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Noninformative Priors for the Stress-Strength Reliability in the Generalized Exponential Distributions

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Abstract

This paper develops the noninformative priors for the stress-strength reliability from one parameter generalized exponential distributions. When this reliability is a parameter of interest, we develop the first, second order matching priors, reference priors in its order of importance in parameters and Jeffreys' prior. We reveal that these probability matching priors are not the alternative coverage probability matching prior or a highest posterior density matching prior, a cumulative distribution function matching prior. In addition, we reveal that the one-at-a-time reference prior and Jeffreys' prior are actually a second order matching prior. We show that the proposed reference prior matches the target coverage probabilities in a frequentist sense through a simulation study and a provided example.

Keywords: Generalized exponential model, matching prior, reference prior, stress-strength reliability.

1. Introduction

The one parameter generalized exponential distribution was introduced by Gupta and Kundu (1999) as an alternative to the gamma or Weibull distributions for analyzing lifetime data (Gupta and Kundu, 2001). An advantage of employing the generalized exponential distribution is that the distribution function can be obtained in a closed form. Kundu and Gupta (2007) showed that the generalized exponential distribution is quite flexible and can be used very effectively in analyzing positive lifetime data in place of the gamma or Weibull models. Raqab and Madi (2005) studied the Bayeian inference for the parameters and reliability function.

Consider X and Y have independent one parameter generalized exponential distributions with shape parameters λ_1 and λ_2, respectively. Then the probability density functions of generalized exponential distributions of X and Y are given by

$$f(x|\lambda_1) = \lambda_1 e^{-x}(1 - e^{-x})^{\lambda_1 - 1}, \quad x > 0,\ \lambda_1 > 0, \tag{1.1}$$

and

$$f(y|\lambda_2) = \lambda_2 e^{-y}(1 - e^{-y})^{\lambda_2 - 1}, \quad y > 0,\ \lambda_2 > 0, \tag{1.2}$$

respectively. The reliability $R = P(Y < X)$ is given by $\lambda_1/(\lambda_1 + \lambda_2)$. The problem of making inference about R has received a considerable attention in literature. An item is able to perform its intended

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function if its strength is greater than the stress imposed upon it. The probability that an item is strong enough to overcome the stress is the measure of confidence of the item and to make statistical inference about this probability is very important.

The problem of estimating the stress-strength reliability was considered by Kundu and Gupta (2005). Kundu and Gupta (2005) derived the maximum likelihood estimator, uniformly minimum variance unbiased estimator and Bayes estimator based on gamma priors, and showed that their performance are quite similar in nature, and the maximum likelihood estimators are marginally better than the rest. Baklizi (2008) and Wong and Wu (2009) studied interval estimation of the stress-strength reliability based on record data. In their results, the parametric bootstrap percentile method (Baklizi , 2008) performs well only in the cases of equal sample sizes. Following Wong and Wu (2009), the modified signed log-likelihood ratio statistic method gives excellent results even for the equal and unequal small sample sizes.

This paper focuses on noninformative priors for the reliability R. Subjective priors are ideal when sufficient information from past experience, expert opinion or previously collected data exist. However, often even without adequate prior information, one can use Bayesian techniques efficiently with some noninformative or default priors.

The notion of a noninformative prior has attracted much attention in recent years. There are different notions of noninformative prior. One is a probability matching prior introduced by Welch and Peers (1963) which matches the posterior and frequentist probabilities of confidence intervals. Interest in such priors revived with the work of Stein (1985) and Tibshirani (1989). Among others, we may cite the work of Mukerjee and Dey (1993), DiCiccio and Stern (1994), Datt and Ghosh (1995a, 1995b, 1996), Mukerjee and Ghosh (1997), Kim *et al.* (2009), Kang *et al.* (2011). The other is the reference prior introduced by Bernardo (1979) which maximizes the Kullback-Leibler divergence between the prior and the posterior. Ghosh and Mukerjee (1992) and Berger and Bernardo (1989, 1992) give a general algorithm to derive a reference prior by splitting the parameters into several groups according to their order of inferential importance. This approach is very successful in various practical problems. Quite often, reference priors satisfy the matching criterion described earlier.

The outline of the remaining sections is as follows. In Section 2, we develop the first order and second order probability matching priors for R. We reveal that the second order matching prior is not a highest posterior density (HPD) matching prior or a cumulative distribution function (CDF) matching prior, and does not match the alternative coverage probabilities up to the second order. We also derive the reference priors for the parameter of interest. It turns out that the one-at-a-time reference prior and Jeffreys' prior are the second order matching prior. We provide the propriety of the posterior distribution for the general prior including the reference and matching priors. Section 4 shows and includes an example of the simulated frequentist coverage probabilities under the proposed prior.

2. The Noninformative Priors

2.1. The probability matching priors

For a prior π, let $\theta_1^{1-\alpha}(\pi; \mathbf{X})$ denote the $(1-\alpha)^{th}$ posterior quantile of θ_1, that is,

$$P^{\pi}\left[\theta_1 \leq \theta_1^{1-\alpha}(\pi; \mathbf{X}) | \mathbf{X}\right] = 1 - \alpha, \tag{2.1}$$

where $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_t)^T$ and θ_1 is the parameter of interest. We want to find priors π for which

$$P\left[\theta_1 \leq \theta_1^{1-\alpha}(\pi; \mathbf{X}) | \boldsymbol{\theta}\right] = 1 - \alpha + o\left(n^{-r}\right) \tag{2.2}$$

for some $r > 0$, as n goes to infinity. Priors π satisfying (2.2) are called matching priors. If $r = 1/2$, then π is referred to as a first order matching prior, while if $r = 1$, π is referred to as a second order matching prior.

In order to find such matching priors π, let

$$\theta_1 = \frac{\lambda_1}{\lambda_1 + \lambda_2} \quad \text{and} \quad \theta_2 = \lambda_1^{n_1}\lambda_2^{n_2}.$$

With this parametrization, the likelihood function of parameters (θ_1, θ_2) for the models (1.1) and (1.2) is given by

$$L(\theta_1,\theta_2) \propto \theta_2 \left[\prod_{i=1}^{n_1}(1-e^{-x_i})^{\theta_1^{\frac{n_2}{n_1+n_2}}(1-\theta_1)^{\frac{-n_2}{n_1+n_2}}\theta_2^{\frac{1}{n_1+n_2}}-1}\right]\left[\prod_{i=1}^{n_2}(1-e^{-y_i})^{\theta_1^{\frac{-n_1}{n_1+n_2}}(1-\theta_1)^{\frac{n_1}{n_1+n_2}}\theta_2^{\frac{1}{n_1+n_2}}-1}\right]. \tag{2.3}$$

Based on (2.3), the Fisher information matrix is given by

$$\mathbf{I}(\theta_1,\theta_2) = \begin{pmatrix} \frac{n_1 n_2}{n_1+n_2}\theta_1^{-2}(1-\theta_1)^{-2} & 0 \\ 0 & \frac{1}{n_1+n_2}\theta_2^{-2} \end{pmatrix}. \tag{2.4}$$

From the above Fisher information matrix $\mathbf{I}$, θ_1 is orthogonal to θ_2 in the sense of Cox and Reid (1987). Following Tibshirani (1989), the class of first order probability matching prior is characterized by

$$\pi_m^{(1)}(\theta_1,\theta_2) \propto \theta_1^{-1}(1-\theta_1)^{-1}d(\theta_2), \tag{2.5}$$

where $d(\theta_2) > 0$ is an arbitrary function differentiable in its argument.

The class of prior given in (2.5) can be narrowed down to the second order probability matching priors as given in Mukerjee and Ghosh (1997). A second order probability matching prior is of the form (2.5), and also d must satisfy an additional differential equation (2.10) of Mukerjee and Ghosh (1997), namely

$$\frac{1}{6}d(\theta_2)\frac{\partial}{\partial\theta_1}\left\{I_{11}^{-\frac{3}{2}}L_{1,1,1}\right\} + \frac{\partial}{\partial\theta_2}\left\{I_{11}^{-\frac{1}{2}}L_{112}I^{22}d(\theta_2)\right\} = 0, \tag{2.6}$$

where

$$\begin{aligned} L_{1,1,1} &= E\left[\left(\frac{\partial \log L}{\partial\theta_1}\right)^3\right] = -\frac{2n_1n_2(n_2-n_1)}{(n_1+n_2)^2}\theta_1^{-3}(1-\theta_1)^{-3}, \\ L_{112} &= E\left[\frac{\partial^3 \log L}{\partial\theta_1^2\partial\theta_2}\right] = -\frac{n_1n_2}{(n_1+n_2)^2}\theta_1^{-2}(1-\theta_1)^{-2}\theta_2^{-1}, \\ I_{11} &= \frac{n_1n_2}{n_1+n_2}\theta_1^{-2}(1-\theta_1)^{-2}, \qquad I^{22} = (n_1+n_2)\theta_2^2. \end{aligned}$$

Then (2.6) simplifies to

$$\frac{\partial}{\partial\theta_2}\left\{-\frac{(n_1n_2)^{\frac{1}{2}}}{(n_1+n_2)^{\frac{1}{2}}}\theta_1(1-\theta_1)^{-1}\theta_2 d(\theta_2)\right\} = 0. \tag{2.7}$$

Therefore, the set of solution of (2.7) is of the form $d(\theta_2) = \theta_2^{-1}$. Thus the resulting second order probability matching prior is

$$\pi_m^{(2)}(\theta_1, \theta_2) \propto \theta_1^{-1}(1-\theta_1)^{-1}\theta_2^{-1}. \tag{2.8}$$

There are alternative ways through which matching can be accomplished. Datta *et al.* (2000) provided a theorem which establishes the equivalence of second order matching priors and HPD matching priors (DiCiccio and Stern, 1994; Ghosh and Mukerjee, 1995) within the class of first order matching priors. The equivalence condition is that $I_{11}^{-3/2}L_{111}$ does not depend on θ_1. Since

$$L_{111} = E\left[\frac{\partial^3 \log L}{\partial \theta_1^3}\right] = -\frac{2n_1n_2}{(n_1+n_2)^2}\frac{(n_1+n_2)(3\theta_1-1)-n_1}{\theta_1^3(1-\theta_1)^3}.$$

From the above equation, one can easily verify that $I_{11}^{-3/2}L_{111}$ depends on θ_1. Therefore the second order probability matching prior (2.8) is not a HPD matching prior. In addition, since

$$\frac{\partial}{\partial\theta_1}\left\{I_{11}^{-\frac{3}{2}}L_{111}\right\} \neq 0,$$

then the second order matching prior (2.8) does not match the alternative coverage probabilities (Mukerjee and Reid, 1999). Now,

$$\frac{\partial}{\partial\theta_1}\left\{\left(I^{11}\right)^2 L_{111}\pi_m^{(2)}\right\} \neq 0,$$

so the second order matching prior (2.8) is not a CDF matching prior (Mukerjee and Ghosh, 1997).

2.2. The reference priors

Reference priors introduced by Bernardo (1979), and extended further by Berger and Bernardo (1992) have become very popular over the years for the development of noninformative priors. From now on, we derive the reference priors for different groups of ordering of (θ_1, θ_2). Then due to the orthogonality of the parameters, following Datta and Ghosh (1995b), choosing rectangular compacts for each θ_1 and θ_2 when θ_1 is the parameter of interest, the reference priors are given as follows.

For the the stress-strength reliability model (2.3), if θ_1 is the parameter of interest, then the reference prior for group of ordering of $\{(\theta_1, \theta_2)\}$ is, which is also a Jeffreys' prior,

$$\pi_1(\theta_1, \theta_2) \ \propto \ \theta_1^{-1}(1-\theta_1)^{-1}\theta_2^{-1}.$$

For group of ordering of $\{\theta_1, \theta_2\}$, which means θ_1 is more important than θ_2, the reference prior, which is also called the one-at-a-time reference prior, is

$$\pi_2(\theta_1, \theta_2) \ \propto \ \theta_1^{-1}(1-\theta_1)^{-1}\theta_2^{-1}.$$

From the above reference priors, we know that the one-at-a-time reference prior π_2 and Jeffreys' prior π_1 are the second order matching prior, and all priors are the same. We will simply refer it as a reference prior.

3. Implementation of the Bayesian Procedure

We investigate the propriety of posteriors for a general class of priors which include the reference prior and the matching prior. We consider the class of priors

$$\pi(\theta_1, \theta_2) \propto \theta_1^{-a}(1-\theta_1)^{-a}\theta_2^{-b}, \tag{3.1}$$

where $a > 0$, $b > 0$ and $b \neq 2$. The following general theorem can be proved.

Theorem 1. *The posterior distribution of* (θ_1, θ_2) *under the prior* π *in (3.1) is proper if* $2n_1 - bn_1 - a + 1 > 0$, $2n_2 - bn_2 - a + 1 > 0$ *or* $2n_1 - bn_1 + a - 1 < 0$, $2n_2 - bn_2 + a - 1 < 0$ *when* $a \geq 1$, *and* $2n_1 - bn_1 + a - 1 > 0$, $2n_2 - bn_2 + a - 1 > 0$ *or* $2n_1 - bn_1 - a + 1 < 0$, $2n_2 - bn_2 - a + 1 < 0$ *when* $a < 1$.

Proof: Note that the joint posterior for θ_1 and θ_2 given $\mathbf{x}$ and $\mathbf{y}$ is

$$\begin{aligned}\pi(\theta_1, \theta_2|\mathbf{x},\mathbf{y}) \propto \theta_1^{-a}(1-\theta_1)^{-a}\theta_2^{-b+1}&\left[\prod_{i=1}^{n_1}(1-e^{-x_i})^{\theta_1^{\frac{n_2}{n_1+n_2}}(1-\theta_1)^{\frac{-n_2}{n_1+n_2}}\theta_2^{\frac{1}{n_1+n_2}}-1}\right]\\ \times&\left[\prod_{i=1}^{n_2}(1-e^{-y_i})^{\theta_1^{\frac{-n_1}{n_1+n_2}}(1-\theta_1)^{\frac{n_1}{n_1+n_2}}\theta_2^{\frac{1}{n_1+n_2}}-1}\right].\end{aligned} \tag{3.2}$$

Let $\theta_1 = \lambda_1/(\lambda_1 + \lambda_2)$ and $\theta_2 = \lambda_1^{n_1}\lambda_2^{n_2}$. Then we get

$$\pi(\lambda_1, \lambda_2|\mathbf{x},\mathbf{y}) \propto (\lambda_1+\lambda_2)^{2a-2}\lambda_1^{2n_1-bn_1-a}\lambda_2^{2n_2-bn_2-a}\prod_{i=1}^{n_1}(1-e^{-x_i})^{\lambda_1}\prod_{i=1}^{n_2}(1-e^{-y_i})^{\lambda_2}. \tag{3.3}$$

If $a \geq 1$, then

$$\begin{aligned}\pi(\lambda_1, \lambda_2|\mathbf{x},\mathbf{y}) &\leq c_1\lambda_1^{2n_1-bn_1+a-2}\lambda_2^{2n_2-bn_2-a}\prod_{i=1}^{n_1}(1-e^{-x_i})^{\lambda_1}\prod_{i=1}^{n_2}(1-e^{-y_i})^{\lambda_2}\\ &\quad + c_1\lambda_1^{2n_1-bn_1-a}\lambda_2^{2n_2-bn_2+a-2}\prod_{i=1}^{n_1}(1-e^{-x_i})^{\lambda_1}\prod_{i=1}^{n_2}(1-e^{-y_i})^{\lambda_2}\\ &\equiv \pi'(\lambda_1, \lambda_2|\mathbf{x},\mathbf{y}),\end{aligned} \tag{3.4}$$

where c_1 is a constant. Thus the function (3.4) is finite if $2n_1 - bn_1 + a - 1 > 0$, $2n_2 - bn_2 - a + 1 > 0$ or $2n_1 - bn_1 + a - 1 < 0$, $2n_2 - bn_2 - a + 1 < 0$, and $2n_1 - bn_1 - a + 1 > 0$, $2n_2 - bn_2 + a - 1 > 0$ or $2n_1 - bn_1 - a + 1 < 0$, $2n_2 - bn_2 + a - 1 < 0$. If $a < 1$, then

$$\begin{aligned}\pi(\lambda_1, \lambda_2|\mathbf{x},\mathbf{y}) &\leq c_2\lambda_1^{2n_1-bn_1-a}\lambda_2^{2n_2-bn_2+a-2}\prod_{i=1}^{n_1}(1-e^{-x_i})^{\lambda_1}\prod_{i=1}^{n_2}(1-e^{-y_i})^{\lambda_2}\\ &\equiv \pi''(\lambda_1, \lambda_2|\mathbf{x},\mathbf{y}),\end{aligned} \tag{3.5}$$

where c_2 is a constant. Thus the function (3.5) is finite if $2n_1 - bn_1 - a + 1 > 0$, $2n_2 - bn_2 + a - 1 > 0$ or $2n_1 - bn_1 - a + 1 < 0$, $2n_2 - bn_2 + a - 1 < 0$. This completes the proof. □

Remark 1. The conditions of propriety of posterior in Theorem 1 is not a rigid condition. For the general prior with $a = 1$ and $b = 1$, that is, the reference prior, the condition reduced to $n_1 \geq 1$ and

$n_2 \geq 1$. This means that the propriety is easily achieved when the sample size of each population is greater than or equal to 1.

Theorem 2. *Under the prior (3.1), the marginal posterior density of θ_1 is given by*

$$\pi(\theta_1|\mathbf{x},\mathbf{y}) \propto \theta_1^{-a}(1-\theta_1)^{-a}\left[\left(\frac{1-\theta_1}{\theta_1}\right)^{-\frac{n_2}{n_1+n_2}}\sum_{i=1}^{n_1}-\log(1-e^{-x_i}) + \left(\frac{1-\theta_1}{\theta_1}\right)^{\frac{n_1}{n_1+n_2}}\sum_{i=1}^{n_2}-\log(1-e^{-y_i})\right]^{-(n_1+n_2)(2-b)}. \tag{3.6}$$

Note that the marginal density of θ_1 required an one dimensional integration. Therefore we have the marginal posterior density of θ_1, and so it is easy to compute the marginal moment of θ_1.

4. Numerical Studies

We evaluate the frequentist coverage probability by investigating the credible interval of the marginal posterior density of θ_1 under the reference prior given in the previous section for several configurations of (λ_1, λ_2) and (n_1, n_2). That is to say, the frequentist coverage of a $(1-\alpha)^{th}$ posterior quantile should be close to $1-\alpha$. This is done numerically. Table 1 gives numerical values of the frequentist coverage probabilities of 0.05 (0.95) posterior quantiles for the our prior. The computation of these numerical values is based on the following algorithm for any fixed true (λ_1, λ_2) and any prespecified probability value α. Here α is 0.05 (0.95). Let $\theta_1^\pi(\alpha|\mathbf{X},\mathbf{Y})$ be the posterior α-quantile of θ_1 given $\mathbf{X}$ and $\mathbf{Y}$. That is,

$$F\left(\theta_1^\pi(\alpha|\mathbf{X},\mathbf{Y})\,|\mathbf{X},\mathbf{Y}\right) = \alpha,$$

where $F(\cdot\,|\mathbf{X},\mathbf{Y})$ is the marginal posterior distribution of θ_1. Then the frequentist coverage probability of this one sided credible interval of θ_1 is

$$P_{(\theta_1,\theta_2)}(\alpha;\theta_1) = P_{(\theta_1,\theta_2)}\left(0 < \theta_1 \leq \theta_1^\pi(\alpha|\mathbf{X},\mathbf{Y})\right). \tag{4.1}$$

The estimated $P_{(\theta_1,\theta_2)}(\alpha;\theta_1)$ when $\alpha = 0.05(0.95)$ is shown in Table 1. In particular, for fixed n and (λ_1, λ_2), we take 10,000 independent random samples of $\mathbf{X} = (X_1, \ldots, X_{n_1})$ and $\mathbf{Y} = (Y_1, \ldots, Y_{n_2})$ from the generalized exponential distributions, respectively.

In Table 1, we can observe that the reference prior meets the target coverage probabilities very well even for the small sample sizes. In addition, note that the results of table are not very sensitive to the change of the values of (θ_1, λ_1).

Example 1. This example taken from Kundu and Gupta (2005), and the data has been generated using $n_1 = n_2 = 30$, $\lambda_1 = 1.5$ and $\lambda_2 = 2.5$ with common scale parameter 0.5. Thus the stress-strength reliability is 0.625. The Y values are 2.58, 3.61, 0.96, 5.55, 6.31, 0.47, 2.30, 0.08, 0.88, 2.90, 2.13, 4.01, 2.01, 1.22, 2.51, 0.92, 1.06, 1.02, 0.66, 1.76 and the corresponding X values are 1.70, 2.11, 2.50, 3.77, 1.41, 3.67, 3.00, 2.59, 1.29, 1.86, 0.64, 0.93, 3.28, 2.69, 0.64, 5.17, 12.24, 1.91, 3.09, 3.21.

For this data, the maximum likelihood estimate(MLE) of θ_1 is 0.615 and the corresponding 95% asymptotic confidence interval of θ_1 is (0.460, 0.750) (Kundu and Gupta, 2005). Bayes estimate and the 95% credible interval based on the reference prior are 0.612 and (0.459, 0.749), respectively. The Bayes estimate based on the reference prior and the MLE give almost same results.

Table 1: Frequentist Coverage Probability of 0.05 (0.95) Posterior Quantiles of θ_1

θ_1	n_1	n_2	$\lambda_1 = 1.0$	$\lambda_2 = 10.0$	$\lambda_3 = 100.0$
0.1	3	3	0.053(0.954)	0.048(0.953)	0.055(0.950)
		5	0.052(0.952)	0.047(0.947)	0.048(0.953)
		10	0.046(0.950)	0.051(0.951)	0.053(0.952)
	5	3	0.048(0.945)	0.049(0.955)	0.053(0.951)
		5	0.049(0.951)	0.053(0.954)	0.052(0.952)
		10	0.052(0.948)	0.051(0.950)	0.044(0.946)
	10	3	0.048(0.948)	0.050(0.948)	0.049(0.954)
		5	0.045(0.951)	0.048(0.953)	0.045(0.951)
		10	0.050(0.945)	0.054(0.951)	0.050(0.948)
0.3	3	3	0.051(0.948)	0.045(0.948)	0.050(0.951)
		5	0.049(0.951)	0.052(0.950)	0.049(0.950)
		10	0.046(0.953)	0.049(0.948)	0.054(0.945)
	5	3	0.054(0.951)	0.049(0.949)	0.046(0.950)
		5	0.052(0.949)	0.053(0.952)	0.046(0.950)
		10	0.052(0.949)	0.050(0.955)	0.051(0.955)
	10	3	0.051(0.950)	0.049(0.949)	0.053(0.949)
		5	0.049(0.951)	0.048(0.951)	0.051(0.954)
		10	0.050(0.950)	0.054(0.950)	0.043(0.948)
0.5	3	3	0.050(0.948)	0.048(0.953)	0.053(0.947)
		5	0.052(0.954)	0.048(0.951)	0.049(0.950)
		10	0.046(0.950)	0.050(0.952)	0.048(0.949)
	5	3	0.052(0.951)	0.047(0.950)	0.053(0.953)
		5	0.047(0.950)	0.054(0.947)	0.055(0.949)
		10	0.048(0.947)	0.050(0.954)	0.050(0.944)
	10	3	0.053(0.949)	0.050(0.947)	0.050(0.947)
		5	0.049(0.949)	0.050(0.954)	0.052(0.956)
		10	0.052(0.951)	0.050(0.953)	0.047(0.947)
0.7	3	3	0.052(0.951)	0.050(0.948)	0.051(0.949)
		5	0.049(0.948)	0.047(0.950)	0.052(0.949)
		10	0.055(0.949)	0.046(0.949)	0.051(0.947)
	5	3	0.054(0.951)	0.050(0.951)	0.053(0.947)
		5	0.045(0.954)	0.049(0.953)	0.051(0.953)
		10	0.052(0.949)	0.051(0.947)	0.051(0.953)
	10	3	0.054(0.948)	0.052(0.949)	0.051(0.947)
		5	0.049(0.950)	0.050(0.948)	0.048(0.947)
		10	0.047(0.951)	0.051(0.949)	0.049(0.956)
0.9	3	3	0.050(0.952)	0.053(0.951)	0.049(0.952)
		5	0.048(0.947)	0.048(0.950)	0.051(0.950)
		10	0.052(0.955)	0.049(0.951)	0.051(0.948)
	5	3	0.052(0.949)	0.047(0.952)	0.051(0.951)
		5	0.054(0.950)	0.050(0.951)	0.051(0.953)
		10	0.047(0.947)	0.051(0.952)	0.052(0.950)
	10	3	0.052(0.949)	0.050(0.945)	0.053(0.947)
		5	0.052(0.945)	0.046(0.948)	0.049(0.951)
		10	0.048(0.953)	0.051(0.951)	0.049(0.948)

5. Concluding Remarks

In the paper, we have found noninformative priors for the stress-strength reliability in generalized exponential distributions. We revealed that the second order matching prior is not a HPD matching prior and is not a CDF matching prior, and also does not match the alternative coverage probabilities up to the second order. It turns out that the reference prior and Jeffreys' prior are the second order matching prior. As illustrated in our numerical study, the reference prior meets very well with the target coverage probabilities. We recommend the use of the reference prior for Bayesian inference of the stress-strength reliability in two independent generalized exponential distributions.

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