Signal-to-Noise Ratio Formulas of a Scalar Gaussian Quantizer Mismatched to a Laplacian Source

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ABSTRACT

The paper derives formulas for the mean-squared error distortion and resulting signal-to-noise (SNR) ratio of a fixed-rate scalar quantizer designed optimally in the minimum mean-squared error sense for a Gaussian density with the standard deviation σ_q when it is mismatched to a Laplacian density with the standard deviation σ_p. The SNR formulas, based on the key parameter and Bennett's integral, are found accurate for a wide range of $\rho\left(=\frac{\sigma_p}{\sigma_q}\right) \geq 0.25$. Also an upper bound to the SNR is derived, which becomes tighter with increasing rate R and indicates that the SNR behaves asymptotically as $\frac{20\sqrt{3\ln 2}}{\rho \ln 10}\sqrt{R}$ dB.

Key Words : Gaussian quantizer, Laplacian source, the mean-squared error distortion, shape mismatch, SNR formulas

I. Introduction

Consider an N-point fixed-rate scalar quantizer Q_N that is designed optimally in the minimum mean-squared error (MSE) sense for a probability density function $q(x)$ but is applied to a source with another density function $p(x)$, where

$$q(x) = \frac{1}{\sigma_q\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_q^2}} \text{ and } p(x) = \frac{1}{\sigma_p\sqrt{2}} e^{-\frac{\sqrt{2}|x|}{\sigma_p}}. \quad (1)$$

These density functions are plotted in Fig. 1 in the case of $\sigma_q = 1$ and $\sigma_p = 2$. In this "shape/variance-mismatched" quantization we are interested in finding formulas for the MSE distortion

$$D(Q_N) = \int_{-\infty}^{\infty} (x - Q_N(x))^2 p(x)dx \quad (2)$$

and the resulting signal-to-noise ratio $\text{SNR} = 10\log_{10}\frac{\sigma_q^2}{D(Q_N)}$ dB. The standard deviation ratio $\rho \equiv \sigma_p/\sigma_q$ is a measure of variance mismatch.

Certainly, the best possible SNR is attainable when $p(x) = q(x)$. In this matched case, we get the minimum distortion $D(Q_N)$ from [1] as

$$D(Q_N) = \frac{1}{12N^2}\left(\int_{-\infty}^{\infty} p^{\frac{1}{3}}(x)dx\right)^3.$$

In the case of optimal Q_N for a zero-mean Gaussian density with variance σ_p^2, $D(Q_N) = \frac{\sqrt{3}\pi\sigma_p^2}{2N^2}$ and therefore the $\text{SNR}(Q_N)$ reduces to approximately $6.02R - 4.35$, where $R = \log_2 N$.

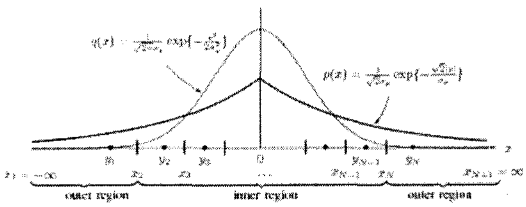

Fig. 1. A Gaussian quantizer mismatched to a Laplacian source: $\sigma_q = 1$ and $\sigma_p = 2$.

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Since, throughout the paper, Q_N is optimal for zero-mean Gaussian $q(x)$, its thresholds $x_1,\dots,x_{N+1}$ and quantization points $y_1,\dots,y_N$, depicted in Fig. 1, are symmetric, i.e., for $i=1,\dots,N$, $x_i=x_{N+2-i}$ and $y_i=-y_{N+1-i}$. The region $[x_2,x_N]$ will be called the *inner* region and the distortion from it the inner distortion, denoted D_i. Similarly the union $(x_1,x_2)\cup(x_N,x_{N+1})$ will be called the *outer* region and the distortion from it the outer distortion, denoted D_o. The region-defining threshold x_N (of optimal Q_N) is called the *key parameter.*

The main result of the paper is a set of formulas for the MSE distortion and the resulting SNR, namely SNR_1 and SNR_2. They are derived using the methodology developed in [2], a formula from [3] and Bennett's integral [4] for approximating the key parameter and the inner distortion, respectively. The inner distortion is further approximated using Lether and Wenston's approach [5] to Dawson's integral that results from evaluating Bennett's integral. Similarly the outer distortion is approximated using Börjesson and Sundberg's approximation [6] to the Q function. Numerical results show that, for a wide range of $\rho\approx 0.25\sim 4$, these formulas predict SNRs approximately within 1% from the true values for bit rate $R\gtrsim 5$. Also found is that an upper bound SNR^U to the SNR derived using only the outer distortion is tight (approximately within 1% of error) for $\rho\gtrsim 0.6$ and $R\gtrsim 8.7/\rho$. The significance of the paper is in the derivation of the closed-form expression for the SNR due to "shape / variance- mismatched" quantization. For example, the comparison of SNR^U with the above optimum from [1] shows the loss of 15.4 and 47.6 dB in the case of $R=8$ and 16 when $\rho=1$, and 29.6 and 70.0 dB when $\rho=2$.

The upper bound SNR^U simplifies to $\frac{20\sqrt{3\ln 2}}{\rho\ln 10}\sqrt{R}=\frac{12.53}{\rho}\sqrt{R}$ dB for large R, which reveals rather an interesting relationship that, upon accepting its accuracy, the SNR is eventually proportional to the square-root of R and inversely proportional to ρ. To the best of the authors' knowledge the results presented herein have not been previously reported in the literature.

The rest of the paper is organized as follows. Section II explains the methodology and derives the principal formulas for the distortion. Section III presents numerical results, assesses the accuracy of the principal formulas, and discusses their implications. Finally Section IV summarizes and concludes.

Ⅱ. Approximation Formulas for Distortion

To derive an approximation formula for $D(Q_N)$, we have taken the following approach [2]: (a) use an approximation formula for the key parameter x_N; (b) approximate the inner distortion D_i by Bennett's integral and the outer distortion D_o by a formula derived herein; and (c) add these approximations for the total distortion. The details follow.

2.1 Approximation Formula for the Key Parameter x_N

The following formula for x_N from [3, Eq. (17)]

$$x_N\approx\sigma_q\sqrt{6\ln N}\left(\begin{array}{l}1-\frac{\ln\ln N}{4\ln N}-\frac{\ln(9\pi)}{4\ln N}-\frac{1}{32}\left(\frac{\ln\ln N}{\ln N}\right)^2\\+\frac{2-\ln(9\pi)}{16}\frac{\ln\ln N}{(\ln N)^2}\end{array}\right)\tag{3}$$

yields values within 1% from the true x_N for $N\geqq 8$.

2.2 Approximation Formula for the Inner Distortion

The inner distortion D_i is often approximated by Bennett's integral[4]:

$$D_i=\int_{x_2}^{x_N}(x-Q_N(x))^2p(x)\,dx\approx\frac{1}{12(N-2)^2}\int_{x_2}^{x_N}\frac{p(x)}{\lambda^2(x)}dx$$

where $\lambda(x)$ being the limiting optimal point density of Q_N, is given by $\lambda(x)=\frac{q^{1/3}(x)}{\int_{-\infty}^{\infty}q^{1/3}(x)dx}$. Using the even symmetry of $p(x)$, $q(x)$ and Q_N, and noting that $(N-2)\approx N$ for large N lead to the approximation $D_i\approx\widetilde{D}_i$ for large N, where

$$\widetilde{D}_i\equiv\frac{1}{6N^2}\int_0^{x_N}\frac{p(x)}{\lambda^2(x)}dx=\frac{\pi\sigma_q}{\sqrt{2}\rho N^2}\int_0^{x_N}e^{\frac{x^2}{3\sigma_q^2}-\frac{\sqrt{2}x}{\sigma_p}}dx$$

which, by completing the square and changing variables, can be rewritten as

$$\widetilde{D}_i = \frac{\sqrt{3}\pi\sigma_q^2}{\sqrt{2}\rho}\frac{e^{-\frac{3}{2\rho^2}}}{N^2}\int_{-\frac{\sqrt{3}}{\sqrt{2}}}^{\frac{x_N}{\sqrt{3}\sigma_q}-\frac{\sqrt{3}}{\sqrt{2}\rho}} e^{t^2}dt$$

In terms of Dawson's integral $F(u) \equiv e^{u^2}\int_0^u e^{t^2}dt$

$$\widetilde{D}_i = \frac{\sqrt{3}\pi\sigma_q^2 e^{-\frac{3}{2\rho^2}}}{\sqrt{2}\rho N^2}\left\{e^{\frac{3}{2\rho^2}}F\left(\frac{\sqrt{3}}{\sqrt{2}\rho}\right)+ e^{\left(\frac{x_N}{\sqrt{3}\sigma_q}-\frac{\sqrt{3}}{\sqrt{2}\rho}\right)^2}F\left(\frac{x_N}{\sqrt{3}\sigma_q}-\frac{\sqrt{3}}{\sqrt{2}\rho}\right)\right\}. \quad (4)$$

2.3 Approximation Formula for the Outer Distortion

The outer distortion $D_o = 2\int_{x_N}^{\infty}(x-y_N)^2 p(x)dx$ (also denoted $\widetilde{D}_o$) can be shown in a straightforward manner, with the substitution for $p(x)$ and integration by parts, to be

$$\widetilde{D}_o = \sigma_p^2 e^{-\frac{\sqrt{2}}{\sigma_p}x_N}\left(1-\sqrt{2}\left(\frac{y_N-x_N}{\sigma_p}\right)+\left(\frac{y_N-x_N}{\sigma_p}\right)^2\right). \quad (5)$$

Since the quantization point y_N, being optimal, is the centroid of $[x_N, \infty)$ with respect to $q(x)$, it is related to x_N through

$$y_N = \frac{\int_{x_N}^{\infty} xq(x)dx}{\int_{x_N}^{\infty} q(x)dx} = \frac{\frac{\sigma_q}{\sqrt{2\pi}}e^{-\frac{x_N^2}{2\sigma_q^2}}}{Q\left(\frac{x_N}{\sigma_q}\right)}, \quad (6)$$

where $Q(u) = \frac{\int_u^{\infty} e^{-\frac{t^2}{2}}dt}{\sqrt{2\pi}}$ is the Q function.

2.4 Formulas for the Signal-to-Noise Ratio

The SNR of Q_N, will be approximated by the following SNR_1:

$$\mathrm{SNR}_1 \equiv 10\log_{10}\frac{\sigma_p^2}{\widetilde{D}}, \quad (7)$$

where $\widetilde{D} = \widetilde{D}_i + \widetilde{D}_o$, the sum of (4) and (5). One can see that, upon inspecting (4), (5) and (6), the key parameter x_N is the only Q_N-related quantity needed to compute SNR_1 and, when it is evaluated using (3), SNR_1 is completely determined. The two special functions, Dawson's integral and the Q function, need to be evaluated in the process.

Another formula proposed is SNR_2 (formally defined in (13) below) obtained from (7) substituting Lether and Wenston's approximation for Dawson's integral and Börjesson and Sundberg's approximation for the Q function. Lether and Wenston [5] have proposed the following approximation to Dawson's integral:

$$F(u) \approx \frac{1-e^{-\alpha u^2}+(2-\alpha)u^2 e^{-\beta u^2}}{2u}, \quad (8)$$

where $\alpha = 173/107 \approx 1.6168$ and $\beta = 237/514 \approx 0.4611$. The maximum error of the approximation is 3.08% from the true value over all real u. With (8) in (4)

$$\widetilde{D}_i \approx \widehat{D}_i \equiv \frac{\sqrt{3}\pi\sigma_q^2 e^{-\frac{3}{2\rho^2}}}{\sqrt{2}\rho N^2}\left(e^{\frac{3}{2\rho^2}}\frac{1-e^{-\frac{3\alpha}{2\rho^2}}+(2-\alpha)\frac{3}{2\rho^2}e^{-\frac{3\beta}{2\rho^2}}}{2\frac{\sqrt{3}}{\sqrt{2}\rho}} + e^{\left(\frac{x_N}{\sqrt{3}\sigma_q}-\frac{\sqrt{3}}{\sqrt{2}\rho}\right)^2}\frac{1-e^{-\alpha\left(\frac{x_N}{\sqrt{3}\sigma_q}-\frac{\sqrt{3}}{\sqrt{2}\rho}\right)^2}+(2-\alpha)\left(\frac{x_N}{\sqrt{3}\sigma_q}-\frac{\sqrt{3}}{\sqrt{2}\rho}\right)^2 e^{-\beta\left(\frac{x_N}{\sqrt{3}\sigma_q}-\frac{\sqrt{3}}{\sqrt{2}\rho}\right)^2}}{2\left(\frac{x_N}{\sqrt{3}\sigma_q}-\frac{\sqrt{3}}{\sqrt{2}\rho}\right)}\right). \quad (9)$$

Börjesson and Sundberg's approximation [6] to the Q function

$$Q(u) \approx \frac{e^{-\frac{u^2}{2}}}{\left(\left(1-\frac{1}{\pi}\right)u+\frac{u}{\pi}\sqrt{1+\frac{2\pi}{u^2}}\right)\sqrt{2\pi}} \quad (10)$$

is accurate for $u > 0$ with the maximum error of 1.17% at $u \approx 0.591$. With (6) and (10)

$$\frac{y_N - x_N}{\sigma_p} \approx \frac{2}{\rho\frac{x_N}{\sigma_q}\left(1+\sqrt{1+\frac{2\pi\sigma_q^2}{x_N^2}}\right)}. \quad (11)$$

Then combining (11) with (5) yields

$$\widetilde{D}_o \approx \widehat{D}_o \equiv \rho^2\sigma_q^2 e^{-\frac{\sqrt{2}}{\rho\sigma_q}x_N}\left\{1-\frac{2\sqrt{2}}{\rho\frac{x_N}{\sigma_q}\left(1+\sqrt{1+\frac{2\pi\sigma_q^2}{x_N^2}}\right)}+\left(\frac{2}{\rho\frac{x_N}{\sigma_q}\left(1+\sqrt{1+\frac{2\pi\sigma_q^2}{x_N^2}}\right)}\right)^2\right\}. \tag{12}$$

Therefore, SNR_2 is now formally defined as

$$\mathrm{SNR}_2 \equiv 10\log_{10}\frac{\sigma_p^2}{\widehat{D}}, \tag{13}$$

where $D \equiv D_i + D_o$, the sum of (9) and (12). Compared with SNR_1, the convenience of SNR_2 consists in the fact that the evaluation of the special functions is eliminated and the mechanism by which ρ and x_N (hence N) affect the distortion is shown more clearly.

An upper bound to SNR is obtained noting that the total distortion $D(Q_N)$ is always greater than the outer distortion D_o and therefore $\mathrm{SNR}^U \equiv 10\log_{10}\frac{\sigma_p^2}{D_o}$ is an upperbound to the SNR. The following approximation for SNR^U follows from using the approximation (12) for D_o and keeping the three most significant terms in (3) for x_N with $N=2^R$ and also inside the logarithm term in the resulting expression:

$$\mathrm{SNR}^U \approx \frac{20\sqrt{3\ln 2}}{\rho\ln 10}\sqrt{R}\left(1-\frac{\ln R}{4R\ln 2}-\frac{\ln(9\pi\ln 2)}{4R\ln 2}\right) - 10\log_{10}\left(1-\frac{1}{\rho\sqrt{3R\ln 2}}+\frac{1}{6R\rho^2\ln 2}\right). \tag{14}$$

Ⅲ. Numerical Results

In this section the accuracy of the derived formulas are evaluated. Toward this goal, optimal quantizers for $q(x)$ with $\sigma_q=1$ are designed for the bit rate R ranging from 1 to 16, using the Lloyd-Max algorithm [7, 8]. The algorithm stops when the candidate quantization point $x_N+(x_N-y_{N_1})$ is within 10^{-9} of the best quantization point of $[x_N,\infty)$, i.e., $\frac{\int_{x_N}^{\infty} xq(x)dx}{\int_{x_N}^{\infty} q(x)dx}$.

Table 1. Optimal Gaussian quantizers: the key parameter x_N and SNRs when mismatched to Laplacian sources

R	x_N ($N=2^R$)	SNR						
		$\rho=0.25$	$\rho=0.5$	$\rho=0.75$	$\rho=1$	$\rho=1.5$	$\rho=2$	$\rho=4$
1	0	-8.24	-1.10	2.03	2.94	2.75	2.26	1.21
2	0.981598821	-2.29	4.52	7.05	7.18	5.70	4.47	2.30
3	1.747927491	4.03	10.24	12.20	11.39	8.46	6.49	3.29
4	2.400803398	10.19	15.99	17.40	15.40	10.97	8.32	4.19
5	2.975926035	16.18	21.82	22.54	19.10	13.22	9.97	5.00
6	3.492269162	22.14	27.72	27.47	22.42	15.25	11.47	5.75
7	3.962315400	28.11	33.66	32.06	25.38	17.11	12.85	6.43
8	4.395065527	34.10	39.62	36.23	28.06	18.82	14.13	7.07
9	4.797249491	40.10	45.58	39.96	30.52	20.41	15.32	7.66
10	5.173991021	46.12	51.53	43.30	32.80	21.91	16.44	8.22
11	5.529245868	52.13	57.45	46.34	34.95	23.33	17.50	8.75
12	5.866110888	58.15	63.31	49.15	36.98	24.67	18.51	9.26
13	6.187035222	64.17	69.07	51.78	38.91	25.96	19.47	9.74
14	6.493992892	70.19	74.67	54.28	40.76	27.19	20.39	10.20
15	6.788802792	76.21	80.02	56.67	42.54	28.37	21.28	10.64
16	7.070054370	82.23	84.99	58.94	44.23	29.50	22.13	11.06

3.1 Accuracy of the Formulas

Table 1 lists the key parameters x_N and the SNRs of optimal Q_N mismatched to Laplacian densities for various values of ρ. (They are also plotted as a shaded smooth surface in Fig. 2.) These true SNRs are evaluated numerically using the designed optimal quantizers. A general trend in the studied rate range of 1 to 16 is that at a fixed R, as ρ increases from 0.25, the SNR increases before it eventually decreases. This phenomenon reflects the fact that for a fixed N, a small ρ results in "idling" of a large portion of Q_N so that an increase in ρ causes a wider portion of Q_N to be active, effecting a reduced distortion, and a further increase in ρ results in heavy tail probability that contributes to a large outer distortion and hence a larger total distortion. Another observation is that, for a given ρ, the rate of increase in the SNR slows down with R, which is especially noticeable in the case of $\rho \geqq 0.75$.

The formulas SNR_1 (7) and SNR_2 (13) are evaluated using (3) for x_N to assess their accuracy and SNR^U (14) is also evaluated. The relative errors of SNR_1 and SNR_2 from the true SNRs are approximately 1% or less for $R \geqq 5$, 0.5% or less for $R \geqq 6$, and 0.28% or less for $R \geqq 9$. Fig. 3 shows the plots of the true SNR, SNR_2 (the values for SNR_1 and SNR_2 are so close that SNR_1 is omitted in Fig. 3 to avoid clutter), and SNR^U. It is noted that the plots of the true SNR and SNR_2 are so close that they virtually overlap, making them indistinguishable except for $\rho \leqq 0.75$ and $R \leqq 5$. The plots of SNR^U in dashed lines are also indistinguishable from those of the true SNR and SNR_2 in the case of $\rho \leqq 1.5$ and $R \geqq 3$ and are noticeable only for $\rho \leqq 1$, which shows that SNR^U is tight for large $\rho \leqq 1.5$ even at low R. The tightness of SNR^U indicates that for large ρ the total distortion is dominated by the outer distortion. For $\rho = 0.6 \sim 1.5$ a rough numerical fitting shows that SNR^U is approximately at 2% or less above the true values if $R \gtrsim 6.81/\rho^{1.25}$ and 1% or less if $R \gtrsim 8.7/\rho$. At small values of $\rho \leqq 0.5$ the accuracy of SNR^U as an approximation of the SNR suffers greatly from the ignored inner distortion in the studied bit range, as evidenced in the case of $\rho = 0.5$ in Fig. 3, and it is necessary to use SNR_1 or SNR_2 for improved accuracy.

Based on these observations it is concluded that the formulas for SNR_1 and SNR_2 are accurate overall and that SNR^U can be useful when accompanied with a proper condition.

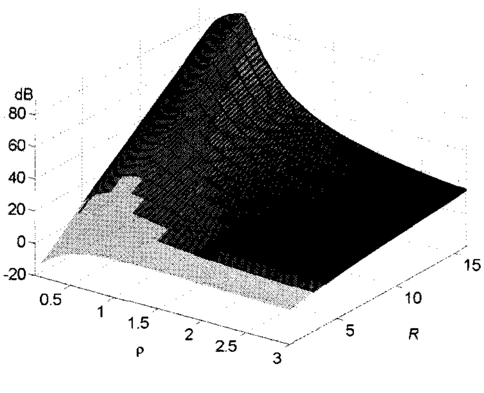

Fig. 2. The SNR(shaded surface), SNR_2(thin mesh lines), and SNR^U(thick mesh lines) with respect to ρ and R.

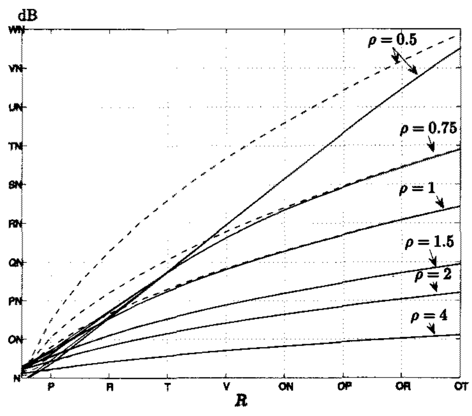

Fig. 3. The SNR(—), SNR_2(—), and SNR^U(---)

3.2 Further Discussion

The overall profiles of the SNR, SNR_2 and SNR^U are given in Fig. 2 with ρ ranging from 0.25 to 3 in the interval of 0.025 and R ranging from 1 to 16. The smooth shaded surface represents the SNR, whereas the thin and thick mesh lines respectively represent SNR_2 and SNR^U that are within 1% from the SNR. The 1% error region of SNR_2 includes that of SNR^U. As discussed in 3.1, these regions are roughly specified by $R \gtrsim 5$ and $\rho \geq 0.25$ for SNR_2 and $R \gtrsim 8.7/\rho$ for SNR^U.

The SNR surface in Fig. 2 can be divided into the small-, medium-, and large-ρ regions corresponding to $\rho \lesssim 0.6$, $0.3 \lesssim \rho \lesssim 1.5$, and $\rho \gtrsim 1.5$, respectively. In the small-ρ region the SNR appears to increase almost linearly with R in the studied bit range. In the medium-ρ region the SNR seems to start with a linear growth but end up with a sublinear growth that is suggestive of a square-root law as in the case of the large-ρ region. In the large-ρ region the SNR increases slowly and levels off, which can be explained inspecting SNR^U in (14). It shows that SNR^U is asymptotically (large R) proportional to the square root of R and inversely proportional to ρ, as it simplifies to $\frac{12.53}{\rho}\sqrt{R}$ dB. (However, this formula turns out rather loose in the studied bit range.) Therefore, it is not very surprising to observe the inverse proportionality of the (true) SNR to ρ for $R \gtrsim 8 \sim 9$, e.g., from the last row of Table 1 one gets $58.94 : 44.23 : 22.13 : 11.06 \sim \frac{1}{0.75} : \frac{1}{1} : \frac{1}{2} : \frac{1}{4}$ for $R = 16$.

Ⅳ. Summary and Conclusion

The paper has derived approximation formulas for the MSE distortion and the SNR of optimal Gaussian quantizers mismatched to a Laplacian source. The derivation uses a formula for the key parameter, Bennett's integral for the inner distortion, Lether-Wenston's approximation for Dawson's integral and Börjesson-Sundberg's approximation for the Q function. The derived formulas for the SNR are accurate overall. An upper bound to the SNR is also derived using the outer distortion only and found to be useful, e.g., in the case of $0.6 \lesssim \rho \lesssim 1.5$ if $R \gtrsim 8.7/\rho$ for the relative error of 1% or less, as well as the case of $\rho \geq 1.5$ if $R \geq 3$. This upper bound reduces asymptotically to $\frac{12.53}{\rho}\sqrt{R}$ dB, whose discovery appears to be rather interesting in that it shows that the SNR is eventually proportional to the square root of the rate and inversely proportional to the deviation ratio. This paper can be useful when one wants to evaluate and predict the SNR of a Gaussian quantizer applied to Laplacian distribution.

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