

# Analysis and Optimization of Cooperative Spectrum Sensing with Noisy Decision Transmission

**Quan Liu, Jun Gao, Yunwei Guo and Siyang Liu**

Department of Communication Engineering, Naval University of Engineering  
Wuhan, 430033 - China

[e-mail: liuquan.hjgc@gmail.com; {alex-hjgc, sims\_23, liusiyang514}@163.com]

\*Corresponding author: Quan Liu

*Received November 4, 2010; revised March 22, 2011; accepted March 23, 2011;  
published April 29, 2011*

---

## **Abstract**

Cooperative spectrum sensing (CSS) with decision fusion is considered as a key technology for tackling the challenges caused by fading/shadowing effects and noise uncertainty in spectrum sensing in cognitive radio. However, most existing solutions assume an error-free decision transmission, which is obviously not the case in realistic scenarios. This paper extends the general decision-fusion-based CSS scheme by considering the fading/shadowing effects and noise corruption in the common control channels. With this more practical model, the fusion centre first estimates the local decisions using a binary minimum error probability detector, and then combines them to get the final result. Theoretical analysis and simulation of this CSS scheme are performed over typical channels, which suggest some performance deterioration compared with the pure case that assumes an error-free decision transmission. Furthermore, the fusion strategy optimization in the proposed cooperation model is also investigated using the Bayesian criteria. The numerical results show that the total error rate of noisy CSS is higher than that of the pure case, and the optimal values of fusion parameter in the counting rule under both cases decrease as the local detection threshold increases.

---

**Keywords:** Cognitive radio, cooperative spectrum sensing, energy detection, decision fusion, minimum error probability criteria, Bayesian criteria

---

This research was supported by the National High-tech Research and Development Program of China [No.2009AAJ208, 2009AAJ116].

**DOI: 10.3837/tiis.2011.04.002**

## 1. Introduction

Cognitive radio (CR) is widely considered as the next big thing in wireless communication because of its promising paradigm for solving the increasingly severe spectrum scarcity problem [1][2]. In an opportunistic manner, secondary (unauthorized) users (SU) in a cognitive radio network (CRN) can dynamically exploit the precious white or grey spectrum holes [3], which are underutilized by the current primary (authorized) users (PU) in multiple dimensions of time, space, code, and so on [4]. Under a hierarchical dynamic access model [5], this kind of spectrum sharing is only allowed when interference is not introduced to the PUs, which entails periodic or continuous spectrum identification by SUs. Direct spectrum sensing has received more attention than the other candidates for spectrum identification, such as database or beacons, owing to the relatively low infrastructure cost and compatibility with the existing legacy primary systems [6].

As the fundamental issue of CRN, spectrum sensing generally means to detect quickly and reliably the presence of the primary signals [7]. Among various possibilities, energy detection (ED) [8][9] is the optimal sensing algorithm and has been widely applied thanks to its relatively low complexity and lack of requirements for prior knowledge of the network. However, ED performance is very susceptible to multipath fading/shadowing and noise uncertainty, which necessitates cooperative spectrum sensing (CSS) among different SUs in the MAC layer of CRN [10][11][12]. Recently, numerous ED-based CSS schemes that take advantage of spatial diversity in centralized or decentralized scenarios have been proposed in the literature. On the basis of the fusion methods in cooperation, the existing schemes can be categorized mainly into data fusion [13][14][15][16] and decision fusion [12][13][17][18][19]. With regard to the former, typical algorithms, such as maximal ratio combination [13], equal gain combination [13], deflection-criteria based combination [14], and optimal weigh-setting combination [15][16], have been investigated. With regard to the latter, CSS with OR-rule decision fusion has been demonstrated to improve the detection performance, relax the local sensitivity requirements, and increase the agility and efficiency of the secondary access [12]. Furthermore, the optimization of CSS with the *K out of N rule* (also called the counting rule) has been discussed [18][19]. Many researchers have also proposed some softened decision fusion schemes that require sharing more information on the reliability and accuracy of different SUs [13][20]. For example, two bits are used to represent each local decision, and then all the decisions are summed up with different weights at the base station [13]. More details about these two categories of fusion-based CSS schemes can be found in the literature [1]. Some other related studies on spectrum sensing include sensing scheduling [21][22], sensing security [23][24], relay-based CSS schemes [25][26][27], and consensus-based CSS schemes [24][28].

In employing decision fusion instead of data fusion, obvious performance degradation occurs because of the information loss during the combination. However, in practice, decision fusion may still be the better choice because of its significantly lower communication overhead and narrower bandwidth in the common control channels. These characteristics coincide well with the tenet of CRN: high spectrum efficiency.

Most CSS schemes with decision fusion in published papers assume an error-free decision transmission. However, such assumption may result in misleading performance interpretations because decision transmission is hardly perfect in realistic scenarios [29]. In this paper, we extend this pure cooperation model by acknowledging the fading/shadowing effects and noise

corruption in the common control channels, which are collectively called as noisy CSS. Using numerical and simulation results, we analyze the local performance of energy detection over several typical channels. Moreover, we extend the CSS scheme based on energy detection and the *K out of N rule* by considering the fading/shadowing/ correlated shadowing effects and noise corruption not only in the sensing channels but also in the control channels [12][17][29]. Using the proposed model, we verify if performance deterioration occurs by comparing the extended CSS scheme and the pure case with ideal decision transmission. Finally, we investigate further the optimal fusion strategy in the proposed scheme. We recognize that the optimal value of K in the counting rule depends primarily on the channel characteristics and the detection threshold in local sensing.

The rest of this paper is organized as follows. In Section 2, the system model that considers an imperfect decision transmission is developed. In Section 3, the performance of local sensing, pure CSS, and noisy CSS over different channels are analyzed and simulated. In Section 4, the optimization problem is investigated. In Section 5, the conclusions are drawn.

## 2. System Model

In the present study, the system setup is a CRN composed of  $N$  SUs and a fusion centre (FC), as shown in Fig. 1. We assume that each SU performs spectrum sensing simultaneously by detecting the primary signal in the sensing channel. Local binary decisions are sent through the control channels to the FC for combination. The primary signal and decision signals may be faded and noise corrupted during transmission, respectively, due to the imperfections of the communication mediums.

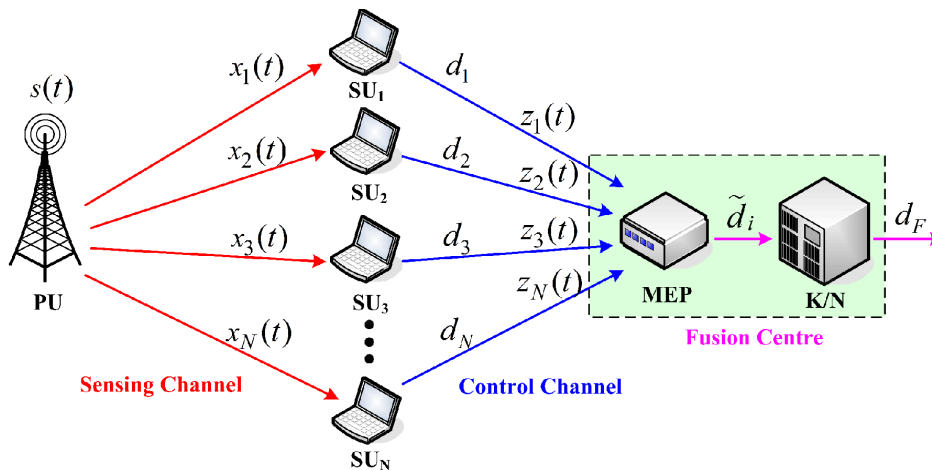


Fig. 1. System setup

### 2.1 Local Spectrum Sensing

We choose energy detectors for local sensing without obscuring the analysis by employing more sophisticated sensing algorithms, since our goal is to characterize the performance gains achieved by cooperation. Fig. 2 illustrates the block diagram of an energy detector [8][9], where  $x_i(t)$ ,  $\lambda_{si}$ ,  $d_i$ , and  $\delta_{si}^2$  denote the observed signal, the detection threshold, the local binary decision ('0'/'1'), and the average noise power at  $SU_i$  ( $i \in [1, N]$ ), respectively.

According to the detection theory developed by Urkowitz[8], the detection process can be formulated as a binary hypothesis testing problem:

$$Y_i = \frac{1}{\delta_{si}^2} \sum_{n=1}^{2m} x_i^2(n) = \begin{cases} \frac{1}{\delta_{si}^2} \sum_{n=1}^{2m} w_{si}^2(n), & H_0 \\ \frac{1}{\delta_{si}^2} \sum_{n=1}^{2m} (h_{si}(n)s(n) + w_{si}(n))^2, & H_1 \end{cases} \quad (1)$$

where  $m$  is the time-bandwidth product,  $m=TW$ ,  $T$  is the observation time,  $W$  is the sensing channel bandwidth,  $s(n)$  is the sampled primary signal, and  $Y_i$ ,  $h_{si}(n)$ , and  $w_{si}(n)$  represent the normalized test statistic, the sampled amplitude gain, and the sampled additive white Gaussian noise at  $SU_i$ , respectively.

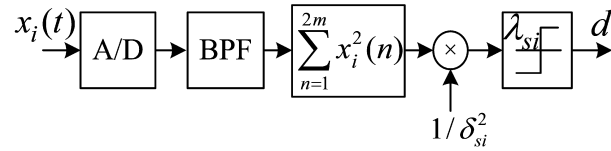


Fig. 2. Block diagram of the  $i$ -th energy detector

$Y_i$  has been derived as having a central and a non-central chi-square distributions under  $H_0$  and  $H_1$ , respectively [8][9]:

$$Y_i \sim \begin{cases} \chi_{2m}^2, & H_0 \\ \chi_{2m}^2(2m\gamma_{si}), & H_1 \end{cases} \quad (2)$$

where  $\gamma_{si}$  is the local sensing SNR with the definition

$$\gamma_{si} = \frac{P_{si}}{\delta_{si}^2} = \frac{\frac{1}{2m} \sum_{n=1}^{2m} |h_{si}(n)s(n)|^2}{N_{0i}W} \quad (3)$$

$N_{0i}$  denotes the one-sided noise power spectral density, and  $P_{si}$  is the received primary signal power. The following generic probabilistic model is typically used in characterizing the local sensing performance:

$$P_{ij} = \Pr\{d_i = H_i | H_j\}, \quad i, j \in \{0,1\}$$

In particular, we use  $P_{di} = P_{11}$  and  $P_{fi} = P_{10}$  to denote the probabilities of detection and false alarm at  $SU_i$ , respectively.

## 2.2 Decision Transmission

The control channels and the sensing channels are always assumed to have similar characteristics. Thus, the decision transmission model is

$$z_i(t) = h_{ci}(t)Ad_i + w_{ci}(t) \quad (4)$$

where  $A$ ,  $z_i(t)$ ,  $h_{ci}(t)$ , and  $w_{ci}(t)$  represent the amplification factor, the received signal, the channel gain, and the additive white Gaussian noise with zero mean in the  $i$ -th control channel, respectively. Hence, the SNR of the  $i$ -th control channel is

$$\gamma_{ci} = \frac{\frac{1}{T_1} \int_0^{T_1} |h_{ci}(t)|^2 A^2 dt}{\delta_{ci}^2} \quad (5)$$

where  $\delta_{ci}^2$  is the average noise power.

### 2.3 Decision Fusion

To realize the spatial diversity, the fusion centre has to accomplish the following tasks [29]:

(1) Estimate the initial local decisions from the received signals, which might have been faded and noise corrupted in the control channels.

(2) Fuse the estimated decisions with the proper combination rules.

For the first step, we use a detector based on the minimum error probability (MEP) criteria [30]. The recovery of the decisions can also be modeled as a binary signal detection problem:

$$z_i(t) = \begin{cases} w_{ci}(t), & D_{i0} \\ h_{ci}(t)A + w_{ci}(t), & D_{i1} \end{cases} \quad (6)$$

where  $D_{i0}$  and  $D_{i1}$  denote the initial  $i$ -th decision '0' and '1', respectively. The costs of right estimations are always considered to be zero and that of error estimations equal to one, i.e.,  $c_{00} = c_{11} = 0$ ,  $c_{01} = c_{10} = 1$ ,  $c_{00} = c_{11} = 0$ , and  $c_{01} = c_{10} = 1$ . Thus, the classical Bayesian average risk is reduced to the average error probability

$$\bar{R}_i = P_{ie} = P(D_{i0})P(\text{say } D_{i1} | D_{i0}) + P(D_{i1})P(\text{say } D_{i0} | D_{i1}) \quad (7)$$

where  $P(D_{i0})$  and  $P(D_{i1})$  are the probabilities of  $D_{i0}$  and  $D_{i1}$ , respectively. Then,

$$P(D_{i0}) = P\{d_i = 0\} = P_{01}P_{H_1} + P_{00}P_{H_0} = (1 - P_{d_i})P_{H_1} + (1 - P_{f_i})P_{H_0} \quad (8)$$

$$P(D_{i1}) = P\{d_i = 1\} = P_{11}P_{H_1} + P_{10}P_{H_0} = P_{d_i}P_{H_1} + P_{f_i}P_{H_0} \quad (9)$$

where  $P_{H_1}$  and  $P_{H_0}$  denote the probabilities of the presence and the absence of the primary user, respectively. Hence, the MEP-rule based detection can be represented as [30]

$$\Lambda(z_i) = \frac{f(z | D_{i1})_{D_{i1}}}{f(z | D_{i0})_{D_{i0}}} \stackrel{D_{i1}}{\geq} \frac{P(D_{i0})}{P(D_{i1})} = \Lambda_{i0} \quad (10)$$

In the AWGN control channels, the detection process has been derived [30], which can be simplified as

$$\tilde{d}_i = \begin{cases} 0, & \text{if } z_i < \lambda_{ci} \\ 1, & \text{if } z_i \geq \lambda_{ci} \end{cases} \quad (11)$$

where  $\lambda_{ci} = \frac{A}{2} + \frac{\delta_{ci}^2 \ln \Lambda_{i0}}{A}$  is the threshold of the MEP detector, and  $\tilde{d}_i$  is the estimate of the  $i^{\text{th}}$  local decision. The MEP detector requires the knowledge of  $P(D_{i0})$  and  $P(D_{i1})$  that may be unknown at FC. Therefore, we use the approximation method similar to that given in the literature [29]. With the application of the Strong Law of Large Numbers,

$$\lim_{N \rightarrow \infty} \bar{Z} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N z_i = AP(D_{i1}) \quad (12)$$

An unbiased estimate of  $P(D_{i1})$  is  $\tilde{P}(D_{i1}) = \frac{\bar{Z}}{A}$  because  $N$  is finite in practice. Then,  $\tilde{P}(D_{i0}) = 1 - \frac{\bar{Z}}{A}$ . Thus,  $\lambda_{ci}$  can be approximated as

$$\tilde{\lambda}_{ci} = \frac{A}{2} + \frac{\delta_{ci}^2}{A} \ln \frac{1 - \tilde{P}(D_{i1})}{\tilde{P}(D_{i1})} = \frac{A}{2} + \frac{\delta_{ci}^2}{A} \ln \frac{A - \bar{Z}}{\bar{Z}} = \frac{A}{2} + \frac{A}{\gamma_{ci}} \ln \frac{A - \bar{Z}}{\bar{Z}} \quad (13)$$

After the estimation of all the binary decisions  $\tilde{d}_1, \tilde{d}_2, \dots, \tilde{d}_N$ , FC continues to combine them to obtain the final decision  $d_F$  for the current sensing period, and then broadcast it to all the cooperative SUs. For simplicity, we choose the *K out of N rule* for the decision combination:

$$d_F = \begin{cases} 0, & \sum_{i=1}^N \tilde{d}_i < K \\ 1, & \sum_{i=1}^N \tilde{d}_i \geq K \end{cases} \quad (14)$$

which means that  $d_F = 1$  if  $K$  or more estimated local decisions are equal to 1. In special cases, the fusion rule is reduced to the OR rule, the AND rule, and the MAJ rule when  $K$  is equal to 1,  $N$ , and  $\lceil N/2 \rceil$ , respectively.

### 3. Performance Analysis and Simulation

#### 3.1 Local Sensing Performance

Over the AWGN sensing channels,  $h_{si}(n)$  is deterministic. Thus, the probabilities of detection, false alarm, and miss detection at SU<sub>*i*</sub> may be written as [9]

$$P_{di} = P\{Y_i > \lambda_i | H_1\} = Q_m\left(\sqrt{2m\gamma_{si}}, \sqrt{\lambda_i}\right) \quad (15)$$

$$P_{fi} = P\{Y_i > \lambda_i | H_0\} = 1 - \Gamma(\lambda_i/2, m) \quad (16)$$

$$P_{mi} = 1 - P_{di} \quad (17)$$

respectively, where  $Q_m(\cdot, \cdot)$  and  $\Gamma(\cdot, \cdot)$  represent the Marcum Q-function and the incomplete gamma function, respectively, with the definitions [31]

$$Q_m(a, b) = \int_b^\infty \frac{x^m}{a^{m-1}} \exp\left(-\frac{x^2 + a^2}{2}\right) I_{m-1}(ax) dx, \quad (18)$$

$$\Gamma(x, a) = \frac{1}{\Gamma(a)} \int_0^x e^{-t} t^{a-1} dt, \quad (19)$$

where  $\Gamma(a)$  is the gamma function  $\Gamma(a) = \int_0^\infty e^{-t} t^{a-1} dt$ .

The local performance can be extended to account for different channel effects, such as fading/shadowing [9]. As expected,  $P_f$  remains constant under any channel because it is considered when the primary signal is absent, and is thus independent of the sensing SNR. When the sensing channel gain varies due to fading/shadowing, the average probability of

detection can be derived by averaging  $P_{di}$  in (15) over the probability density function (PDF) of  $\gamma_{si}$  [9],

$$\bar{P}_{di} = \int_{\gamma_{si}} Q_m(\sqrt{2mx}, \sqrt{\lambda_i}) f_{\gamma_{si}}(x) dx \tag{20}$$

In general, the SNR of a practical channel can be statistically represented via the combination of three terms [17]:

$$SNR_{dB} = \overline{SNR}_{dB} + Shadow_{dB} + Fading_{dB} \tag{21}$$

where the three terms in dB on the right denote the mean SNR, the shadowing (also called large scale fading) effects, and the small scale fading effects, respectively. The characteristics of several typical channels often used in wireless communication are summarized as follows:

*Log-normal shadowing channel:* After attenuation by the obstacles in the propagation path, the empirical measurements suggest that the variation of the received power follows a normal distribution when represented in dB. Thus, the PDF of the linear SNR  $\gamma$  (as opposed to dB) is [17]

$$f_{\gamma}(x) = \frac{10}{\ln(10)\sigma_{dB}} \frac{1}{\sqrt{2\pi x}} \exp\left(-\frac{(10\log(x) - 10\log(\bar{\gamma}))^2}{2\sigma_{dB}^2}\right) \tag{22}$$

where  $\sigma_{dB}$  is the dB-spread of the shadowing effects, and  $\bar{\gamma}$  is the average SNR.

*Rayleigh channel:* The signal amplitude in this kind of channel follows a Rayleigh distribution, and  $\gamma$  follows an exponential PDF [9]:

$$f_{\gamma}(x) = \frac{1}{\gamma} \exp\left(-\frac{x}{\gamma}\right) \tag{23}$$

*Nakagami channel:* In this channel, the signal amplitude follows a Nakagami distribution. Then,  $\gamma$  follows a gamma PDF [9]:

$$f_{\gamma}(x) = \frac{1}{\Gamma(g)} \left(\frac{g}{\gamma}\right)^g x^{g-1} \exp\left(-\frac{g}{\gamma}x\right) \tag{24}$$

where  $g$  is the Nakagami parameter, and the channel is reduced to either AWGN when  $g = \infty$ , or Rayleigh when  $g = 1$ .

*Suzuki channel:* This channel is a mixture of Rayleigh and Shadowing effects, and  $\gamma$  follows a Suzuki distribution [17]:

$$f_{\gamma}(x) = \int_0^{\infty} \frac{1}{t^2} \frac{10}{\ln(10)} \frac{1}{\sqrt{2\pi\sigma_{dB}}} \exp\left(-\frac{x}{t} - \frac{(10\log(t) - 10\log(\bar{\gamma}))^2}{2\sigma_{dB}^2}\right) dt \tag{25}$$

Closed expressions and numerical results of the performance in some of the above channels have been given in the literature [9][12][17]. To validate the analysis above, we perform a simulation study on the local sensing. First, we choose a BPSK signal as the primary signal. After its transmission in the fading/shadowing channels listed above, an energy detector is used to sense its presence via Monte Carlo simulation with 100,000 trials. The simulation parameters are given as follows:  $m = 5$ ,  $\bar{\gamma}_s = 5dB$ ,  $\sigma_{dB} = 6dB$ , and  $g = 3$ . Fig. 3 shows the complementary receiver operating characteristic (CROC) curves (plot of  $P_m$  vs  $P_f$ ) over these channels. The corresponding theoretical results calculated by numerical integration methods in (20) are also given for comparison. The simulation results clearly coincide well with that in

theory. The detection over the AWGN channel outperforms other cases most of the time. In contrast, the Suzuki channel provides the worst-case scenario among all of the listed channels. Further simulations are also performed with some primary signals that are more complicated, such as a mixed signal. The results are very similar with that in Fig. 3, which verifies that the energy detector solely depends on signal power, rather than on signal forms or any other prior knowledge.

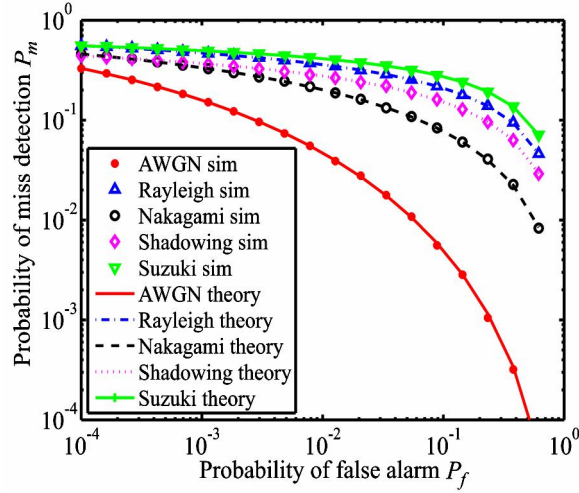


Fig. 3. Complementary receiver operating characteristic (CROC) curves of local energy detection over typical channels

### 3.2 Collective Sensing Performance of CSS

For simplicity, the relative distances between any two SUs are assumed to be smaller than their distances to the PU. Thus, we first consider that all  $N$  SUs experience independent and identically distributed channel effects, with the same  $\bar{\gamma}_s$  in all sensing channels and the same  $\bar{\gamma}_c$  in all control channels. Spatially correlated shadowing is discussed later.

For the AWGN scenario, the primary signal and the local decisions are assumed to be only corrupted by noise without any fading or shadowing,  $w_s \sim N(0, \delta_s^2)$  and  $w_c \sim N(0, \delta_c^2)$ . Intuitively, the equivalent local performance at FC after decision estimation may deteriorate to some extent. For  $SU_i$ , the equivalent local probabilities of false alarm and detection can be derived as follows:

$$\begin{aligned} \tilde{P}_{fi} &= P\{\tilde{d}_i = 1 | H_0\} \\ &= P\{\tilde{d}_i = 1 | d_i = 1\}P\{d_i = 1 | H_0\} + P\{\tilde{d}_i = 1 | d_i = 0\}P\{d_i = 0 | H_0\} \\ &= P\{z_i > \lambda_{ci} | D_{i1}\}P_{fi} + P\{z_i > \lambda_{ci} | D_{i0}\}(1 - P_{fi}) \\ &= P\{w_{ci} > \lambda_{ci} - A\}P_{fi} + P\{w_{ci} > \lambda_{ci}\}(1 - P_{fi}) \\ &= Q\left(\frac{\lambda_{ci} - A}{\delta_{ci}}\right)P_{fi} + Q\left(\frac{\lambda_{ci}}{\delta_{ci}}\right)(1 - P_{fi}) \end{aligned} \quad (26)$$

$$\tilde{P}_{di} = P\{\tilde{d}_i = 1 | H_1\} = Q\left(\frac{\lambda_{ci} - A}{\delta_{ci}}\right)P_{di} + Q\left(\frac{\lambda_{ci}}{\delta_{ci}}\right)(1 - P_{di}) \quad (27)$$



where  $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-t^2/2) dt$ .

After fusion by the  $K$  out of  $N$  rule, the collective probabilities of false alarm, detection, and miss detection are

$$\tilde{Q}_f = \sum_{n=K}^N \binom{N}{n} \tilde{P}_{fi}^n (1 - \tilde{P}_{fi})^{N-n} = 1 - B_F(K-1, N, \tilde{P}_{fi}) \quad (28)$$

$$\tilde{Q}_d = \sum_{n=K}^N \binom{N}{n} \tilde{P}_{di}^n (1 - \tilde{P}_{di})^{N-n} = 1 - B_F(K-1, N, \tilde{P}_{di}) \quad (29)$$

$$\tilde{Q}_m = B_F(K-1, N, \tilde{P}_{di}) \quad (30)$$

respectively, because the noise statistics of all SUs are the same. In (30),

$B_F(m, M, p) = \sum_{n=0}^m \binom{M}{n} p^n (1-p)^{M-n}$  is the Binomial cumulative distribution function [31].

Furthermore, we can also obtain the average performance in fading/shadowing scenarios with variational  $\gamma_s$  and  $\gamma_c$  by integrating (28)–(30) with the PDFs of  $\gamma_s$  and  $\gamma_c$ :

$$\tilde{Q}_{f,avg} = \int_{\gamma_c} \int_{\gamma_s} \tilde{Q}_f f_{\gamma_s}(x) f_{\gamma_c}(y) dx dy \quad (31)$$

$$\tilde{Q}_{d,avg} = \int_{\gamma_c} \int_{\gamma_s} \tilde{Q}_d f_{\gamma_s}(x) f_{\gamma_c}(y) dx dy \quad (32)$$

$$\tilde{Q}_{m,avg} = 1 - \tilde{Q}_{d,avg} \quad (33)$$

The CROC curves of CSS with noisy decisions over several typical channels for various  $\bar{\gamma}_c$  are plotted in Fig. 4(a)–(d). These plots are obtained by numerical computation and are also verified with Monte Carlo simulation according to Section 2 with 100,000 trials. The simulation process is performed with a BPSK primary signal and 10 SUs ( $N = 10$ ), and setting other parameters as  $m = 10$ ,  $\bar{\gamma}_s = 0dB$ ,  $\sigma_{dB} = 2dB$ , and  $K = 1$  (the OR rule). For comparison, the corresponding simulated and theoretical performance of local sensing and pure CSS (i.e., the CSS scheme without any fading or noise corruption in the control channels [12]) are also shown. Thus, in all below figures, we use ‘noisy’, ‘local,’ and ‘pure’ as shorthand representations of the three schemes, and also, we use  $Q_f$  and  $Q_m$  to denote their general probabilities of false alarm and miss detection, respectively.

As expected, CSS can effectively counteract the deleterious impacts of fading/shadowing in sensing channels because the decision fusion can result in a higher chance of having a user with its sensing SNR well above the average. However, there is a performance drop in the noisy CSS compared with that in the pure case. This drop is due to the transmission error of the local decisions caused by fading/shadowing effects and noise corruption in the common control channels. The collective performance of noisy CSS is even worse than the local performance when  $\bar{\gamma}_c$  is lower than some value, for example 10dB over AWGN channel, and 5dB over Rayleigh channel. This implies no necessity for cooperation under such condition. Moreover, the performance of noisy CSS improves as  $\bar{\gamma}_c$  increases. It is almost equivalent to that of the pure case when  $\bar{\gamma}_c$  is higher than 20dB over all listed channels, except for the Suzuki channel where there is always a gap between the two cases even when  $\bar{\gamma}_c$  is very high [Fig. 4-(d)].

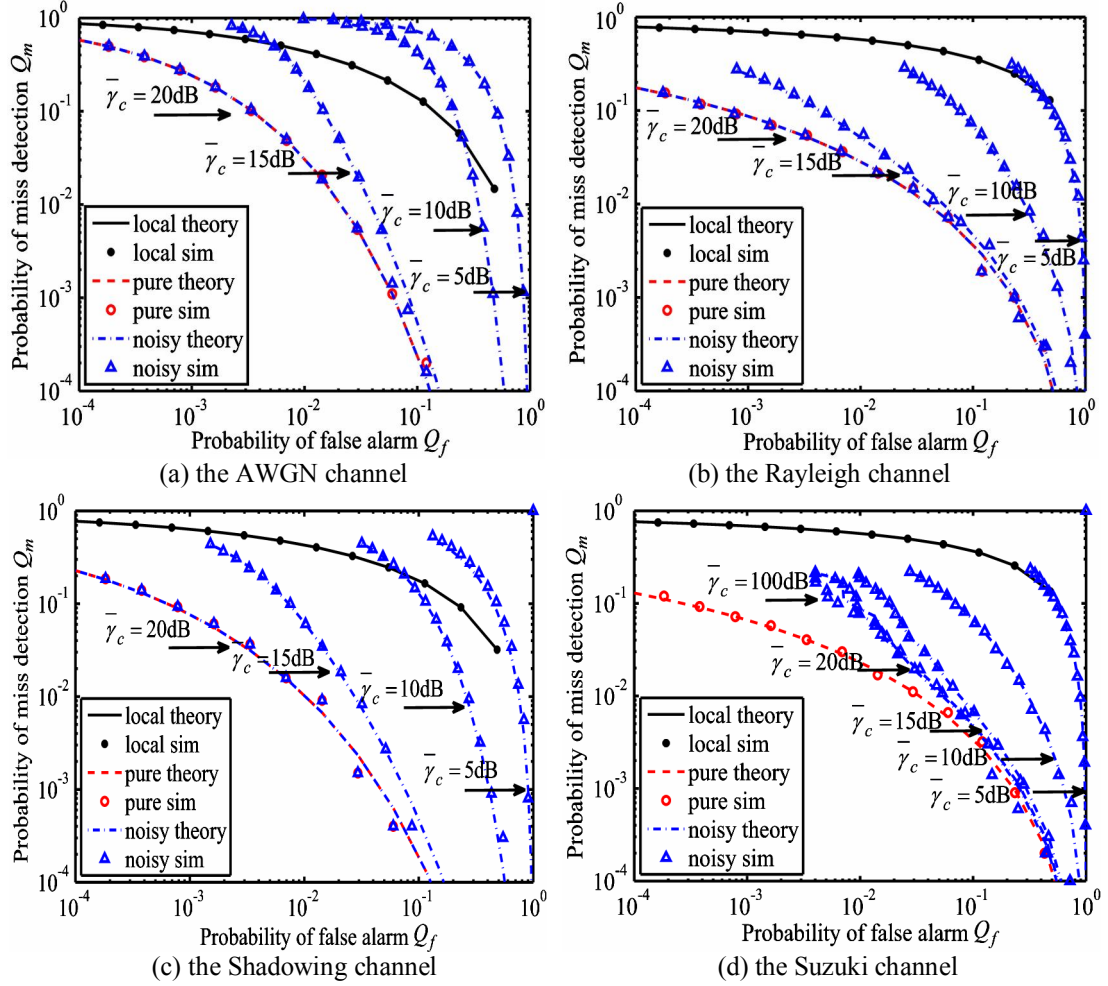


Fig. 4. Complementary receiver operating characteristic (CROC) curves of CSS with noisy decisions for various average control SNRs over different channels

### 3.3 Spatially Correlated Shadowing Effects

The analysis and simulations above are based on the condition that all SUs experience independent channel effects. This is safely assumed for scenarios with only multipath fading like Rayleigh and Nakagami [32]. However, in practice, there is usually a degree of spatial correlation associated with the log-normal shadowing, which will degrade the performance of CSS intuitively because similar shadowing effects of different SUs may partly counter the collaboration gain [12][32]. To examine the impact of correlated shadowing, we employ an exponential correlation model given in the literature [32]:

$$R_{ij} = \exp(-\alpha b_{ij}) \quad (34)$$

where  $R_{ij}$  and  $b_{ij}$  ( $i, j \in [1, N]$ ) denote the correlation factor and distance between the  $SU_i$  and  $SU_j$ , respectively; and  $\alpha$  is an environment-dependent parameter, with  $\alpha \approx 0.1204$  in urban areas and  $\alpha \approx 0.002$  in suburban areas [32]. The random variable (RV) for shadowing without correlation in the  $i$ -th sensing or control channel can be represented as

$$Shadow_{i,\text{dB}} = \sigma_{\text{dB}} X_i \quad (35)$$

where  $X_i$  is an RV with standard Gaussian distribution. If the correlation is considered, the RV becomes

$$\begin{aligned} \text{Shadow}_{i,\text{dB}} &= \sqrt{R_{i1}}\sigma_{\text{dB}}X_1 + \sqrt{R_{i2}}\sigma_{\text{dB}}X_2 + \cdots + \sqrt{R_{ii}}\sigma_{\text{dB}}X_i + \cdots + \sqrt{R_{iN}}\sigma_{\text{dB}}X_N \\ &= \sum_{j=1}^N \sqrt{R_{ij}}\sigma_{\text{dB}}X_j = \sum_{j=1}^N \sqrt{C_{ij}}X_j \end{aligned} \quad (36)$$

where  $C_{ij}$  denotes the element of the covariance matrix,

$$C_{ij} = \text{Cov}(\text{Shadow}_{i,\text{dB}}, \text{Shadow}_{j,\text{dB}}) = R_{ij}\sigma_{\text{dB}}^2 \quad (37)$$

The correlated shadowing effect is obviously closely bound up with the topology of the CRN. In the following simulation, we consider a centralized case where 10 SUs are uniformly distributed around a circle with FC as the centre and  $r$  as the radius. Fig. 5 shows the CROC curves of CSS with correlated shadowing over the Suzuki channels for various values of  $r$ . Both the pure and noisy cooperation cases are considered with the parameters set as  $m = 10$ ,  $\bar{\gamma}_s = 0\text{dB}$ ,  $\bar{\gamma}_c = 20\text{dB}$ ,  $\sigma_{\text{dB}} = 2\text{dB}$ ,  $\alpha \approx 0.002$ ,  $N = 10$ , and  $K = 1$  (the OR rule). The spatial correlation degrades the performance of CSS under both the noisy and pure models for all values of  $r$ . Furthermore, this effect becomes significant when the users are dispersed over a smaller circle, which suggests that cooperation over a large distance is more feasible than a dense case confined to a small area.

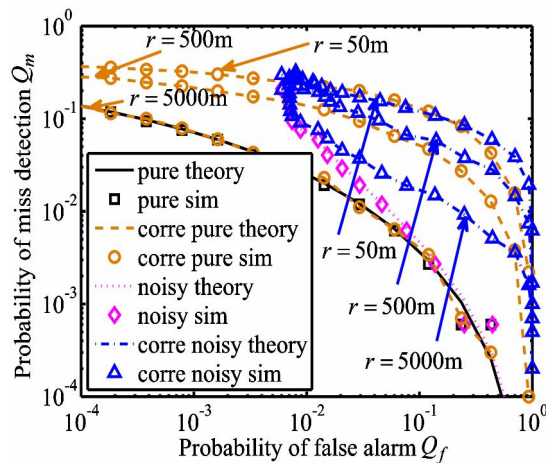


Fig. 5. Complementary receiver operating characteristic (CROC) curves of the CSS with correlated shadowing over the Suzuki channels for various values of  $r$

#### 4. Optimization of the Cooperative Spectrum Sensing

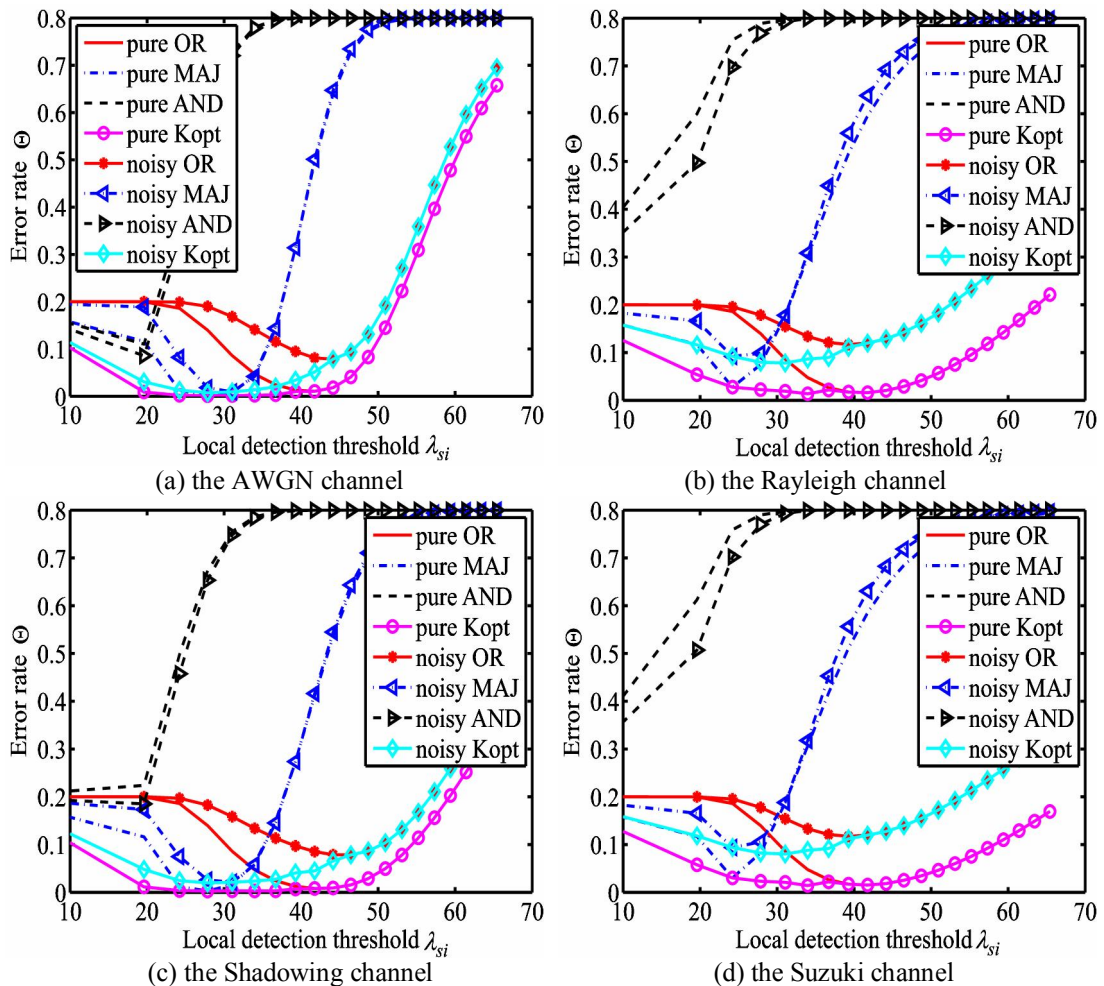
In this section, we investigate the optimal  $K$  in the counting rule used for the decision fusion at FC. Under the Bayesian criteria, we aim at minimizing the total error rate  $\Theta$  denoted by

$$\text{Min}_{1 \leq K \leq N} \Theta = W_f Q_f + W_m Q_m \quad (38)$$

where  $W_f$  is the cost of false alarm, and  $W_m = 1 - W_f$  corresponds to the miss detection cost.

Both the pure and noisy CSS are studied using the general numerical search method according to (28)–(33) and (38), and their optimal fusion strategies (i.e., optimal values of  $K$ )

can be obtained. Given  $m=10$ ,  $\bar{\gamma}_s=0dB$ ,  $\bar{\gamma}_c=10dB$ ,  $\sigma_{dB}=2dB$ ,  $N=10$ ,  $W_f=0.2$ , and  $W_m=0.8$ , **Figs. 6(a)–(d)** show the error rate curves as the function of the local sensing threshold  $\lambda_{si}$  for several typical values of  $K$  ( $K=1,5,10,K_{opt}$ ) over the AWGN, Rayleigh, Shadowing, and Suzuki channels, respectively. The optimal fusion strategy compares quite favorably with the non-optimal cases over these channels. As expected, there is always a higher error rate in the noisy CSS because the fading/shadowing effects or noise corruption during decision transmission, aside from the local detection algorithms limitation, may also lead to error decisions at FC. **Fig. 7** shows the exact optimal values of  $K$  for different  $\lambda_{si}$  over the AWGN and Suzuki channels. Although the detailed values of  $K_{opt}$  for specified  $\lambda_{si}$  under different scenarios are clearly not the same, their variation trends coincide well with each other, that is, as the detection threshold increases, the optimal value decreases.



**Fig. 6.** Comparison of the total error rate in the noisy and pure CSS for different  $K$  over different channels

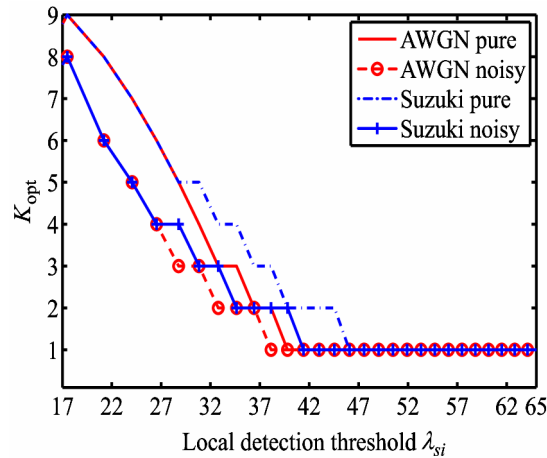


Fig. 7. Optimal fusion strategy (versus local detection threshold) of CSS over the AWGN and Suzuki channels

## 5. Conclusion and Future Work

In this paper, we have investigated the CSS scheme based on energy detection and the  $K$  out of  $N$  rule under an extended system model that considers an imperfect decision transmission in the control channels. Before the final combination, the fusion centre estimates the initial decisions utilizing an MEP binary detector because the received decisions may fade and be noise corrupted during transmission. As demonstrated by both the numerical and simulation results over several typical channels, the performance gains of CSS relative to the local sensing under the proposed model have some degradation compared with that in the ideal case that assumes an error-free decision transmission. The performance gains are even insignificant if the average SNR in the control channel is very low. Moreover, we have also investigated the optimization problem under this practical model to minimize the total error rate. The results suggest a decreased variation trend in the optimal value of  $K$  with the increasing local threshold.

Although we have studied the effect of imperfect control channels on the performance of the traditional  $K/N$ -rule-based CSS, more studies must be conducted to develop efficient CSS schemes in such an environment [33]. For example, designing a joint local decision recovery and final decision fusion scheme is a helpful tool. In addition, characterizing the performance of CSS schemes under different spatial distributions of cooperative users and different correlation models of shadowing effects is of great importance. Other interesting issues for future research include extending the uncertainty models of the control channels by error probability, which can generally accommodate any distorting effects, and also, developing some online algorithms [34] for maximizing the throughput of CRN when the sensing outcome is not always reliable.

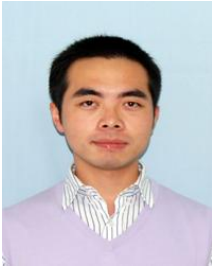
## References

- [1] Y. Zeng, Y. Liang, A. T. Hoang and R. Zhang, "A review on spectrum sensing for cognitive radio: challenges and solutions," *EURASIP Journal on Advances in Signal Processing*, vol. 2010, ID:381465, pp. 1-15, Nov. 2010. [Article \(CrossRef Link\)](#).
- [2] S. Byun, I. Balasingham and A. V. Vasilakos, "A market-clearing model for spectrum trade in cognitive radio networks," in *Proc. of ACM Mobihoc*, Paris, France, pp. 16-20, May 2011.



- [3] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201-220, Feb. 2005. [Article \(CrossRef Link\)](#).
- [4] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Communications Surveys & Tutorials*, vol. 11, no. 1, pp. 116-130, Jan. 2009. [Article \(CrossRef Link\)](#).
- [5] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Processing Magazine*, vol. 24, no. 3, pp. 79-89, May 2007. [Article \(CrossRef Link\)](#).
- [6] A. Ghasemi and E. S. Sousa, "Spectrum sensing in cognitive radio networks: requirements, challenges and design trade-offs," *IEEE Communications Magazine*, vol. 46, no. 4, pp. 32-39, Apr. 2008. [Article \(CrossRef Link\)](#).
- [7] Q. Liu, J. Gao, J. Guan and Y. Guo, "A survey on linker layer key technologies in cognitive radio networks (in Chinese)," *Telecommunication Engineering*, vol. 50, no. 3, pp. 90-98, Mar. 2010. [Article \(CrossRef Link\)](#).
- [8] H. Urkowitz, "Energy detection of unknown deterministic signals," *Proceedings of the IEEE*, vol. 55, no. 4, pp. 523-531, Apr. 1967. [Article \(CrossRef Link\)](#).
- [9] F. F. Digham, M. Alouini and M. K. Simon, "On the energy detection of unknown signals over fading channels," in *Proc. of IEEE International Conference on Communications, ICC*, Alaska, USA, pp. 3575-3579, May 2003.
- [10] D. Cabric, A. Tkachenko and R. W. Brodersen, "Spectrum sensing measurements of pilot, energy, and collaborative detection," in *Proc. of IEEE Military Communications Conference, MILCOM*, Washington, D.C., USA, pp. 1-7, Oct. 2006.
- [11] R. Tandra and A. Sahai, "Fundamental limits on detection in low SNR under noise uncertainty," in *Proc. of International Conference on Wireless Networks, Communications and Mobile Computing*, Maui, Hawaii, USA, pp. 464-469, Jun. 2005. [Article \(CrossRef Link\)](#).
- [12] A. Ghasemi and E. S. Sousa, "Opportunistic spectrum access in fading channels through collaborative sensing," *Journal of Communications*, vol. 2, no. 2, pp. 71-82, Mar. 2007. [Article \(CrossRef Link\)](#).
- [13] J. Ma and Y. Li, "Soft combination and detection for cooperative spectrum sensing in cognitive radio networks," in *Proc. of IEEE Global Telecommunications Conference, GLOBECOM*, Washington, D.C., USA, pp. 3139-3143, Nov. 2007.
- [14] Z. Quan, S. Cui and A. H. Sayed, "Optimal linear cooperation for spectrum sensing in cognitive radio networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 28-40, Feb. 2008. [Article \(CrossRef Link\)](#).
- [15] B. Shen and K. S. Kwak, "Soft combination schemes for cooperative spectrum sensing in cognitive radio networks," *ETRI Journal*, vol. 31, no. 3, pp. 263-270, Jun. 2009. [Article \(CrossRef Link\)](#).
- [16] B. Shen, S. Ullah and K. Kwak, "Deflection coefficient maximization criterion based optimal cooperative spectrum sensing," *AEU - International Journal of Electronics and Communications*, vol. 64, no. 9, pp. 819-827, Sep. 2010. [Article \(CrossRef Link\)](#).
- [17] S. Kyperountas, N. Correal, Q. Shi and Z. Ye, "Performance analysis of cooperative spectrum sensing in suzuki fading channels," in *Proc. of 2nd International Conference on Cognitive Radio Oriented Wireless Networks and Communications, CrownCom*, Orlando, USA, pp. 428-432, Jun. 2007.
- [18] W. Zhang, R. K. Mallik and K. B. Letaief, "Optimization of cooperative spectrum sensing with energy detection in cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 8, no. 12, pp. 5761-5766, Dec. 2009. [Article \(CrossRef Link\)](#).
- [19] J. Shen, S. Liu, L. Zeng, G. Xie, J. Gao and Y. Liu, "Optimization of cooperative spectrum sensing in cognitive radio network," *IET Communications*, vol. 3, no. 7, pp. 1170-1178, Mar. 2009. [Article \(CrossRef Link\)](#).
- [20] X. Zhou, J. Ma, G. Li, K. Young and A. C. K. Soong, "Probability-based combination for cooperative spectrum sensing," *IEEE Transactions on Communications*, vol. 58, no. 2, pp. 463-466, Feb. 2010. [Article \(CrossRef Link\)](#).

- [21] Y. Xu, Y. Li, Y. Zhao, H. Zou and A. V. Vasilakos, "Selective sensing and transmission for multi-channel cognitive radio networks," in *Proc. of INFOCOM workshop on Cognitive and Cooperative Networks*, Shanghai, China, pp. 1-6, Apr. 2011.
- [22] T. H. Anh, L. Ying-Chang and Z. Yonghong, "Adaptive joint scheduling of spectrum sensing and data transmission in cognitive radio networks," *IEEE Transactions on Communications*, vol. 58, no. 1, pp. 235-246, January, 2010. [Article \(CrossRef Link\)](#).
- [23] R. Chen, J. Park, Y. T. Hou and J. H. Reed, "Toward secure distributed spectrum sensing in cognitive radio networks," *IEEE Communications Magazine*, vol. 46, no. 4, pp. 50-55, Apr. 2008. [Article \(CrossRef Link\)](#).
- [24] Q. Liu, J. Gao, Y. Guo and S. Liu, "Attack-proof cooperative spectrum sensing based on consensus algorithm in cognitive radio networks," *KSII Transactions on Internet and Information Systems*, vol. 4, no. 6, pp. 1042-1062, Dec. 2010. [Article \(CrossRef Link\)](#).
- [25] L. Song, Y. Li, M. Tao and A. V. Vasilakos, "A Hybrid Relay Selection Scheme Using Differential Modulation," in *Proc. of IEEE Wireless Communications and Networking Conference, WCNC*, Budapest, Hungary, pp. 1-6, Apr. 2009.
- [26] L. Song, Y. Li, A. Huang, B. Jiao and A. V. Vasilakos, "Differential modulation for bi-directional relaying with analog network coding," *IEEE Transaction on Signal Processing*, vol. 58, no. 7, pp. 3933-3938, Jul. 2010. [Article \(CrossRef Link\)](#).
- [27] G. Ganesan and Y. Li, "Cooperative spectrum sensing in cognitive radio, Part II: multiuser networks," *IEEE Transactions on Wireless Communications*, vol. 6, no. 6, pp. 2214-2222, Jun. 2007. [Article \(CrossRef Link\)](#).
- [28] Z. Li, F. R. Yu and M. Huang, "A distributed consensus-based cooperative spectrum-sensing scheme in cognitive radios," *IEEE Transactions on Vehicular Technology*, vol. 59, no. 1, pp. 383-393, Jan. 2010. [Article \(CrossRef Link\)](#).
- [29] T. C. Aysal, S. Kandeepan and R. Piesiewicz, "Cooperative spectrum sensing with noisy hard decision transmissions," in *Proc. of IEEE International Conference On Communication, ICC*, Dresden, Germany, pp. 1-5, Jun. 2009.
- [30] M. Zhang and M. Lv, "Signal detection and estimation (in Chinese), 2nd Edition," Publishing House of Electronics Industry, Beijing, 2005.
- [31] M. Abramowitz and I. A. Stegun, "Handbook of Mathematical Functions, National Bureau of Standards, Applied Math. Series #55," *Dover Publications*, New York, 1965.
- [32] M. Gudmundson, "Correlation model for shadow fading in mobile radio systems," *Electronics Letters*, vol. 27, no. 23, pp. 2145-2146, Dec. 1991. [Article \(CrossRef Link\)](#).
- [33] D. Oh and Y. Lee, "Cooperative spectrum sensing with imperfect feedback channel in the cognitive radio systems," *International Journal of Communication Systems*, vol. 23, no. 3, pp. 763-779, Mar. 2010. [Article \(CrossRef Link\)](#).
- [34] R. Kannan, S. Wei, J. Zhang and A. V. Vasilakos, "Throughput optimization for cognitive radio under sensing uncertainty," in *Proc. of IEEE Military Communications Conference, MILCOM*, San Jose, C.A., USA, pp. 1-6, 2010.



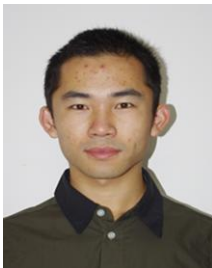
**Quan Liu**, Student Member, IEEE, received his B.S. degree in Communication Engineering from Naval University of Engineering, Wuhan, China, in 2006. Since 2008, he has been with the Software Radio Lab, Naval University of Engineering, where he is a Ph.D. candidate. His research interests include the key technologies in the Link Layer of cognitive radio network and multirate signal processing.



**Jun Gao** received the B.S. degree in Communication Engineering from Naval Electronic College of Engineering, in 1982, and the Ph.D. degree in Electronic Engineering from Beijing Institute of Technology in 1989. Since 1996, he has been a professor at Naval University of Engineering, and his research interests are digital communications and software radio systems.



**Yunwei Guo** received the B.S. degree in Communication Engineering from Naval University of Engineering, Wuhan, China, in 2004. Since 2008, he has been with the Software Radio Lab, Naval University of Engineering, where he is a Postgraduate. His current interest is spectrum sensing technology in cognitive radio network.



**Siyang Liu** received the B.S. degree in Underwater Acoustic Engineering from Harbin Engineering University, Heilongjiang, China, in 2009. Since 2009, he has been with the Software Radio Lab, Naval University of Engineering, as a Postgraduate. His current interest is dynamic spectrum allocation in cognitive radio network.