n is the number of nodes. It gives the fundamental limits of the throughput with multi-hop routing. Since then, extensive research has been conducted to understand the throughput of random WAHNs for different types of communications such as broadcast and multicast, and with several new techniques such as network coding (NC), multi-packet reception (MPR), and multi-packet transmission (MPT). It has been demonstrated that, in static WAHNs, a plain routing cannot scale the throughput order for broadcast and multicast [1], [2], while MPR and MPT [3], [4] can increase the throughput order over routing, NC [4]–[6] can only achieve the same throughput order as routing.

The previous studies of NC for the problem of throughput capacity scaling (such as the works in [4]–[6]), however, only focused on the schemes which are relying on single-signal reception at the physical layer (called further on "digital network coding (DNC)"). Recently, by allowing simultaneous signal access and signal superposition reception at the physical layer, new NC schemes such as physical-layer network coding (PNC) [7] and analog network coding (ANC) [8] have been proposed. As mentioned in [4], PNC and ANC may implicitly offer the ability of MPR and MPT. Thus, it may be possible that PNC and ANC, unlike DNC, have the potential to improve the throughput order. Now, in order to fill the gap left by previous works, the following two questions need to be answered: (1) How will the throughput of random WAHNs scale with PNC or ANC? (2) Will PNC and ANC improve the throughput order over former routing and DNC schemes [1], [5], [6]? This paper is motivated by the above questions. For simplicity, we term the NC scheme that directly deals with a superposition of signals at physical layer (such as PNC [7], ANC [8], etc.), as the "PNC" scheme.[2] We study the role of PNC in terms of scaling laws, as compared to the DNC, MPR, and MPT schemes whose throughput scaling laws were previously addressed. In this paper, we use the protocol model in [1]–[6] for throughput capacity analysis and consider an n-node general multicast network, where the nodes in the network are randomly distributed in a unit area, and there are n multicast sessions, each of which consists of one source and m destinations ($m \leq n$).

The main result of this paper can be summarized as follows.

- In a random n-node WAHN, when each multicast session consists of m sinks, the per-session throughput order with PNC is

$$\lambda = \begin{cases} \Theta\left(\frac{1}{n\sqrt{m}R(n)}\right), & m = O(R^{-2}(n)), \\ \Theta\left(\frac{1}{n}\right), & m = \Omega(R^{-2}(n)). \end{cases}$$

This result implies that the throughput order improvement of multicast traffic with PNC over the former routing or network coding techniques is $\Theta(1)$. Similar to plain routing and DNC schemes, PNC also does not improve the throughput order of random WAHNs. Nevertheless, the derivation of the throughput upper bound with PNC in this paper also shows that, PNC may provide an extra constant factor of the throughput improvement over the former routing or network coding techniques because it embraces the wireless interference.

The remainder of this paper is organized as follows. We provide a short review of related works in Section II. The model and preliminaries for throughput capacity analysis of PNC are presented in Section III. The throughput order bounds of PNC

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[1] O, Ω, and Θ describe the upper, lower, and tight order bounds, respectively.

[2] From the point of view of information theory, the ANC-like operations are similar to the PNC operations.

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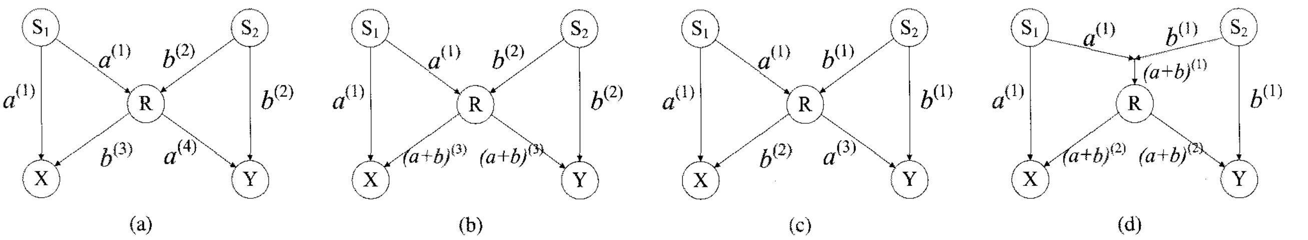

Fig. 1. Transmission models (examples) of traditional, DNC, MPR, and PNC schemes: (a) Traditional routing scheme, (b) network coding (DNC) scheme, (c) MPR scheme, and (d) PNC scheme.

in WAHNs are derived in Section IV. We conclude the paper in Section V.

II. RELATED WORKS

Gupta and Kumar's seminal work [1] derived the unicast throughput capacity of traditional routing in WAHNs, where the nodes in the network are identical with single packet reception. After that, broadcast capacity [9], [10] and multicast capacity [2], [11] were further studied by researchers. These pioneering literatures showed that, without exploiting node mobility [12], in a static n-node random WAHN, when each data dissemination session consists of m sinks, the per-session throughput with the traditional routing scheme is

$$\lambda = \begin{cases} \Theta\left(\frac{1}{\sqrt{mn}\log n}\right), & m = O(R^{-2}(n)), \\ \Theta\left(\frac{1}{n}\right), & m = \Omega(R^{-2}(n)). \end{cases} \tag{1}$$

As the generalization of routing, the benefit of network coding is first discovered by Ahlswede *et al.* [13]. They proved that by using network coding, for single-session communication with one source, the maximum throughput of the session can achieve the upper bound on the theoretical capacity of the network, which is the minimum-cut capacity. In contrast, traditional routing schemes may not always achieve this capacity. However, the above result of NC are derived in wired networks, in which multiple messages from different nodes can simultaneous access one node and then this node gets the mixture of the messages. Due to the insufficiency of system resources of wireless communication systems, in recent advancement of network coding, researchers turned to utilizing NC in wireless networks and studying its benefit.

Liu *et al.* [5], [14] first considered the throughput scaling problem of DNC in random WAHNs. They showed that by exploiting NC and wireless broadcast, the order of scaling laws for multi-source unicasts cannot be improved. Recently, using a similar analysis method to Liu *et al.* [5], [14], Karande *et al.* [6], [15] further demonstrated that the throughput orders for multi-source multicasts also cannot be improved by exploiting DNC. Thus, the throughput scaling laws with DNC in random WAHNs are also expressed as in (1).

On the contrary, Garcia-Luna-Aceves *et al.* [3] and Wang *et al.* [4] successively showed that the throughput order of unicast and multicast traffics can be increased by exploiting non-interference transmission abilities, i.e., by exploiting MPR and MPT, on the wireless nodes. This result implies that the scaling laws are improved if the interference is eliminable in WAHNs. We note that, to confront the multiple-access interference in wireless networks, the concept of PNC ([7], [8], etc.) was recently proposed, where the relay nodes in the PNC transmission scheme deal with the interfered signals at physical layer, instead of mixing the information at upper layers. Thus, it is intuitive that PNC may improve the throughput order as compared to DNC because it utilizes the additive nature of the electromagnetic (EM) waves and allows multiple messages to access at the same time for coding.

To explicitly show the difference of the above schemes, we present transmission examples in Fig. 1, i.e., the transmission examples of traditional routing [1], [2], [9]–[11], DNC [5], [6], [14], [15], MPR [3], [4] (MPT is similar with MPR), and PNC schemes, where S_1 and S_2 are two sources, and X and Y are two destination nodes in the network which could as well be S_1 and S_2 themselves. The two sources transmit two bits of data a and b where the expression $a^{(i)}, b^{(i)}$ means that the bit a or b is transmitted in time slot i of the channel. We have

a) By the traditional routing scheme, the two bits are transmitted successfully in four time slots.
b) By the NC scheme, this transmission consumes three time slots. The router in the NC scheme sends the combination of the two bits to both of the destinations simultaneously.
c) By the MPR scheme, this transmission also consumes three time slots. The router in the MPR scheme is able to receive the two bits simultaneously and send them to the two destinations in different time slots.
d) By the PNC scheme, only two time slots are needed to finish this two-bit transmission, in which the router can receive and send a superposition of the interfered signals from/to the two channels simultaneously by employing physical-layer processing techniques.

The receiver nodes in both the schemes of DNC and PNC get the information they need by decoding the combinations that they have received.

From the above literature review in this section, we can find that prior works have derived the throughput scaling laws in random WAHNs for traditional routing (unicast, multicast), DNC (unicast, multicast), and MPR/MPT (unicast, multicast) schemes by using the same model [1] and similar approaches (e.g., the approaches in [1], [5], and [11]). However, the throughput scaling laws for PNC in random WAHNs remain elusive. A comprehensive theoretical treatment of PNC for single-source multicast was done in [16] and it was further generalized to multi-source multicast for AWGN and general discrete memoryless networks in [17], but the results of the achievable rate and cut-set bound in these literatures are based on the topology properties of the network and not for the throughput scaling laws in random WAHNs with respect to the node number n. In this paper, based on the model used in [1]–[6], we study the unified throughput scaling laws for unicast and multicast with PNC in random WAHNs.

III. MODEL AND PRELIMINARIES

In this paper, we consider a static random WAHN where there are n nodes uniformly distributed in a unit square area. The analysis of this paper is based on the protocol model proposed in [1]. We analyze the throughput order of PNC and compare it with DNC, MPR, and MPT schemes in static random WAHNs.

Definition 1: The protocol model of point-to-point communication [1]. All the nodes in a network have a common communication range $R(n)$. The transmission from a node (say a_1) to another node (say a_2) is successful if these two nodes are within a range of $R(n)$, and there is no other overlapped transmission from any node within a range of $(1+\Delta)R(n)$ to node a_2, where $\Delta > 0$ is a range boundary of multiple access interference depending on the properties of wireless medium.

The protocol model for a point-to-point (PTP) communication was used to analyze the throughput order of traditional routing schemes [1], [2] and DNC schemes [5], [6]. In [3] and [4], it was further extended to take into account the capability of ideal MPR or MPT at receivers and transmitters. It was assumed that any transmitter (receiver) node can transmit (receive) different information simultaneously to (from) multiple nodes within a circle centered at the transmitter (resp., receiver) with a range of $R(n)$. In this paper, we also use the same protocol model under which PNC can be ideally implemented. It is assumed that any receiver node with the PNC scheme can receive a superposition of the simultaneously transmitted signals from multiple nodes within a range of $R(n)$. Similarly, any transmitter node can broadcast its information to multiple nodes within its communication range of $R(n)$, but it cannot send different packets at the same time. In addition, we assume that no node can transmit and receive at the same time, which is equivalent to half-duplex communications [1].

Note that as n goes to infinity, the density of the network also goes to infinity. To observe the throughput scalability of WAHNs, our analysis only focuses on dense networks, where the node number n is a large number. For the n-node random network, we also utilize the following well known properties [18], [19].

Lemma 1: Connectivity criteria. The network connectivity under the protocol model can be guaranteed with high probability (w.h.p.)[3] if and only if

$$R(n) \geq \sqrt{\frac{3\log n}{n}} = \Theta\left(\sqrt{\frac{\log n}{n}}\right). \quad (2)$$

Lemma 2: Chernoff bounds. Let N_s be a random variable representing the number of nodes in a region A_s, and let $|A_s|$ denote the area of A_s. Then, we have the following result as the Chernoff bounds.

[3]In this paper, w.h.p. denotes "with probability 1 when $n \to \infty$".

1) For any $\delta > 0$, $P[N_s > (1+\delta)n|A_s|] < (e^{\delta}/(1+\delta)^{1+\delta})^{n|A_s|}$.
2) For any $0 < \delta < 1$, $P[N_s < (1-\delta)n|A_s|] < e^{-\frac{1}{2}n|A_s|\delta^2}$.

Combining the above two inequalities, we can get that, for any $0 < \delta < 1$, we have [3]

$$P[|N_s - n|A_s|| > \delta n|A_s|] < e^{-\theta n|A_s|} \quad (3)$$

where $\theta = \min\{(1+\delta)\log(1+\delta) - \delta, \frac{1}{2}\delta^2\}$.

We assume that the nodes are connected w.h.p. in the network under the protocol model. We can use a connected undirected graph $G = (V, E)$ to represent the network, where V and E are sets of vertices and edges in the graph G, respectively. Then in the following, we present the definitions of multicast throughput of random WAHNs.

Definition 2: Feasible throughput for multicast [1], [2], [4]. In a WAHN of n nodes, where each node is a source of a multicast session and transmits its packets to m random destinations, a throughput of $C_m(n)$ bits per second for each session is feasible if there is a joint spatial and temporal scheduling transmission scheme, such that every source node can successfully deliver $C_m(n)$ bits per second on average to its m chosen destinations. That is, there is a $T < \infty$ such that in every time interval $[(i-1)T, iT]$ every source node can send $TC_m(n)$ bits to its destination nodes.

Definition 3: Throughput order [1]. $C_m(n)$ is said to be of order $\Theta(f(n))$ bits per second if there exist deterministic positive constants c and c' such that

$$\begin{cases} \lim_{n\to\infty} \text{Prob}(C_m(n) = cf(n) \text{ is feasible}) = 1, \\ \lim_{n\to\infty} \text{Prob}(C_m(n) = c'f(n) \text{ is feasible}) < 1. \end{cases}$$

An upper bound on the throughput order with PNC can be achieved if the optimal spatial and temporal transmission scheduling schemes are used in the network. To describe such scheduling schemes, we label all the nodes as $a_1, a_2, \cdots, a_n$ and let t_i represent the time interval $[(i-1)T, iT]$. We denote the signal transmitted by transmitter node a_j ($j = 1, \cdots, n$) during time slot t_i by $S(a_j, X(a_j, t_i))$, where $X(a_j, t_i)$ is the source packet or combination packet sent by a_j during t_i. Here, $X(a_j, t_i)$ can be regarded as a packet of binary data and $S(a_j, X(a_j, t_i))$ is the modulated signal (can be represented by complex numbers) of $X(a_j, t_i)$. We also denote the received packet at receiver node a_k ($k = 1, \cdots, n$) during t_i by $Y(a_k, t_i, S(a_{k_1}, X(a_{k_1}, t_i)), \cdots, S(a_{k_l}, X(a_{k_l}, t_i)), N(t_i))$, where $a_{k_1}, \cdots, a_{k_l}$ are the neighboring transmitter nodes of a_k (within the communication range of a_k), and $N(t_i)$ is the noise[4]. Thus, $Y(a_k, t_i, \cdots)$ is a packet of binary data obtained from the superposition of the transmitted signals $S(a_{k_1}, X(a_{k_1}, t_i)), \cdots, S(a_{k_l}, X(a_{k_l}, t_i))$, which is the combination (e.g., after XOR computation) of $X(a_{k_1}, t_i), \cdots, X(a_{k_l}, t_i)$.

We consider an example of the original PNC signal mapping [7] to further demonstrate the details of the PNC operations stated above. With the assumption of the QPSK modulation (for the details, see Table 1 in [7]), in each of the I & Q dimensions, the source bit $X = \{0; 1\}$ is mapped to $S = \{1; -1\}$,

[4]For simplicity, we assume the additive white gaussian noise (AWGN) as the noise model in each time of reception.

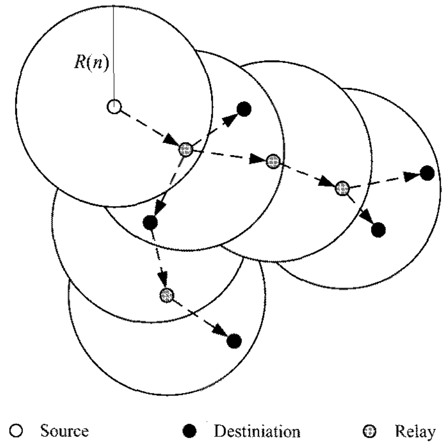

Fig. 2. Total area of a minimum area multicast tree.

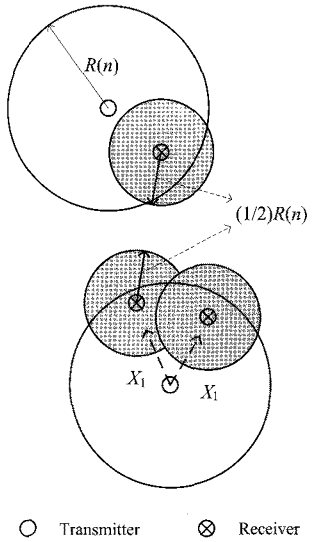

Fig. 3. Total active area for DNC based on protocol model.

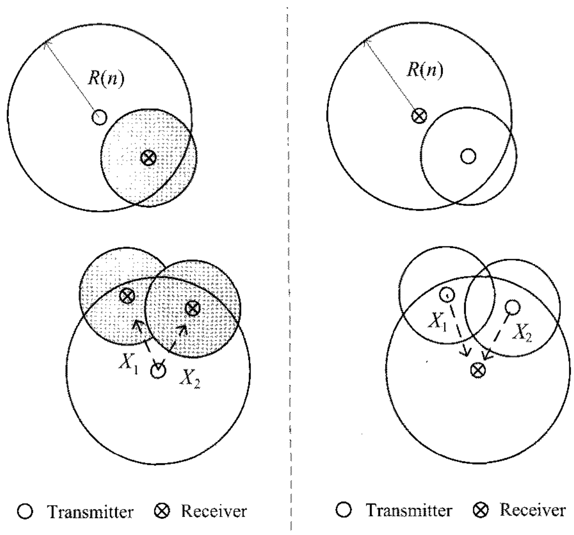

Fig. 4. Available total active area for DNC+MPT and DNC+MPR schemes: (a) DNC+MPT and (b) DNC+MPR.

and we can see that if two signals are added in the air and received we get $2, 0$, or -2 since S of one signal is 1 or -1 and S of the other signal is 1 or -1. Then, with the PNC modulation mapping, $2, 0$, or -2 will be mapped to $-1, 1$, or -1, respectively, and the output bit after the PNC operation is $Y = 0, 1$, or 0, respectively. Thus, we have $Y^{(I)} = X_1^{(I)} \oplus X_2^{(I)}$ and $Y^{(Q)} = X_1^{(Q)} \oplus X_2^{(Q)}$. Similar to the above, the PNC operations with other kinds of signal mapping schemes can also be described. We assume that every node in the network has an infinite buffer. The destination nodes with the PNC scheme will buffer the received packets (i.e., $Y(a_k, t_i, \cdots)$) and try to decode them by solving an equation system. A PNC scheme for multicast is called *feasible* if every destination node can decode successfully and can get all the source information of the multicast session(s) it belongs to within a finite time. In the remainder of the paper, we will consider only the network throughput with feasible PNC schemes.

IV. THROUGHPUT ORDER BOUNDS FOR PNC

In this section, we analyze the throughput scaling law of PNC in random WAHNs and compare it with the previous results of DNC, MPR, and MPT schemes [3]–[6]. We will first derive the upper bound on the average number (or area) of simultaneous transmissions that can be supported in the network, and then calculate the lower bound on the average number (or area) of transmissions that are needed for one multicast session. Once they are found, the throughput upper bound can be obtained as the ratio between the number of simultaneous transmissions supported in the network and the number of transmissions needed for one session.

Definition 4: Minimum area multicast tree. The minimum area multicast tree (as shown in Fig. 2) in the multicast tree with m destinations for each source is a multicast tree that has a minimum total area. The area of a multicast tree is defined as the total area covered by the circles centered around each source or relay in the multicast tree with a range of $R(n)$.

Definition 5: Total active area (TAA). The total active area (as shown in Figs. 3–5) is the total valid area where simultaneous transmissions or receptions (in different nodes) can be supported within the whole area of the network.

The above two definitions are based on existing works (e.g., [4], [11], etc.). Let $\overline{A_{taa}}$ be the statistical mean of the total active area in a random network, and $\overline{A_m}$ be the average of the minimum area consumed to multicast a packet to m destinations. From the protocol model, each transmission covers an area of $\Theta(R^2(n))$. Thus, the average number of simultaneous transmissions supported in the network is upper bounded by $\overline{A_{taa}}/\Theta(R^2(n))$, while the number of transmissions of one session is lower bounded by $\overline{A_m}/\Theta(R^2(n))$. We have the following results.

Lemma 3: Under the optimal transmission scheduling, the per-session throughput order for multicast of a random WAHN of n nodes is upper bounded by $O\left(\frac{1}{n} \times \frac{\overline{A_{taa}}}{\overline{A_m}}\right)$.

Proof: From Definitions 4 and 5, the ratio between $\overline{A_{taa}}$ and $\overline{A_m}$ represents the average number of simultaneous multicasts that can occur in the network. Normalizing this ratio by n provides per-session throughput order. □

Thus, we can obtain the upper bound of the throughput order if we find both the lower bound of $\overline{A_m}$ and the upper bound of $\overline{A_{taa}}$. In the sequel, we derive the bounds of $\overline{A_m}$ and $\overline{A_{taa}}$, respectively.

A. Lower Bound on $\overline{A_m}$.

Definition 6: Euclidean minimum spanning tree (EMST). Consider a connected undirected graph $G = (V, E)$. The EMST of G is a spanning tree of G with the total minimum Euclidean distance between connected vertices of this tree.

Definition 7: m-cast Tree. An m-cast tree is a minimum set of the nodes that connect a source node of a multicast session with all its intended m destinations.

Note that to achieve the throughput upper bound, the transmissions of each multicast session in the considered network should be taken along with an m-cast tree. Let $\overline{D_{\mathrm{emst}}}$ denote the total average Euclidean distance of each m-cast tree in the network. We can have the following results for $\overline{D_{\mathrm{emst}}}$.

Lemma 4: $\overline{D_{\mathrm{emst}}}$ for a network of m nodes is tightly bounded as $\overline{D_{\mathrm{emst}}} = \Theta(\sqrt{m})$.

Proof: Let $f(x)$ denote the node probability distribution function in the network area. Then, Steele [20] has shown that for large values of m and $d > 1$, $\overline{D_{\mathrm{emst}}}$ is tightly bounded as

$$\overline{D_{\mathrm{emst}}} = \Theta\left(c(d)m^{\frac{d-1}{d}} \int_{R^d} f(x)^{\frac{d-1}{d}} dx\right) \qquad (4)$$

where d is the dimension of the network, and both $c(d)$ and the integral term are not functions of m. Thus, as we set $d = 2$ for a 2-dimensional network, we have $\overline{D_{\mathrm{emst}}} = \Theta(\sqrt{m})$. □

Lemma 5: With the transmission range of $R(n)$, $\overline{A_m}$ has the following lower bound.

$$\overline{A_m} = \begin{cases} \Omega\left(\sqrt{m}R(n)\right), & m = O(R^{-2}(n)), \\ \Omega(1), & m = \Omega(R^{-2}(n)). \end{cases} \qquad (5)$$

Proof: If we assume the transmission range of the nodes is arbitrarily large, then all the nodes in each m-cast tree are connected in one hop. In this case, Li [21] has proven that the union of the transmission disks of the multicast session (which consists of one source node and m destination nodes), will cover at least a constant area, say $0 < \rho < 1$. Thus, in this case $\overline{A_m}$ is lower bounded as $\Omega(1)$. If the transmission range is not large enough to connect any two adjacent nodes in the m-cast tree in one hop, then we need to create a connected m-cast tree for the information relay. In this case, from Li [21] we can also find that $\overline{A_m}$ is lower bounded as $\Omega\left(\overline{D_{\mathrm{emst}}}R(n)\right)$. Then from Lemma 4, $\overline{A_m}$ is then lower bounded as $\Omega\left(\sqrt{m}R(n)\right)$. We note that in the above two cases, the number of multicast destinations m is different, and there exists a threshold value m_{th} between these two bounds. This threshold is derived by computing m_{th} such that the two limits are equal, i.e., $\Omega\left(\sqrt{m}R(n)\right) = \Omega(1)$. Thus, we have $m_{th} = \Theta(R^{-2}(n))$. From the above results, it can be deduced that

$$\overline{A_m} = \begin{cases} \Omega\left(\sqrt{m}R(n)\right), & m = O(R^{-2}(n)), \\ \Omega(1), & m = \Omega(R^{-2}(n)). \end{cases}$$

□

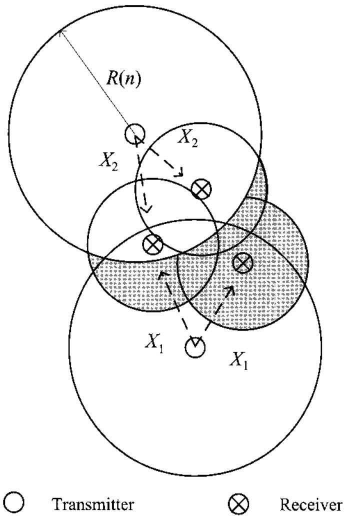

Fig. 5. Available total active area for PNC.

B. Upper Bound on $\overline{A_{taa}}$.

Note that Lemmas 3 and 5 can be applied to all the DNC, MPR, MPT, and PNC schemes. Thus, the difference of throughput upper bounds for different schemes only lies on the difference of their $\overline{A_{taa}}$. Now, we come to calculate $\overline{A_{taa}}$ by counting the total valid area of the circles covered by transmissions that can be simultaneously supported in different schemes. Some examples for this calculation are illustrated in Figs. 3–5.

Gupta and Kumar [1] showed that the necessary condition which is that a disk of radius $(1/2)\Delta R(n)$ centered at each receiver is disjoint under which there is no multiple access interference in the network. Then, recently Liu *et al.* [5] and Karande *et al.* [6] further demonstrated that for the transmission scheme exploiting DNC and broadcast, the receiver disks can overlap, but the union of the disks centered at the receivers of one transmission should be disjoint from the union of the disks centered at the receivers of another transmission (as shown in Fig. 3). Thus, the value of $\overline{A_{taa}}$ for DNC with broadcast scheme is upper bounded by $(4/\pi((1/2)\Delta R(n))^2)\pi R^2(n) = \Theta(1)$.

For DNC+MPT mode, the above restriction of the spatial distribution of the receiver disks still exists, but different receivers in the transmission circle of one transmitter can get different information now (see Fig. 4(a)). Thus, if we sum up the active transmission areas to calculate $\overline{A_{taa}}$, then $\overline{A_{taa}}$ is upper bounded by $\Theta\left(nR^2(n)\right)$, which is the average number of nodes in the transmission range of one transmitter [4].

$\overline{A_{taa}}$ of the DNC+MPR mode is the same as the DNC+MPT mode if we interchange the transmitters and receivers (see Fig. 4(b)). The upper bound on $\overline{A_{taa}}$ is also found as $\Theta\left(nR^2(n)\right)$.

For the PNC scheme, the above restrictions of the node distribution no longer exist. However, during each time slot, a transmitter node can only transmit one packet, while a receiver node in the overlapped transmission circles will receive a combination of the information transmitted from its neighboring transmitter nodes as one packet. Therefore, we have the following important observation of $\overline{A_{taa}}$ for PNC.

Lemma 6: For PNC, $\overline{A_{taa}}$ is upper bounded by $O(1)$.

Proof: Different from the DNC scheme, in the PNC scheme the disks of receiver nodes can overlap, and a receiver can have a superposition of multiple signals from different transmitters (see Fig. 5). However, from the model introduced in Section III, with feasible PNC schemes, the destination nodes will have the information they demand by decoding the packets that they have received and buffered (which can be the source packets or combination packets of a single source, or the combination packets of multiple sources). During any arbitrary time interval $[(i-1)T, iT]$, each node can use all packets received during the time interval $[0, iT]$ to decode (solve the equation system) and obtain the source data it needs. For a node (such as a_k), to decode and have h source packets in any scheduling scheme, h different source packets or combination packets that are independent to each other (such as $Y(a_k, t_{i_1}, \cdots), \cdots, Y(a_k, t_{i_h}, \cdots)|_{(t_{i_1}, \cdots, t_{i_h} \in [0, iT])})$ are required to solve the equations. It is obvious that we need at least h times of receptions and information exchanges among the nodes for collecting such packets. Thus, regardless of receiving one signal or a superposition of multiple signals, the average number of packets of information obtained by a receiver in one time slot is equally upper bounded by $\Theta(1)$.

In addition, in the transmission range of one transmitter node, all receiver nodes will receive identical information from this common transmitter (for example, in one time slot, the receiver nodes in Fig. 5 have X_1, X_2, and the combination $X_1 + X_2$, respectively, but in this time slot, the available transmitted information is only X_1 and X_2, and in the union of the transmission areas of Fig. 5, only two transmissions can be counted for $\overline{A_{taa}}$). Thus, the overlapped transmission circles can only contribute to the total active area at most *once*.

Furthermore, we note from Definition 2 that the throughput order of multicast can be calculated by the average amount of information received by all the destination nodes of each multicast session during a unit of time. Thus, the average active area of each transmission is only $1\times$ the area of the communication range. Since we normalize the area of the whole network to unity, from the above results, the upper bound on $\overline{A_{taa}}$ becomes $O(1)$ with PNC. □

Therefore, by Lemmas 3, 5, and 6, we can obtain the following theorem, which is the main result of this paper.

Theorem 1: Suppose that there are m destinations in each multicast session in an n-node random WAHN. The upper bound on the per-session throughput order of multicast with PNC is

$$\lambda = \begin{cases} O\left(\frac{1}{n\sqrt{m}R(n)}\right), & m = O(R^{-2}(n)), \\ O\left(\frac{1}{n}\right), & m = \Omega(R^{-2}(n)). \end{cases} \quad (6)$$

From the connectivity criteria ([1], [18]) as shown in (2), from the result of Theorem 1 we can find that if we guarantee the connectivity of random WAHNs, the per-session throughput upper bound for PNC multicast is

$$\lambda = \begin{cases} O\left(\frac{1}{\sqrt{mn\log n}}\right), & m = O(R^{-2}(n)), \\ O\left(\frac{1}{n}\right), & m = \Omega(R^{-2}(n)). \end{cases} \quad (7)$$

The order of this upper bound converges to the order of both the upper bounds and the lower bounds (as shown in (1)) for traditional routing [1], [2] and DNC schemes [4]–[6]. In addition, it can be deduced from [2], [4], and [6] that the throughput order bounds for the case of multi-source multicasting also serve for the case when multiple unicasts are considered. Furthermore, note that PNC is a generalization of former routing and DNC scheme. If we choose a spatial and temporal scheduling scheme that avoids the simultaneous signal superposition reception and combination, the PNC scheme becomes identical to the former schemes. Hence, based on a similar approach as in the existing works [1], [2], [4], and by utilizing the Chernoff bounds [19] and (3), we can find a scheduling scheme for multicast or unicast such as a time division multiple access (TDMA) scheme to achieve the throughput order found in Theorem 1, which means that the achievable lower bound with PNC for multicast or unicast is also $\Omega(1/(n\sqrt{m}R(n)))$ when $m = O(R^{-2}(n))$ and $\Omega(1/n)$ when $m = \Omega(R^{-2}(n))$. The above results can be interpreted by a more formal way as follows.

Theorem 2: For multicast with m destinations in an n-node random static WAHN with PNC, the per-session throughput order is $\lambda = \Theta(1/n\sqrt{m}R(n))$ when $m = O(R^{-2}(n))$, while $\lambda = \Theta(1/n)$ when $m = \Omega(R^{-2}(n))$. The gain of PNC over the former routing or DNC schemes is $\Theta(1)$.

Theorem 2 also holds for both multi-source unicasts and broadcasts cases when we set m as $m = \Theta(1)$ and $m = n$.

Theorems 1 and 2 show that the throughput order improvement with PNC over former routing and DNC schemes is $\Theta(1)$. This result disagrees with the naive intuition that PNC can improve the throughput order. The reason is that although PNC embraces the interference, the nodes are still only able to perform single-packet reception and transmission as the traditional schemes.

Nevertheless, the proof of Lemma 6 implies that PNC may provide an extra constant factor on the increase of the total active area, because the transmission/reception areas can be overlapped and closely crammed into the whole area of the network. For instance, as compared to the DNC scheme, where each of the receiver disks with radius $(1/2)\Delta R(n)$ is disjoint and can be regularly and closely packed in a network of a unit square area [5], a feasible factor on the increase of $\overline{A_{taa}}$ with PNC can be $4/\pi$ (as the transmission/reception areas can be overlapped in the PNC scheme, $4/\pi$ is the ratio between the area of a square and a disk with the same side length/diameter). This increase of the total active area results in constant improvement of the network throughput. The relation between the constants that are multiplying to the scaling throughput of PNC and DNC needs to be further investigated.

V. CONCLUSION

In this paper, we have shown that in an n-node random WAHN, the throughput order improvement with PNC over former routing and DNC schemes is $\Theta(1)$ for multi-source multicasts when the number of destinations of each session is m. This result breaks the naive intuition that PNC can improve the throughput order of random WAHNs. Nevertheless, the derivation of the throughput upper bound with PNC in this paper also shows that, PNC may provide an extra constant improvement over the former routing or network coding techniques because it embraces the wireless interference.

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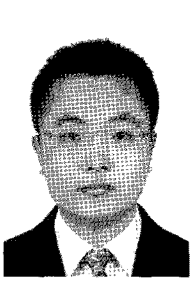

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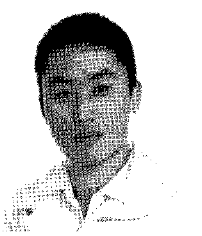

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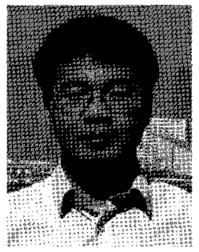

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