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BUYING AND SELLING RULES FOR A SIMPLE TRANSACTION OF A MEAN-REVERTING ASSET

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Abstract. We consider an optimal trading rule in this paper. We assume that the underlying asset follows a mean-reverting process and the transaction consists of one buying and one selling. To maximize the profit, we find price levels to buy low and to sell high. Associated HJB equations are used to formulate the value function. A verification theorem is provided for sufficient conditions. We conclude the paper with a numerical example.

1. Introduction

A simplest trading strategy in financial market is to buy law and to sell high a share. However, it is controvertial to decide appropriate price levels of the share to buy or sell in practical market. This paper contributes to find the price levels of the share when it is governed by a mean-reverting model. A mean reversion process is one of the often and widely used models in financial markets to formulate the movement of an asset price that tends to converge an equibrilium of the prices. Poterba and Summers [9] applied variance ratio test to detect mean reversion in stock prices and market returns for US and seventeen countries. Frankel and Rose [4] examined derivations from purchasing power parity to show strong evidence of mean-reversion in real exchange rates of various countries. Fama and French [3] tested a standard economic argument that profitability is mean reverting. A *mean reversion process*, or *Ornstein-Uhlenbeck* process (named after Ornstein and Uhlenbeck [8]), has a drift term that brings the variable being modeled back to an equilibrium level. The variable tends to oscillate around this equilibrium.

Consider the price of stock $S(t) = e^{X(t)}$ where $X(t)$ is the mean-reversion process which satisfies the following:

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$$dX(t) = a(b - X(t))dt + \sigma dw(t),$$

where a is the rate of reversion, b is the equilibrium level, σ is the volatility, and $w(t)$ is a standard Brownian motion. We consider a trading rule consisting of buying and selling a share.

The problem is to find good price levels to buy and sell the share. We aim at maximizing our profit through transactions. A basic strategy is to buy low and sell high. However, picking an appropriate buying and selling price levels is difficult in practice.

Prior to this study, Guo and Zhang [2] used a smooth-fit technicque to characterize the optimal selling rule under a model with hybrid GMB. Zhang and Zhang [10] studied the problem finding the buying and selling levels assuming infinitively buying and selling. Song, Yin and Zhang [7] calculated the level pairs with a stochastic approximation methods.

In this article, we consider the trading rule that allows a round trip transactions, i.e., only buy once and sell once. Let the thresholds pair (x_1, x_2) be the pair of buying price level and selling price level. We will determine the value of (x_1, x_2) based on the parameters, a, b, σ, a discount factor ρ, and transaction discount factor K.

The rest of this paper is arranged as follows. Section 2 begins with the mean reversion model and the setup of the problem. Section 3 presents some properties of the value functions. Section 4 introduces a verification theorem which demonstrates the optimality of the trading rule. Section 5 includes numerical examples to show the dependency of (x_1, x_2) and values of value functions on various parameters. In the last section, we compare the result of [10] with our results.

2. Problem Formulation

Let $X(t) \in \mathbb{R}$ denote a mean-reverting process governed by

$$dX(t) = a(b - X(t))dt + \sigma dw(t), \qquad X(0) = x \tag{1}$$

where $a > 0$ is the rate of reversion, b is the equilibrium level, $\sigma > 0$ is the volatility, and $w(t)$ is a standard Brownian motion. Let

$$0 \leq \tau \leq \delta$$

denote stopping times of a buying decision τ and a selling decision δ. The net position is flat(no stock holding) at $t \in [0, \tau) \cup [\delta, \infty)$ and long(one share of stock

holding) at $t \in [\tau, \delta)$.

The asset price is given by $S(t) = \exp X(t)$. Suppose that $0 < K < 1$ is the percentage of slippage or commission per transaction and the discount factor $\rho > 0$. We aim to maximize the reward functions

$$\begin{aligned} J_1(x, \Lambda_1) &= E[e^{-\rho\delta} S(\delta)(1-K) - e^{-\rho\tau} S(\tau)(1+K)], \\ J_2(x, \Lambda_2) &= E[e^{-\rho\delta} S(\delta)(1-K)] \end{aligned}$$

where $\Lambda_1 = \{\tau, \delta\}$ and $\Lambda_2 = \{\delta\}$. For $i = 1, 2$, let $V_i(x)$ be the value functions with $X(0) = x$ and initial net position i. Therefore, we have

$$V_i(x) = \sup_{\Lambda_i} J_i(x, \Lambda_i).$$

3. Properties of the Value Functions

In this section, we discuss various properties of the value functions and solve the stopping problem with a smooth fit principle. Note that

$$V_1(x) \geq J_1(x, \Lambda_1) = J_2(x, \Lambda_2) - Ee^{-\rho\tau} S(\tau)(1+K)$$

In particular, setting $\tau = 0$, we have the inequality

$$V_1(x) \geq V_2(x) - e^x(1+K). \tag{2}$$

Otherwise, we similarly obtain

$$V_2(x) \geq e^x(1-K). \tag{3}$$

The next lemma guarantees the bounds of $V_i(x)$.

Lemma 3.1. *There exist constants K_1, K_2 and K_3 such that*

$$\begin{aligned} 0 \leq V_1(x) &\leq K_1 \text{ and} \\ 0 \leq V_2(x) &\leq K_2 e^x + K_3. \end{aligned}$$

Proof. It is clear that $V_i(x)$ are nonnegative by definition. We first find the upper bounds. From Dynkin's formula, we have

$$Ee^{-\rho\delta} S(\delta) - Ee^{-\rho\tau} S(\tau) = E \int_\tau^\delta e^{-\rho t} e^{X(t)} (A - aX(t)) dt,$$

where $A = \sigma^2/2 + ab - \rho$. Note that $e^x(A - ax)$ is bounded above on $\mathbb{R}$. Let C be an upper bound. It follows that

$$Ee^{-\rho\delta} S(\delta) - Ee^{-\rho\tau} S(\tau) \leq CE \int_\tau^\delta e^{-\rho t} dt. \tag{4}$$

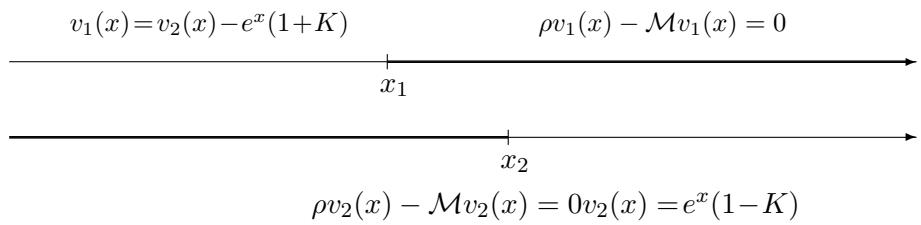

Figure 1. Thicken lines are Continuation Regions

Using the definition of $J_1(x,\Lambda_1)$, we have

$$\begin{aligned} J_1(x,\Lambda_1) &\le Ee^{-\rho\delta}S(\delta) - Ee^{-\rho\tau}S(\tau) \\ &\le CE\int_\tau^\delta e^{-\rho t}dt \\ &\le C\int_0^\infty e^{-\rho t}dt = \frac{C}{\rho} := K_1 \end{aligned}$$

This implies that $0 \le V_1(x) \le K_1$.

Similarly, we have the inequality

$$J_2(x,\Lambda_2) \le Ee^{-\rho\delta}S(\delta)(1-K).$$

As in (4), we can take $\tau = 0$ and show that

$$Ee^{-\rho\delta}e^{X(\delta)} - e^x \le \frac{C}{\rho}.$$

It implies that

$$V_2(x) \le (1-K)\Big(e^x + \frac{C}{\rho}\Big) := K_2e^x + K_3.$$

This completes the proof. □

Let $\mathcal{M}$ denote the generator of $X(t)$. That is,

$$\mathcal{M} = a(b-x)\frac{\partial}{\partial x} + \frac{\sigma^2}{2}\frac{\partial^2}{\partial x^2}.$$

The associated HJB equations have the form:

$$\begin{cases} \min\{\rho v_1(x) - \mathcal{M}v_1(x), v_1(x) - v_2(x) + e^x(1+K)\} = 0 \\ \min\{\rho v_2(x) - \mathcal{M}v_2(x), v_2(x) - e^x(1-K)\} = 0 \end{cases} \tag{5}$$

Recall that we decide to buy when the price is low and sell when the price is high. We say the low price as S_1 and the high price as S_2. Let $x_j = \log(S_j), j = 1, 2$. Then the continuation region for $i = 1$ should be (x_1, ∞) on which $\rho v_1(x) - \mathcal{M}v_1(x) = 0$. If $x < x_1$, $v_1(x) = v_2(x) - e^x(1+K)$. Similarly, the continuation region for $i = 2$ should be $(-\infty, x_2)$ on which $\rho v_2(x) - \mathcal{M}v_2(x) = 0$. If $x > x_2$, $v_2(x) = e^x(1-K)$.

Note that we need two restrictions for the values of x_1 and x_2 from the HJB equation. The stock trade, buying at $S_1 = e^{x_1}$ and selling at $S_2 = e^{x_2}$, is expected to be positive with the slippage cost. Therefore, it implies $e^{x_2}(1-K) > e^{x_1}(1+K)$, i.e.,

$$x_2 - x_1 > \log \frac{1+K}{1-K}.$$

In addition, $(\rho - \mathcal{M})v_2(x) = 0$ on $(-\infty, x_1)$. Since $v_1(x) = v_2(x) - e^x(1+K)$, and $(\rho - \mathcal{M})v_1(x) \geq 0$ on $(-\infty, x_1)$, we have $(\rho - \mathcal{M})e^x \leq 0$ on $(-\infty, x_1)$. Similarly, since $v_2(x) = e^x(1-K)$, and $(\rho - \mathcal{M})v_2(x) \geq 0$ on (x_2, ∞), we have $(\rho - \mathcal{M})e^x \geq 0$ on (x_2, ∞). These implies

$$x_1 \leq \frac{1}{a}\Big(\frac{\sigma^2}{2} + ab - \rho\Big) \leq x_2.$$

For the next step, we solve the solution $\rho v_i(x) - \mathcal{M}v_i(x) = 0$ with $i = 1, 2$. Let $\beta = \sqrt{2a}/\sigma$ and $\eta(t) = t^{\rho/a-1}\exp(-t^2/2)$. Then the general solution is given by

$$C_1 \int_0^\infty \eta(t)e^{-\beta(b-x)t}dt + C_2 \int_0^\infty \eta(t)e^{\beta(b-x)t}dt$$

for some constants C_1, C_2. (See [1] for the detail.) In view of the above lemma, $v_2(-\infty)$ should be bounded which implies $C_2 = 0$. Similarly, the bound of $v_1(\infty)$ implies $C_1 = 0$. Therefore, we have

$$\begin{aligned} v_1(x) &= C_2 \int_0^\infty \eta(t)e^{\beta(b-x)t}dt \\ v_2(x) &= C_1 \int_0^\infty \eta(t)e^{-\beta(b-x)t}dt. \end{aligned} \tag{6}$$

Note that both v_1 and v_2 are C^2 functions on (x_1, ∞) and $(-\infty, x_2)$, respectively. Therefore, we apply the smooth fit principle at x_1 and x_2 which requires

$$\begin{cases} v_1(x_1) = v_2(x_1) - e^{x_1}(1+K), \\ v_1'(x_1) = v_2'(x_1) - e^{x_1}(1+K), \\ v_2(x_2) = e^{x_2}(1-K), \\ v_2'(x_2) = e^{x_2}(1-K). \end{cases} \tag{7}$$

From the $v_2(x)$ of (7), we have $v_2(x_2) = v_2'(x_2)$. Combining $v_2(x)$ in (6), we get an equation for x_2 such that

$$\int_0^\infty \eta(t)e^{-\beta(b-x_2)t}dt = \int_0^\infty \eta(t)\beta t e^{-\beta(b-x_2)t}dt. \tag{8}$$

Moreover, it is easy to find C_1 by plugging x_2 into the $v_2(x)$ equation on (7) and finally we get $v_2(x)$ on $(-\infty, x_2)$.

After the $v_2(x)$ is determined from an explicit x_2 and C_1, we use it to solve $v_1(x)$. In particular, the combination of (6) and (7) gives the following:

$$\begin{cases} C_2 \displaystyle\int_0^\infty \eta(t)e^{\beta(b-x_1)t}dt = C_1 \int_0^\infty \eta(t)e^{-\beta(b-x_1)t}dt - e^{x_1}(1+K), \\ C_2 \displaystyle\int_0^\infty -\beta t\eta(t)e^{\beta(b-x_1)t}dt = C_1 \int_0^\infty \beta t\eta(t)e^{-\beta(b-x_1)t}dt - e^{x_1}(1+K). \end{cases} \tag{9}$$

Canceling C_2, we attain

$$\begin{aligned} &\left(\int_0^\infty \eta(t)e^{\beta(b-x_1)t}dt\right)^{-1}\left\{C_1\int_0^\infty \eta(t)e^{-\beta(b-x_1)t}dt - e^{x_1}(1+K)\right\} \\ &= \left(\int_0^\infty -\beta t\eta(t)e^{\beta(b-x_1)t}dt\right)^{-1}\left\{C_1\int_0^\infty \beta t\eta(t)e^{-\beta(b-x_1)t}dt - e^{x_1}(1+K)\right\}. \end{aligned} \tag{10}$$

where $C_1 = e^{x_2}(1+K)(\int_0^\infty \eta(t)e^{\beta(b-x_2)t}dt)^{-1}$. The solution x_1 of the above equation also gives C_2 by plugging x_1 into (7), and we have $v_1(x)$.

4. Verification Theorem

In this section, we give a verification theorem to show that the solutions $v_i(x)$ of (5) are equal to the value functions $V_i(x)$ for $i = 1, 2$ respectively, and the sequences of optimal stopping times are given by (x_1, x_2).

Theorem 4.1. *Let (x_1, x_2) be a solution to (8) and (10) satisfying*

$$x_1 \le \frac{1}{a}\left(\frac{\sigma^2}{2} + ab - \rho\right) \le x_2 \qquad \text{and} \qquad x_2 - x_1 > \log\left(\frac{1+K}{1-K}\right).$$

Let

$$\begin{cases} v_1(x) = \begin{cases} C_2 \displaystyle\int_0^\infty \eta(t)e^{\beta(b-x)t}dt, & \text{if } x \ge x_1, \\ C_1 \displaystyle\int_0^\infty \eta(t)e^{-\beta(b-x)t}dt - e^x(1+K), & \text{if } x < x_1, \end{cases} \\ v_2(x) = \begin{cases} C_1 \displaystyle\int_0^\infty \eta(t)e^{-\beta(b-x)t}dt, & \text{if } x < x_2, \\ e^x(1-K), & \text{if } x \ge x_2, \end{cases} \end{cases}$$

with C_1, C_2 which are determined by (x_1, x_2) as in (9). If the following inequalities

$$\begin{cases} v_1(x) \ge v_2 - e^x(1+K), \\ v_2(x) \ge e^x(1-K), \end{cases}$$

hold on the interval (x_1, x_2), then

$$v_i(x) = V_i(x)$$

for $i = 1, 2$. *Moreover, let*

$$\Lambda_1^* = \{\tau^*, \delta^*\}$$

where the stopping times $\tau^* = \inf\{t \geq 0 : X(t) \leq x_1\}$, $\delta^* = \inf\{t \geq \tau^* : X(t) \leq x_2\}$, *and*

$$\Lambda_2^* = \{\delta^*\}$$

where the stopping times $\delta^* = \inf\{t \geq 0 : X(t) \leq x_2\}$. *Then* Λ_1^* *and* Λ_2^* *are optimal.*

To prove the theorem, we need the following lemma.

Lemma 4.2. *Given* z_1 *and* z_2, *let* $\theta_1 = \inf\{t : X(t) \leq z_1\}$ *and* $\theta_2 = \inf\{t : X(t) \geq z_2\}$. *Then*

$$P(\theta_1 < \infty) = P(\theta_2 < \infty) = 1.$$

Proof. See the proof of Lemma 6 in [10]. □

Proof of the Theorem. We need two steps to prove. First, we show that $v_i(x) \geq J_i(x, \Lambda_i)$ for all Λ_i, $i = 1, 2$. As the second, we show that $v_i(x) = J_i(x, \Lambda_i^*)$. Therefore, $v_i(x) = V_i(x)$ and Λ_i^* is optimal.
It is easy to show that $v_1(x) \geq 0$ on (x_1, ∞) since $C_2 > 0$ in (6). Note that $v_1 \in C^2(\mathbb{R} \setminus \{x_1\})$ and $v_2 \in C^2(\mathbb{R} \setminus \{x_2\})$ satisfying

$$\begin{cases} \rho v_i(x) - \mathcal{M} v_i(x) \geq 0, \\ v_1(x) \geq v_2(x) - e^x(1+K)\}, \\ v_2(x) \geq e^x(1-K), \end{cases}$$

for all $x \in \mathbb{R}$ and,

$$\begin{cases} \rho v_1(x) - \mathcal{M} v_1(x) \geq 0, & \text{on } (x_1, \infty), \\ v_1(x) \geq v_2(x) - e^x(1+K)\}, & \text{on } (-\infty, x_1), \\ \rho v_2(x) - \mathcal{M} v_2(x) \geq 0, & \text{on } (-\infty, x_2), \\ v_2(x) \geq e^x(1-K), & \text{on } (x_2, \infty). \end{cases}$$

Using $\rho v_i(x) - \mathcal{M} v_i(x) \geq 0$, Dynkin's formula and Fatou's lemma as in Øksendal (2003, p. 226), for any stopping times $0 \leq \theta_1 \leq \theta_2$, we have

$$Ee^{-\rho\theta_1} v_i(X_{\theta_1}) \geq Ee^{-\rho\theta_2} v_i(X_{\theta_2}) \tag{11}$$

a.s., for $i = 1$ or 2. Given $\Lambda_1 = (\tau, \delta)$, we have

$$\begin{aligned} v_1(x) &\geq Ee^{-\rho\tau} v_1(X(\tau)) \\ &\geq Ee^{-\rho\tau}(v_2(X(\tau)) - S(\tau)(1+K)) \\ &= Ee^{-\rho\tau}(v_2(X(\tau)) - Ee^{-\rho\tau} S(\tau)(1+K) \\ &\geq Ee^{-\rho\delta}[S(\delta)(1-K)] - Ee^{-\rho\tau}[S(\tau)(1+K)] \\ &= E[e^{-\rho\delta} S(\delta)(1-K) - e^{-\rho\tau} S(\tau)(1+K)] \end{aligned}$$

from (2) and (11). We have $v_1(x) \geq J_1(x, \Lambda_1)$ which implies that $v_1(x) \geq V_1(x)$.

Similarly, we can show that $v_2(x) \geq V_2(x)$.
Next, we construct the equalities. Define a stopping time

$$\tau^* = \begin{cases} 0 & \text{if } x \leq x_1 \\ \inf\{t \geq 0 : X(t) = x_1\}, & \text{if } x > x_1 \end{cases}$$

Then, $\tau^* < \infty$, a.s. by Lemma 4.2. Using Dynkin's formula, we have

$$\begin{aligned} v_1(x) &= Ee^{-\rho\tau^*} v_1(X(\tau^*)) \\ &= Ee^{-\rho\tau^*}(v_2(X(\tau^*)) - S(\tau^*)(1+K)) \\ &= Ee^{-\rho\tau^*}(v_2(X(\tau^*)) - Ee^{-\rho\tau^*} S(\tau^*)(1+K). \end{aligned}$$

Let $\delta^* = \inf\{t \geq \tau^* : X(t) = x_2\}$. Then, $\delta^* < \infty$ a.s. by Lemma 4.2, and

$$\begin{aligned} Ee^{-\rho\tau^*} v_2(X(\tau^*)) &= Ee^{-\rho\delta^*} v_2(X(\delta^*)) \\ &= Ee^{-\rho\delta^*}(S(\delta^*)(1-K)) \end{aligned}$$

Therefore, it follows that

$$v_1(x) = E[e^{-\rho\delta^*} S(\delta^*)(1-K) - e^{-\rho\tau^*} S(\tau^*)(1+K)].$$

and it is positive by the second hypothesis of this theorem.
Similarly, we have

$$v_2(x) = E[e^{-\rho\delta^*} S(\delta^*)(1-K)].$$

This completes the proof. □

5. A Numerical Example

In this section, we demonstrate a numerical example with the following:

$$a = 0.8, \quad b = 2, \quad \sigma = 0.5, \quad \rho = 0.01, \quad K = 0.01$$

As the result of solving (7), we have $(x_1, x_2) = (1.5058, 2.2434)$. It implies that the low stock price $S_1 = \exp x_1 = 4.5078$ and the high price $S_2 = \exp x_2 = 9.4253$.

The probability or the frequency for the price to increase from S_1 to S_2 are closely connected to the price levels S_1 and S_2.
The corresponding value functions $V_1(x)$ and $V_2(x)$ are plotted in Figure 2. As we see in the picture, $V_1(x)$ is uniformly bounded and $V_2(x)$ is increasing with an exponential growth rate.

Now we vary each parameters at a time and observe the dependence of (x_1, x_2). First we vary the equilibrium parameter b to get the associated (x_1, x_2). Intuitively, larger b would result larger rewards and larger threshold pairs (x_1, x_2). Table 1 confirms this observation. For the monotonely increased b, the result gives the

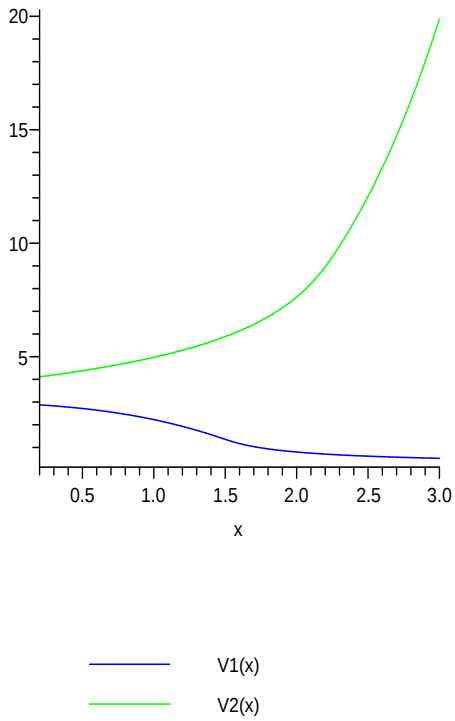

Figure 2. The value functions $V_1(x)$ and $V_2(x)$

b	1	1.5	2	2.5	3
x_1	0.5058	1.0058	1.5058	2.0058	2.5058
x_2	1.2434	1.7434	2.2434	2.7434	3.2434
$V_1(1)$	0.2935	0.8226	2.2266	4.4785	7.9976
$V_2(1)$	2.8066	3.5680	4.9721	7.2239	10.7431

Table 1. The thresholds (x_1, x_2) for varying b

monotonely increased (x_1, x_2). We added the values $(V_1(1), V_2(1))$ to observe the values of the associated value functions.

Next, we vary the rate of reversion a in Table 2. A larger a implies smaller difference between the threshold levels (x_1, x_2). Because the oscillating of the stock price tends to bounce more narrowly on a range for a larger a, it is natural that the gap between buying price and selling price is decreased.

In Table 3, we vary the volatility σ. Larger σ implies greater range for the stock price $S(t) = \exp(x(t))$. Intuitively larger volatility makes further gap between a high price of stock and a low price of stock, and it is showed this as well.

a	0.6	0.7	0.8	0.9	1
x_1	1.5015	1.5026	1.5058	1.5099	1.5143
x_2	2.3695	2.2991	2.2434	2.1982	2.1606
$V_1(1)$	2.6378	2.4044	2.2266	2.0860	1.9714
$V_2(1)$	5.3827	5.1498	4.9721	4.8314	4.7169

Table 2. (x_1, x_2) with varying a

σ	0.3	0.4	0.5	0.6	0.7
x_1	1.5531	1.5242	1.5058	1.5031	1.5177
x_2	1.9374	2.0848	2.2434	2.4134	2.5928
$V_1(1)$	1.3231	1.7414	2.2266	2.8003	3.4828
$V_2(1)$	4.0686	4.4869	4.9721	5.5475	6.2282

Table 3. (x_1, x_2) with varying σ

ρ	0.001	0.005	0.01	0.015	0.02
x_1	1.5080	1.5071	1.5058	1.5045	1.5032
x_2	2.2541	2.2494	2.2434	2.2375	2.2316
$V_1(1)$	2.2813	2.2569	2.2266	2.1966	2.1669
$V_2(1)$	5.0268	5.0024	4.9721	4.9421	4.9123

Table 4. (x_1, x_2) with varying ρ

K	0.001	0.005	0.01	0.015	0.02
x_1	1.5284	1.5183	1.5058	1.4936	1.4815
x_2	2.2434	2.2434	2.2434	2.2434	2.2434
$V_1(1)$	2.2963	2.2653	2.2266	2.1879	2.1492
$V_2(1)$	5.0173	4.9972	4.9721	4.9470	4.9218

Table 5. (x_1, x_2) with varying K

In Table 4, we vary the discount rate ρ. Large ρ implies smaller reward functions, and smaller (x_1, x_2) and Table 4 shows that.

As the last, we vary the slippage rate K in Table 5. The result shows that x_1 is monotonely decreasing in K and x_2 stays constantly. This is because lager K discourages stock translations and has to be returned by smaller x_1. Comparing with the result of the paper [10] under the same condition, we got the result $(x_1, x_2) = (1.1998, 1.6385)$ whose x_1 level is slightly less than the result in [10]. It's because our $V_1(x)$ doesn't associate with $V_1(x)$, and this makes $V_2(x)$ smaller, so even $V_1(x)$. Indeed, our $(V_1(1), V_2(1)) = (0.3846, 3.1303)$ which are both less than the values of $(V_1(1), V_2(1))$ in [10] whose value of $(V_1(1), V_2(1))$ was $(0.607, 3.364)$ under the

five variables condition $a = 0.8, b = 2, \sigma = 0.5, \rho = 0.5, K = 0.01$. In practice, the profit function having one trade is obviously smaller than the profit function having infinitive trades. In advance, studying about a problem having finite multiple trades, or applying this result to real data should be interesting subjects.

6. Conclusion

Through this paper, we solved quasi-algebraic equations for the two threshold levels. We showed that the two thresholds lead to the optimal stopping times to buy and sell a share.

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