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ON FUZZY S-WEAKLY r-M-CONTINUOUS FUNCTIONS ON FUZZY r-MINIMAL SPACES

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Abstract. We introduce the concept of fuzzy S-weakly r-M-continuous functions on fuzzy r-minimal spaces, and investigate some characterizations of such functions and the relations between the continuity and fuzzy r-minimal compactness on fuzzy r-minimal spaces.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh [14]. Chang [2] defined fuzzy topological spaces using fuzzy sets. In [3, 11], Chattopadhyay, Hazra and Samanta introduced a smooth topological space which is a generalization of fuzzy topological space. In [12], Yoo et al. introduced the concept of fuzzy r-minimal space which is an extension of the smooth topological space. The concepts of fuzzy r-open sets, fuzzy r-semiopen sets, fuzzy r-preopen sets, fuzzy r-β-open sets and fuzzy r-regular open sets were introduced in [1, 4, 5, 6], which are various kinds of fuzzy r-minimal structures. The concept of fuzzy r-M-continuity was also introduced and investigated in [12]. In [9], we introduced and studied the concept of fuzzy weak r-M-continuity. In this paper, we introduce the concept of fuzzy S-weakly r-M-continuous function, which is a generalization of fuzzy r-M-continuous function. We investigate some characterizations of such continuity, and show that the relationships between fuzzy S-weakly r-M-continuity and fuzzy r-M-continuity. In particular, we obtain Theorem 3.11: Let $f : X \to Y$ be a fuzzy S-weakly r-M-continuous function between r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. If A is a fuzzy r-minimal semicompact set in X and if $\mathcal{M}_Y$ has the property $(\mathcal{U})$, then $f(A)$ is almost fuzzy r-minimal compact.

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2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member A of I^X is called a fuzzy set of X. By $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}}$ we denote constant maps on X with value 0 and 1, respectively. For any $A \in I^X$, A^c denotes the complement $\tilde{\mathbf{1}} - A$. All other notations are standard notations of fuzzy set theory.

A *fuzzy point* x_α in X is a fuzzy set x_α defined as follows

$$x_\alpha(y) = \begin{cases} \alpha \text{ if } y = x, \\ 0 \text{ if } y \neq x. \end{cases}$$

A fuzzy point x_α is said to belong to a fuzzy set A in X, denoted by $x_\alpha \in A$, if $\alpha \leq A(x)$ for $x \in X$.

A fuzzy set A in X is the union of all fuzzy points which belong to A.

Let $f : X \to Y$ be a function and $A \in I^X$ and $B \in I^Y$. Then $f(A)$ is a fuzzy set in Y, defined by

$$f(A)(y) = \begin{cases} \sup\limits_{z \in f^{-1}(y)} A(z), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise}, \end{cases}$$

for $y \in Y$ and $f^{-1}(B)$ is a fuzzy set in X, defined by $f^{-1}(B)(x) = B(f(x)), x \in X$.

A *smooth topology* [11] on X is a map $\mathcal{T} : I^X \to I$ which satisfies the following properties:

(1) $\mathcal{T}(\tilde{\mathbf{0}}) = \mathcal{T}(\tilde{\mathbf{1}}) = 1$.
(2) $\mathcal{T}(A_1 \wedge A_2) \geq \mathcal{T}(A_1) \wedge \mathcal{T}(A_2)$.
(3) $\mathcal{T}(\vee A_i) \geq \wedge \mathcal{T}(A_i)$.

The pair $(X, \mathcal{T})$ is called a *smooth topological space*.

Definition 2.1 ([12])**.** *Let X be a nonempty set and $r \in (0,1] = I_0$. A fuzzy family $\mathcal{M} : I^X \to I$ on X is said to have a fuzzy r-minimal structure if the family*

$$\mathcal{M}_r = \{A \in I^X \mid \mathcal{M}(A) \geq r\}$$

contains $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}}$.

Then the $(X, \mathcal{M})$ is called a fuzzy r-minimal space (simply r-FMS) if $\mathcal{M}$ has a fuzzy r-minimal structure. Every member of $\mathcal{M}_r$ is called a fuzzy r-minimal open set. A fuzzy set A is called a fuzzy r-minimal closed set if the complement of A is fuzzy r-minimal open.

Let $(X, \mathcal{M})$ be an r-FMS and $r \in I_0$. The fuzzy r-minimal closure and the fuzzy r-minimal interior of A [12], denoted by $mC(A,r)$ and $mI(A,r)$, respectively, are defined as

$$mC(A,r) = \cap\{B \in I^X : B^c \in \mathcal{M}_r \text{ and } A \subseteq B\},$$
$$mI(A,r) = \cup\{B \in I^X : B \in \mathcal{M}_r \text{ and } B \subseteq A\}.$$

Theorem 2.1 ([12]). *Let $(X, \mathcal{M})$ be an r-FMS and A, B in I^X.*

(1) $mI(A, r) \subseteq A$ and if A is a fuzzy r-minimal open set, then $mI(A, r) = A$.

(2) $A \subseteq mC(A, r)$ and if A is a fuzzy r-minimal closed set, then $mC(A, r) = A$.

(3) If $A \subseteq B$, then $mI(A, r) \subseteq mI(B, r)$ and $mC(A, r) \subseteq mC(B, r)$.

(4) $mI(A, r) \cap mI(B, r) \supseteq mI(A \cap B, r)$ and $mC(A, r) \cup mC(B, r) \subseteq mC(A \cup B, r)$.

(5) $mI(mI(A, r), r) = mI(A, r)$ and $mC(mC(A, r), r) = mC(A, r)$.

(6) $\tilde{\mathbf{1}} - mC(A, r) = mI(\tilde{\mathbf{1}} - A, r)$ and $\tilde{\mathbf{1}} - mI(A, r) = mC(\tilde{\mathbf{1}} - A, r)$.

Let $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$ be two r-FMS's. Then a function $f : X \to Y$ is said to be

(1) *fuzzy r-M-continuous* [12] if for every fuzzy r-minimal open set A in Y, $f^{-1}(A)$ is fuzzy r-minimal open in X,

(2) *fuzzy weakly r-M-continuous* [9] if for fuzzy point x_α in X and each fuzzy r-minimal open set V containing $f(x_\alpha)$, there is a fuzzy r-minimal open set U containing x_α such that $f(U) \subseteq mC(V, r)$.

Theorem 2.2 ([9]). *Let $f : X \to Y$ be a function between r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then the following statements are equivalent:*

(1) f is fuzzy weakly r-M-continuous.

(2) $f^{-1}(V) \subseteq mI(f^{-1}(mC(V, r)), r)$ for each fuzzy r-minimal open set V in Y.

(3) $mC(f^{-1}(mI(B, r)), r) \subseteq f^{-1}(B)$ for each fuzzy r-minimal closed set B in Y.

(4) $mC(f^{-1}(V), r) \subseteq f^{-1}(mC(V, r))$ for each fuzzy r-minimal open set V in Y.

3. Main Results

Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then a fuzzy set A is said to be *fuzzy r-minimal semiopen* [9] if $A \subseteq mC(mI(A, r), r)$. We showed that any union of fuzzy r-minimal semiopen sets is fuzzy r-minimal semiopen [9].

For $A \in I^X$, $msC(A, r)$ and $msI(A, r)$, respectively, are defined as the following:

$$msC(A, r) = \cap\{F \in I^X : A \subseteq F, \quad F \text{ is fuzzy } r\text{-minimal semiclosed}\}$$

$$msI(A, r) = \cup\{U \in I^X : U \subseteq A, U \text{ is fuzzy } r\text{-minimal semiopen }\}.$$

Definition 3.1. *Let $f : X \to Y$ be a function between r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then f is said to be fuzzy S-weakly r-M-continuous if for each fuzzy r-minimal open set A of Y, $f^{-1}(A) \subseteq msI(f^{-1}(mC(A, r)), r)$.*

Remark 3.1. *Every fuzzy weakly r-M-continuous function is fuzzy S-weakly r-M-continuous but the converse is not always true as shown in the next example.*

fuzzy r-M-continuous $\Rightarrow$ *fuzzy weakly r-M-continuous* $\Rightarrow$ *fuzzy S-weakly r-M-continuous*

Example 3.1. *Let $X = I$, and let A and B be two fuzzy sets in X defined as*

$$A(x) = -\frac{1}{3}x + \frac{2}{3} \text{ for } x \in I,$$

$$B(x) = \frac{1}{2}x \text{ for } x \in I.$$

Define a fuzzy family $\mathcal{M} : I^X \to I$ by

$$\mathcal{M}(\sigma) = \begin{cases} \frac{1}{2} & \text{if } \sigma = \tilde{\mathbf{0}}, \tilde{\mathbf{1}}, \\ \frac{1}{2} & \text{if } \sigma = A, \\ 0 & \text{otherwise }; \end{cases}$$

and a fuzzy family $\mathcal{N} : I^X \to I$ by

$$\mathcal{N}(\sigma) = \begin{cases} \frac{2}{3} & \text{if } \sigma = \tilde{\mathbf{0}}, \tilde{\mathbf{1}}, \\ \frac{2}{3} & \text{if } \sigma = B, \\ 0 & \text{otherwise.} \end{cases}$$

Consider the identity function $f : (X, \mathcal{M}) \to (X, \mathcal{N})$. Note that:

$$mI(f^{-1}(mC(B, \frac{1}{2})), \frac{1}{2}) = mI(f^{-1}(\tilde{\mathbf{1}} - B), \frac{1}{2}) = A;$$

$$msI(f^{-1}(mC(B, \frac{1}{2})), \frac{1}{2}) = msI(f^{-1}(\tilde{\mathbf{1}} - B), \frac{1}{2}) = \tilde{\mathbf{1}} - B.$$

Clearly f is a fuzzy S-weakly $\frac{1}{2}$-M-continuous function but it is not a fuzzy weakly $\frac{1}{2}$-M-continuous function by Theorem 2.2.

Theorem 3.2. *Let $f : X \to Y$ be a function between r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then f is fuzzy S-weakly r-M-continuous if and only if for every fuzzy point x_α and each fuzzy r-minimal open set V containing $f(x_\alpha)$, there exists a fuzzy r-minimal semiopen set U containing x_α such that $f(U) \subseteq mC(V, r)$.*

Proof. If f is a fuzzy S-weakly r-M-continuous function, then for each fuzzy point x_α in X and each fuzzy r-minimal open set V containing $f(x_\alpha)$, we have $x_\alpha \in f^{-1}(V) \subseteq msI(f^{-1}(mC(V, r)), r)$. Put $U = msI(f^{-1}(mC(V, r)), r)$. Then U is a fuzzy r-minimal semiopen set such that $x_\alpha \in U \subseteq f^{-1}(mC(V, r))$. Thus $f(U) \subseteq mC(V, r)$.

For the converse, let V be a fuzzy r-minimal open set in Y. By hypothesis, for each $x_\alpha \in f^{-1}(V)$, there exists a fuzzy r-minimal semiopen set U containing x_α such that $f(U) \subseteq mC(V, r)$. So $\cup\{U : x_\alpha \in f^{-1}(V)\} \subseteq f^{-1}(mC(V, r))$ for a fuzzy r-minimal semiopen set U containing x_α. Since $\cup\{U : x_\alpha \in f^{-1}(V)\}$ is fuzzy r-minimal semiopen, we have $f^{-1}(V) \subseteq msI(f^{-1}(mC(V, r)), r)$. Hence f is fuzzy S-weakly r-M-continuous. □

Theorem 3.3. *Let $f : X \to Y$ be a function between r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then the following are equivalent:*

(1) f is fuzzy S-weakly r-M-continuous.

(2) $msC(f^{-1}(mI(F, r)), r) \subseteq f^{-1}(F)$ for each fuzzy r-minimal closed set F in Y.

Proof. (1) ⇒ (2) Let F be a fuzzy r-minimal closed subset of Y. Then since $\tilde{\mathbf{1}} - F$ is a fuzzy r-minimal open set in Y,

$$\begin{aligned} f^{-1}(\tilde{\mathbf{1}} - F) &\subseteq msI(f^{-1}(mC(\tilde{\mathbf{1}} - F, r)), r) \\ &= msI(f^{-1}(\tilde{\mathbf{1}} - mI(F, r)), r) \\ &= msI(\tilde{\mathbf{1}} - f^{-1}(mI(F, r)), r) \\ &= \tilde{\mathbf{1}} - msC(f^{-1}(mI(F, r)), r). \end{aligned}$$

Hence $msC(f^{-1}(mI(F, r)), r) \subseteq f^{-1}(F)$.

Similarly, it is proved that (2) ⇒ (1). □

Let X be a nonempty set and $\mathcal{M} : I^X \to I$ a fuzzy family on X. The fuzzy family $\mathcal{M}$ is said to have the property $(\mathcal{U})$ [12] if for $A_i \in \mathcal{M}$ $(i \in J)$,

$$\mathcal{M}(\cup A_i) \geq \wedge \mathcal{M}(A_i).$$

Theorem 3.4 ([12])**.** *Let $(X, \mathcal{M})$ be an r-FMS with the property $(\mathcal{U})$ and $A \in I^X$. Then*

(1) A is fuzzy r-minimal open if and only if $mI(A, r) = A$.

(2) A is fuzzy r-minimal closed if and only if $mC(A, r) = A$.

Theorem 3.5. *Let $f : X \to Y$ be a function between r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. If $\mathcal{M}_Y$ has the property $(\mathcal{U})$, then the following are equivalent:*

(1) f is fuzzy S-weakly r-M-continuous.

(2) $msC(f^{-1}(mI(mC(B, r), r)), r) \subseteq f^{-1}(mC(B, r))$ for every fuzzy set B in Y.

(3) $f^{-1}(mI(B, r)) \subseteq msI(f^{-1}(mC(mI(B, r), r)), r)$ for every fuzzy set B in Y.

(4) $msC(f^{-1}(V), r) \subseteq f^{-1}(mC(V, r))$ for every fuzzy r-minimal open set V in Y.

Proof. (1) ⇒ (2) For $B \in I^Y$, from the property $(\mathcal{U})$ and Theorem 3.4, we have that $mC(B, r)$ is a fuzzy r-minimal closed set in Y. By Theorem 3.3, we easily obtain $msC(f^{-1}(mI(mC(B, r), r)) \subseteq f^{-1}(mC(B, r))$.

(2) ⇒ (3) For $B \in I^Y$,

$$\begin{aligned} f^{-1}(mI(B, r)) &= \tilde{\mathbf{1}} - (f^{-1}(mC(\tilde{\mathbf{1}} - B, r))) \\ &\subseteq \tilde{\mathbf{1}} - msC(f^{-1}(mI(mC(\tilde{\mathbf{1}} - B, r), r)), r) \end{aligned}$$

$$= msI(f^{-1}(mC(mI(B,r),r)),r).$$

Hence,$f^{-1}(mI(B,r)) \subseteq msI(f^{-1}(mC(mI(B,r),r)),r)$.

(3) $\Rightarrow$ (4) Let V be any fuzzy r-minimal open set of Y. Then from $V \subseteq mI(mC(V,r),r)$, it follows

$$\begin{aligned}\tilde{\mathbf{1}} - f^{-1}(mC(V,r)) &= f^{-1}(mI(\tilde{\mathbf{1}} - V,r)) \\ &\subseteq msI(f^{-1}(mC(mI(\tilde{\mathbf{1}} - V,r),r)),r) \\ &= msI(\tilde{\mathbf{1}} - (f^{-1}(mI(mC(V,r),r))),r) \\ &= \tilde{\mathbf{1}} - msC(f^{-1}(mI(mC(V,r),r)),r) \\ &\subseteq \tilde{\mathbf{1}} - msC(f^{-1}(V),r).\end{aligned}$$

Hence we have $msC(f^{-1}(V),r) \subseteq f^{-1}(mC(V,r))$.

(4)$\Rightarrow$ (1) Let V be a fuzzy r-minimal open set in Y. By Theorem 3.4, $\tilde{\mathbf{1}} - mC(V,r)$ is fuzzy r-minimal open. So from $V \subseteq mI(mC(V,r),r)$ and (4), it follows

$$\begin{aligned}f^{-1}(V) &\subseteq f^{-1}(mI(mC(V,r),r)) \\ &= \tilde{\mathbf{1}} - f^{-1}(mC(\tilde{\mathbf{1}} - mC(V,r),r)) \\ &\subseteq \tilde{\mathbf{1}} - msC(f^{-1}(\tilde{\mathbf{1}} - mC(V,r)),r) \\ &= msI(f^{-1}(mC(V,r)),r).\end{aligned}$$

Hence f is fuzzy S-weakly r-M-continuous.

□

Definition 3.2 ([10])**.** *Let $(X,\mathcal{M})$ be an r-FMS and $A \in I^X$. Then a fuzzy set A is said to be*

(1) fuzzy r-minimal preopen if $A \subseteq mI(mC(A,r),r)$;

(2) fuzzy r-minimal regular open (resp., fuzzy r-minimal regular closed if $A = mI(mC(A,r),r)$ (resp., $A = mC(mI(A,r),r)$).

Theorem 3.6. *Let $f : X \to Y$ be a function between r-FMS's $(X,\mathcal{M}_X)$ and $(Y,\mathcal{M}_Y)$. If $\mathcal{M}_Y$ has the property $(\mathcal{U})$, then the following are equivalent:*

(1) f is fuzzy S-weakly r-M-continuous.

(2) $msC(f^{-1}(mI(mC(G,r),r)),r) \subseteq f^{-1}(mC(G,r))$ for each fuzzy r-minimal open set G in Y.

(3) $msC(f^{-1}(mI(mC(V,r),r)),r) \subseteq f^{-1}(mC(V,r))$ for each fuzzy r-minimal preopen set V in Y.

(4) $msC(f^{-1}(mI(K,r)),r) \subseteq f^{-1}(K)$ for each fuzzy r-minimal regular closed set K in Y.

(5) $msC(f^{-1}(mI(mC(G,r),r)),r) \subseteq f^{-1}(mC(G,r))$ for each fuzzy r-minimal semiopen set G in Y.

Proof. (1) $\Rightarrow$ (2) Let G be a fuzzy r-minimal open set of Y; then by Theorem 3.5 (2), $msC(f^{-1}(mI(mC(G,r),r)),r) \subseteq f^{-1}(mC(G,r))$.

(2) $\Rightarrow$ (3) Let $V \subseteq Y$ be fuzzy r-minimal preopen. Then $V \subseteq mI(mC(V,r),r)$. Set $A = mI(mC(V,r),r)$. Then by Theorem 3.6, A is a fuzzy r-minimal open set, and so $msC(f^{-1}(mI(mC(A,r),r)),r,r) \subseteq f^{-1}(mC(A,r))$. From $mC(A,r) = mC(V,r)$, it follows

$$msC(f^{-1}(mI(mC(V,r),r)),r) \subseteq f^{-1}(mC(V,r)).$$

(3)$\Rightarrow$ (4) Let K be a fuzzy r-minimal regular closed set of Y. Since $mI(K,r)$ is a fuzzy r-minimal preopen set,

$$msC(f^{-1}(mI(mC(mI(K,r),r),r)),r) \subseteq f^{-1}(mC(mI(K,r),r)).$$

From $mI(K,r) = mI(mC(mI(K,r),r),r)$ and $K = mC(mI(K,r),r)$, it follows $msC(f^{-1}(mI(K,r)),r) \subseteq f^{-1}(K)$.

(4) $\Rightarrow$ (5) For any fuzzy r-minimal semiopen set G, we know that $G \subseteq mC(mI(mC(G,r),r),r) \subseteq mC(G,r)$, and so $mC(G,r)$ is fuzzy r-minimal regular closed. Hence $msC(f^{-1}(mI(mC(G,r),r)),r) \subseteq f^{-1}(mC(G,r))$.

(5) $\Rightarrow$ (1) Let V be a fuzzy r-minimal open set. Then since V is also a fuzzy r-minimal semiopen set, from $V \subseteq mI(mC(V,r),r)$ and (5), we have

$$\begin{aligned} msC(f^{-1}(V),r) &\subseteq msC(f^{-1}(mI(mC(V,r),r)),r) \\ &\subseteq f^{-1}(mC(V,r)). \end{aligned}$$

Hence, by Theorem 3.5 (4), f is fuzzy S-weakly r-M-continuous. □

We recall that the following notions introduced in [8, 13]: Let $(X, \mathcal{M})$ be an r-FMS and $\mathcal{A} = \{A_i \in I^X : i \in J\}$. $\mathcal{A}$ is called a *fuzzy r-minimal cover* if $\cup\{A_i : i \in J\} = \tilde{\mathbf{1}}$. It is a *fuzzy r-minimal open(semiopen) cover* if each A_i is a fuzzy r-minimal open(semiopen) set. A subcover of a fuzzy r-minimal cover $\mathcal{A}$ is a subfamily of it which also is a fuzzy r-minimal cover. A fuzzy set A in X is said to be *fuzzy r-minimal semicompact* (resp., *almost fuzzy r-minimal compact*) if every fuzzy r-minimal semiopen (resp., open) cover $\mathcal{A} = \{A_i \in \mathcal{M}_r : i \in J\}$ of A, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{k \in J_0} A_k$ (resp., $A \subseteq \cup_{k \in J_0} mC(A_k, r)$).

Lemma 3.7 ([Theorem 3.8 of [7]]). *Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then*
(1) A is fuzzy r-minimal semiopen iff $msI(A,r) = A$.
(2) F is fuzzy r-minimal semiclosed iff $msC(F,r) = F$.
(3) $msI(msI(A,r),r) = msI(A,r)$ and $msC(msC(A,r),r) = msC(A,r)$.

Theorem 3.8. *Let $f : X \to Y$ be a fuzzy weakly r-M-continuous function between r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. If A is a fuzzy r-minimal semicompact set in X and if $\mathcal{M}_Y$ has the property $(\mathcal{U})$, then $f(A)$ is almost fuzzy r-minimal compact.*

Proof. Let $\{B_i \in I^Y : i \in J\}$ be a fuzzy r-minimal open cover of $f(A)$ in Y. Then since f is fuzzy S-weakly r-M-continuous, from Theorem 3.7 (3) and $B_i = mI(B_i)$, we have $f^{-1}(B_i) \subseteq msI(f^{-1}(mC(B_i,r)),r)$ for each $i \in J$.

So $\{msI(f^{-1}(mC(B_i))) : i \in J\}$ is a fuzzy r-minimal semiopen cover of A in X. Since A is fuzzy r-minimal semicompact, there exists a finite subset $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that

$$A \subseteq \cup_{k \in J_0} msI(f^{-1}(mC(B_k, r)), r) \subseteq f^{-1}(mC(B_k, r)).$$

This implies that $f(A) \subseteq \cup_{k \in J_0} mC(B_k, r)$. Hence $f(A)$ is almost fuzzy r-minimal compact. □

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