J. Appl. Math. & Informatics Vol. **29**(2011), No. 3 - 4, pp. 831 - 846
Website: http://www.kcam.biz

APPLICATION OF LINEAR PROGRAMMING FOR SOLVING FUZZY TRANSPORTATION PROBLEMS†

AMIT KUMAR* AND AMARPREET KAUR

Abstract. There are several methods, in the literature, for finding the fuzzy optimal solution of fully fuzzy transportation problems (transportation problems in which all the parameters are represented by fuzzy numbers). In this paper, the shortcomings of some existing methods are pointed out and to overcome these shortcomings, a new method (based on fuzzy linear programming formulation) is proposed to find the fuzzy optimal solution of unbalanced fuzzy transportation problems with a new representation of trapezoidal fuzzy numbers. The advantages of the proposed method over existing method are discussed. Also, it is shown that it is better to use the proposed representation of trapezoidal fuzzy numbers instead of existing representation of trapezoidal fuzzy numbers for finding the fuzzy optimal solution of fuzzy transportation problems. To illustrate the proposed method a fuzzy transportation problem (FTP) is solved by using the proposed method and the obtained results are discussed. The proposed method is easy to understand and to apply for finding the fuzzy optimal solution of fuzzy transportation problems occurring in real life situations.

AMS Mathematics Subject Classification : 90C05, 90C08, 90C70.
Key words and phrases : Fuzzy sets, Linear programming, Ranking function, Trapezoidal fuzzy number, JMD trapezoidal fuzzy number.

1. Introduction

In today's highly competitive market, the pressure on organizations to find better ways to create and deliver value to customers becomes stronger. How and when to send the products to the customers in the quantities, they want in a cost-effective manner, become more challenging. Transportation models provide a powerful framework to meet this challenge. They ensure the efficient movement and timely availability of raw materials and finished goods.

Received June 30, 2010. Revised July 30, 2010. Accepted August 18, 2010. *Corresponding author. †This work was supported by the research grant of the University.

The basic transportation problem was originally developed by Hitchcock [7]. In conventional transportation problems it is assumed that decision maker is sure about the precise values of transportation cost, availability and demand of the product. In real world applications, all the parameters of the transportation problems may not be known precisely due to uncontrollable factors. This type of imprecise data is not always well represented by random variable selected from a probability distribution. Fuzzy numbers introduced by Zadeh [13] may represent this data. So, fuzzy decision making method is needed here.

Zimmermann [14] showed that solutions obtained by fuzzy linear programming are always efficient. Subsequently, Zimmermann's fuzzy linear programming has developed into several fuzzy optimization methods for solving the transportation problems. Oheigeartaigh [10] proposed an algorithm for solving transportation problems where the capacities and requirements are fuzzy sets with linear or triangular membership functions. Chanas et al. [1] presented a fuzzy linear programming model for solving transportation problems with crisp cost coefficients and fuzzy supply and demand values. Chanas and Kuchta [2] proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers, and developed an algorithm for obtaining the optimal solution. Saad and Abbas [12] discussed the solution algorithm for solving the transportation problem in fuzzy environment.

Liu and Kao [9] described a method for solving fuzzy transportation problems based on extension principle. Gani and Razak [5] presented a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. To deal with uncertainties of supply and demand parameters, Gupta and Mehlawat [6] transformed the past data pertaining to the amount of supply of the i^{th} supply point and the amount of demand of the j^{th} demand point using level (λ, ρ) interval-valued fuzzy numbers.

Dinagar and Palanivel [3] investigated fuzzy transportation problem, with the aid of trapezoidal fuzzy numbers and proposed fuzzy modified distribution method to find the optimal solution in terms of fuzzy numbers. Pandian and Natarajan [11] proposed a new algorithm namely, fuzzy zero point method for finding a fuzzy optimal solution for a fuzzy transportation problems, where the transportation cost, supply and demand are represented by trapezoidal fuzzy numbers.

In this paper, a new method (based on fuzzy linear programming formulation) is proposed to find the fuzzy optimal solution of unbalanced fuzzy transportation problems with a new representation of trapezoidal fuzzy numbers. The advantages of the proposed method over existing method are discussed. Also, it is shown that it is better to use the proposed representation of trapezoidal fuzzy numbers instead of existing representation of trapezoidal fuzzy numbers for finding the fuzzy optimal solution of fuzzy transportation problems. To illustrate the proposed method a fuzzy transportation problem (FTP) is solved by using the proposed method and the obtained results are discussed. The proposed method is easy to understand and to apply for finding the fuzzy optimal solution of fuzzy transportation problems occurring in real life situations.

This paper is organized as follows: In Section 2, some basic definitions and arithmetic operations are reviewed. In Section 3, formulation of FTP is presented and the shortcomings of the existing methods are pointed out. Also application of ranking function for solving FTP is presented. In Section 4, a new method is proposed to find fuzzy optimal solution of fuzzy transportation problems. In Section 5, a new representation of trapezoidal fuzzy numbers, named as JMD representation of trapezoidal fuzzy numbers, is proposed and the method, proposed in Section 4, are modified with JMD representation of trapezoidal fuzzy numbers. Also, it is shown that it is better to use the JMD representation of trapezoidal fuzzy numbers instead of existing representation of trapezoidal fuzzy numbers for finding the fuzzy optimal solution of fuzzy transportation problems. The advantages of the proposed method are discussed in Section 6. In Section 7, to illustrate the proposed method a numerical example is solved and the obtained results are discussed. The conclusions are discussed in Section 8.

2. Preliminaries

In this section some basic definitions, arithmetic operations and ranking function are reviewed.

2.1. Basic definitions.

In this section, some basic definitions are reviewed.

Definition 1 ([4]). *A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:*
(i) $\tilde{A}$ is convex i.e., $\mu_{\tilde{A}}(\lambda x_1+(1-\lambda)x_2) \geq$ minimum $(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))\ \forall\, x_1, x_2 \in R,\ \forall\, \lambda \in [0,1]$.
(ii) $\tilde{A}$ is normal i.e., $\exists\, x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$.
(iii) $\mu_{\tilde{A}}$ is piecewise continuous.

Definition 2 ([4]). *A fuzzy number $\tilde{A}$ is said to be non-negative fuzzy number if and only if $\mu_{\tilde{A}}(x) = 0,\ \forall\, x < 0$.*

Definition 3 ([4]). *A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be a trapezoidal fuzzy number if its membership function is given by*

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \frac{m-x}{\alpha}, & m-\alpha \leq x \leq m \\ 1\ , & m \leq x \leq n \\ 1 - \frac{x-n}{\beta}, & n \leq x \leq n+\beta \end{cases}\ , \text{ where } n-m \geq 0,\ \alpha \geq 0,\ \beta \geq 0.$$

Definition 4 ([4]). *A trapezoidal fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be non-negative trapezoidal fuzzy number if and only if $m - \alpha \geq 0$*

Definition 5 ([4]). *A trapezoidal fuzzy number* $\tilde{A} = (m, n, \alpha, \beta)$ *is said to be zero trapezoidal fuzzy number if and only if* $m = 0, n = 0, \alpha = 0, \beta = 0$.

Definition 6 ([4]). *Two trapezoidal fuzzy numbers* $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)$ *and* $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)$ *are said to be equal i.e.,* $\tilde{A}_1 = \tilde{A}_2$ *if and only if* $m_1 = m_2, n_1 = n_2, \alpha_1 = \alpha_2, \beta_1 = \beta_2$.

Definition 7 ([8]). *A ranking function is a function* $\Re : F(R) \to R$*, where* $F(R)$ *is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line.*
Let $\tilde{A} = (m, n, \alpha, \beta)$ *be a trapezoidal fuzzy number then* $\Re(\tilde{A}) = \frac{1}{2}(m + n) + \frac{1}{4}(\beta - \alpha)$.

2.2. Arithmetic operations. In this section addition and multiplication operations of trapezoidal fuzzy numbers are reviewed [4]:
Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)$ be two trapezoidal fuzzy numbers then
(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)$
(ii) $\tilde{A}_1 \otimes \tilde{A}_2 \simeq (m', n', \alpha', \beta')$,
where m' =minimum$(m_1m_2, m_1n_2, n_1m_2, n_1n_2)$,
n' =maximum$(m_1m_2, m_1n_2, n_1m_2, n_1n_2)$,
$\alpha' = m' -$ minimum $((m_1 - \alpha_1)(m_2 - \alpha_2), (m_1 - \alpha_1)(n_2 + \beta_2), (n_1 + \beta_1)$
$(m_2 - \alpha_2), (n_1 + \beta_1)(n_2 + \beta_2))$,
$\beta' =$ maximum $((m_1 - \alpha_1)(m_2 - \alpha_2), (m_1 - \alpha_1)(n_2 + \beta_2), (n_1 + \beta_1)$
$(m_2 - \alpha_2), (n_1 + \beta_1)(n_2 + \beta_2)) - n'$

3. Fuzzy transportation problem

In conventional transportation problems it is assumed that decision maker is sure about the precise values of transportation cost, availability and demand of the product. In real world applications, all these parameters of the transportation problems may not be known precisely due to uncontrollable factors. To deal with such situations, fuzzy set theory is applied in literature to solve the transportation problems.

Several authors [3,9,11] have proposed different methods for solving balanced fuzzy transportation problems by representing the transportation cost, availability and demand as normal fuzzy numbers. The balanced fuzzy transportation problems, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand may be formulated as follows:

$$\text{Minimize} \sum_{i=1}^{p} \sum_{j=1}^{q} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$$

$$\text{subject to} \sum_{j=1}^{q} \tilde{x}_{ij} = \tilde{a}_i, \ \ \sum_{i=1}^{p} \tilde{x}_{ij} = \tilde{b}_j, \ \ \sum_{i=1}^{p} \tilde{a}_i = \sum_{j=1}^{q} \tilde{b}_j, \ \ i, j = 1, 2, 3, ..., p$$

$\tilde{x}_{ij}$ is a non-negative trapezoidal fuzzy number, where
p = total number of sources ; q = total number of destinations
$\tilde{a}_i$ = the fuzzy availability of the product at i^{th} source
$\tilde{b}_j$ = the fuzzy demand of the product at j^{th} destination
$\tilde{c}_{ij}$ = the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination
$\tilde{x}_{ij}$ = the fuzzy quantity of the product that should be transported from i^{th} source to j^{th} destination (or fuzzy decision variables) to minimize the total fuzzy transportation cost
$\sum_{i=1}^{p} \tilde{a}_i$ = total fuzzy availability of the product
$\sum_{j=1}^{q} \tilde{b}_j$ = total fuzzy demand of the product
$\sum_{i=1}^{p} \sum_{j=1}^{q} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$ = total fuzzy transportation cost

Remark 1. *If* $\sum_{i=1}^{p} \tilde{a}_i = \sum_{j=1}^{q} \tilde{b}_j$ *then the FTP is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem.*

3.1. Shortcomings of the existing methods. There are several methods to find the optimal solution of fuzzy transportation problems. But, there are shortcomings in some existing methods which are as follows:
(i) In the existing methods [1,9,10], fuzzy transportation problems are first converted into equivalent crisp transportation problem (CTP) using α-cut method, which are then solved by standard methods. The final results of a FTP are thus real numbers, which represents a compromise in terms of fuzzy numbers.
(ii) Using the existing methods [1,9,10], it is not possible to convert an unbalanced fuzzy transportation problem into balanced fuzzy transportation problem without using α-cut technique.
(iii) It is very difficult to apply existing methods for finding the fuzzy optimal solution of fuzzy transportation problems.
(iv) Dinagar and Palanivel [3] proposed a method for solving fuzzy transportation problems and by applying the proposed method for solving a FTP they obtained the following fuzzy optimal solution:
(a) The fuzzy optimal quantity of the product that should be transported, from second source to third destination, third source to first destination and third source to third destination are trapezoidal fuzzy numbers $(-5, -1, 6, 12)$, $(-5, -1, 3, 7)$, and $(-11, -3, 6, 12)$ respectively.
(b) The minimum total fuzzy transportation cost is $(-122, -2, 139, 257)$.
It is clear that there exist negative part in all the obtained trapezoidal fuzzy numbers, which depicts that quantity of the product and transportation cost may be negative. But the negative quantity of the product and negative transportation cost has no physical meaning.

Similarly the results obtained by using the existing methods [11] has no physical meaning.

3.2. Application of ranking function for solving fuzzy transportation problems. The fuzzy optimal solution of the balanced fuzzy transportation problem:

$$\text{Minimize} \sum_{i=1}^{p}\sum_{j=1}^{q} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$$

$$\text{subject to} \sum_{j=1}^{q} \tilde{x}_{ij} = \tilde{a}_i, \ \ \sum_{i=1}^{p} \tilde{x}_{ij} = \tilde{b}_j, \ \ \sum_{i=1}^{p} \tilde{a}_i = \sum_{j=1}^{q} \tilde{b}_j, \ \ i, j = 1, 2, 3, ..., p$$

$\tilde{x}_{ij}$ is a non-negative trapezoidal fuzzy number is a fuzzy number $\tilde{x}_{ij}$ which satisfies the following characteristics:
(i) $\tilde{x}_{ij}$ is a non-negative fuzzy number
(ii) $\sum_{j=1}^{q} \tilde{x}_{ij} = \tilde{a}_i, \ i = 1, 2, 3, ..., p$ and $\sum_{i=1}^{p} \tilde{x}_{ij} = \tilde{b}_j, \ j = 1, 2, 3, ..., q$
(iii) If there exist any non-negative fuzzy number $\tilde{x}'_{ij}$
such that $\sum_{j=1}^{q} \tilde{x}'_{ij} = \tilde{a}_i, \ i = 1, 2, 3, ..., p$ and $\sum_{i=1}^{p} \tilde{x}'_{ij} = \tilde{b}_j, \ j = 1, 2, 3, ..., q$
then $\Re(\sum_{i=1}^{p}\sum_{j=1}^{q} \tilde{c}_{ij} \otimes \tilde{x}_{ij}) < \Re(\sum_{i=1}^{p}\sum_{j=1}^{q} \tilde{c}_{ij} \otimes \tilde{x}'_{ij})$

4. Proposed method to find the fuzzy optimal solution of balanced fuzzy transportation problems with existing representation of trapezoidal fuzzy numbers

In this section, to overcome all the shortcomings, discussed in Section 3.1, a new method (based on fuzzy linear programming formulation), is proposed to find the fuzzy optimal solution of FTP occurring in real life situations by representing all the parameters as trapezoidal fuzzy numbers. The steps of the proposed method are as follows:

Step 1. Find the total fuzzy availability $\sum_{i=1}^{p} \tilde{a}_i$ and the total fuzzy demand $\sum_{j=1}^{q} \tilde{b}_j$. Let $\sum_{i=1}^{p} \tilde{a}_i = (m, n, \alpha, \beta)$ and $\sum_{j=1}^{q} \tilde{b}_j = (m', n', \alpha', \beta')$. Examine that the problem is balanced or not, i.e., $\sum_{i=1}^{p} \tilde{a}_i = \sum_{j=1}^{q} \tilde{b}_j$ or $\sum_{i=1}^{p} \tilde{a}_i \neq \sum_{j=1}^{q} \tilde{b}_j$.

Case (i) If the problem is balanced, i.e., $\sum_{i=1}^{p} \tilde{a}_i = \sum_{j=1}^{q} \tilde{b}_j$, then Go to Step 2.

Case (ii) If $\sum_{i=1}^{p} \tilde{a}_i \neq \sum_{j=1}^{q} \tilde{b}_j$ then convert the unbalanced problem into balanced problem as follows:

Case (a) If $m-\alpha \leq m'-\alpha'$, $\alpha \leq \alpha'$, $n-m \leq n'-m'$, and $\beta \leq \beta'$ then introduce a dummy source with fuzzy availability $(m'-m, n'-n, \alpha'-\alpha, \beta'-\beta)$. Assume the fuzzy transportation cost for unit quantity of the product from the introduced dummy source to all destinations as zero trapezoidal fuzzy number then Go to Step 2.

Case (b) If $m-\alpha \geq m'-\alpha'$, $\alpha \geq \alpha'$, $n-m \geq n'-m'$, and $\beta \geq \beta'$ then introduce a dummy destination with fuzzy demand $(m-m', n-n', \alpha-\alpha', \beta-\beta')$. Assume the fuzzy transportation cost for unit quantity of the product from all sources to the introduced dummy destination as zero trapezoidal fuzzy number then Go to Step 2.

Case (c) If neither Case(a) nor Case(b) is satisfied then introduce a dummy source with fuzzy availability (maximum $\{0,(m'-\alpha')-(m-\alpha)\}$ + maximum $\{0,(\alpha'-\alpha)\}$, maximum $\{0,(m'-\alpha')-(m-\alpha)\}$ + maximum $\{0,(\alpha'-\alpha)\}$ + maximum $\{0,(n'-m')-(n-m)\}$, maximum $\{0,(\alpha'-\alpha)\}$, maximum $\{0,(\beta'-\beta)\}$) and dummy destination with fuzzy demand (maximum $\{0,(m-\alpha)-(m'-\alpha')\}$ + maximum $\{0,(\alpha-\alpha')\}$, maximum $\{0,(m-\alpha)-(m'-\alpha')\}$ + maximum $\{0,(\alpha-\alpha')\}$ + maximum $\{0,(n-m)-(n'-m')\}$, maximum $\{0,(\alpha-\alpha')\}$, maximum $\{0,(\beta-\beta')\}$). Assume the fuzzy transportation cost for unit quantity of the product from the introduced dummy source to all destinations and from all sources to the introduced dummy destination as zero trapezoidal fuzzy number then Go to Step 2.

Step 2. Now the balanced fuzzy transportation problem may be formulated as follows:

$$\text{Minimize} \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$$

$$\text{subject to} \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i,\ i=1,2,3,...,m;\ m = p \text{ or } p+1$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j,\ j=1,2,3,...,n;\ n = q \text{ or } q+1$$

$$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$$

$\tilde{x}_{ij}$ is a non-negative trapezoidal fuzzy number, where

m = total number of sources ; n = total number of destinations ; $\tilde{c}_{ij} = (m'_{ij}, n'_{ij}, \alpha'_{ij}, \beta'_{ij})$; $\tilde{a}_i = (m_i, n_i, \alpha_i, \beta_i)$; $\tilde{b}_j = (m'_j, n'_j, \alpha'_j, \beta'_j)$; $\tilde{x}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})$

$\sum_{i=1}^{m} \tilde{a}_i$ = total fuzzy availability of the product

$\sum_{j=1}^{n} \tilde{b}_j$ = total fuzzy demand of the product

$\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$ = total fuzzy transportation cost

Step 3. Now our objective is to find $\tilde{x}_{ij}$ such that

$$\begin{aligned}
&\text{Minimize } \Re\Big(\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\otimes\tilde{x}_{ij}\Big)\\
&\text{subject to } \sum_{j=1}^{n}\tilde{x}_{ij}=\tilde{a}_i,\ i=1,2,3,...,m\\
&\sum_{i=1}^{m}\tilde{x}_{ij}=\tilde{b}_j,\ j=1,2,3,...,n\\
&\sum_{i=1}^{m}\tilde{a}_i=\sum_{j=1}^{n}\tilde{b}_j\\
&\tilde{x}_{ij} \text{ is a non-negative trapezoidal fuzzy number.}
\end{aligned}$$

Step 4. Let $\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}\tilde{c}_{ij}\otimes\tilde{x}_{ij}=(m_0,n_0,\alpha_0,\beta_0)$, then the fuzzy linear programming problem (FLPP), obtained in Step 3, may be written as:

Minimize $\Re(m_0,n_0,\alpha_0,\beta_0)$

$$\begin{aligned}
&\text{subject to } \Big(\sum_{j=1}^{n}m_{ij},\sum_{j=1}^{n}n_{ij},\sum_{j=1}^{n}\alpha_{ij},\sum_{j=1}^{n}\beta_{ij}\Big)=(m_i,n_i,\alpha_i,\beta_i),\ i=1,2,3,...,m\\
&\Big(\sum_{i=1}^{m}m_{ij},\sum_{i=1}^{m}n_{ij},\sum_{i=1}^{m}\alpha_{ij},\sum_{i=1}^{m}\beta_{ij}\Big)=(m'_j,n'_j,\alpha'_j,\beta'_j),\ j=1,2,3,...,n\\
&\sum_{i=1}^{m}(m_i,n_i,\alpha_i,\beta_i)=\sum_{j=1}^{n}(m'_j,n'_j,\alpha'_j,\beta'_j)\\
&(m_{ij},n_{ij},\alpha_{ij},\beta_{ij}) \text{ is a non-negative trapezoidal fuzzy number.}
\end{aligned}$$

Step 5. The FLPP, obtained in Step 4, is converted into following crisp linear programming problem:

Minimize $\big(\frac{(m_0+n_0)}{2}+\frac{(\beta_0-\alpha_0)}{4}\big)$

$$\begin{aligned}
&\text{subject to } \sum_{j=1}^{n}m_{ij}=m_i\ ,\ i=1,2,3,...,m; \qquad \sum_{j=1}^{n}n_{ij}=n_i,\ i=1,2,3,...,m\\
&\sum_{j=1}^{n}\alpha_{ij}=\alpha_i,\ i=1,2,3,...,m; \qquad \sum_{j=1}^{n}\beta_{ij}=\beta_i,\ i=1,2,3,...,m\\
&\sum_{i=1}^{m}m_{ij}=m'_j,\ j=1,2,3,...,n; \qquad \sum_{i=1}^{m}n_{ij}=n'_j,\ j=1,2,3,...,n\\
&\sum_{i=1}^{m}\alpha_{ij}=\alpha'_j,\ j=1,2,3,...,n; \qquad \sum_{i=1}^{m}\beta_{ij}=\beta'_j,\ j=1,2,3,...,n\\
&m_{ij}-\alpha_{ij},n_{ij}-m_{ij},m_{ij},n_{ij},\alpha_{ij},\beta_{ij}\geq 0\ \forall i,j.
\end{aligned}$$

Step 6. Find the optimal solution $m_{ij},n_{ij},\alpha_{ij},\beta_{ij}$ by solving the crisp linear programming problem, obtained in Step 5.

Step 7. Find the fuzzy optimal solution $(\tilde{x}_{ij})$ by putting the values of $m_{ij},n_{ij},\alpha_{ij},\beta_{ij}$ in $\tilde{x}_{ij}=(m_{ij},n_{ij},\alpha_{ij},\beta_{ij})$.

Step 8. Find the minimum total fuzzy transportation cost by putting the values of $\tilde{x}_{ij}$ in $\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}\tilde{c}_{ij}\otimes\tilde{x}_{ij}$.

Remark 2. *If* $\sum_{i=1}^{p} \tilde{a}_i \neq \sum_{j=1}^{q} \tilde{b}_j$ *i.e.,* $(m, n, \alpha, \beta) \neq (m', n', \alpha', \beta')$ *and neither Case (a)* $m-\alpha \leq m'-\alpha'$, $\alpha \leq \alpha'$, $n-m \leq n'-m'$, $\beta \leq \beta'$ *nor Case (b)* $m-\alpha \geq m'-\alpha'$, $\alpha \geq \alpha'$, $n-m \geq n'-m'$, $\beta \geq \beta'$ *is satisfied then there may exist infinite non-negative trapezoidal fuzzy numbers* $(m_1, n_1, \alpha_1, \beta_1)$ *and* $(m_1', n_1', \alpha_1', \beta_1')$ *such that* $(m, n, \alpha, \beta) \oplus (m_1, n_1, \alpha_1, \beta_1) = (m', n', \alpha', \beta') \oplus (m_1', n_1', \alpha_1', \beta_1')$ *but we want to find trapezoidal fuzzy numbers* $(m_1, n_1, \alpha_1, \beta_1)$ *and* $(m_1', n_1', \alpha_1', \beta_1')$ *which satisfies the following characteristics:*
(i) $(m_1, n_1, \alpha_1, \beta_1)$ *and* $(m_1', n_1', \alpha_1', \beta_1')$ *are non-negative trapezoidal fuzzy numbers.*
(ii) $(m, n, \alpha, \beta) \oplus (m_1, n_1, \alpha_1, \beta_1) = (m', n', \alpha', \beta') \oplus (m_1', n_1', \alpha_1', \beta_1')$.
(iii) If there exist non-negative trapezoidal fuzzy numbers $(x, y, \alpha'', \beta'')$ *and* $(x_1, y_1, \alpha_1'', \beta_1'')$ *such that* $(m, n, \alpha, \beta) \oplus (x, y, \alpha'', \beta'') = (m', n', \alpha', \beta')$ $\oplus (x_1, y_1, \alpha_1'', \beta_1'')$ *then* $\Re(x, y, \alpha'', \beta'') > \Re(m_1, n_1, \alpha_1, \beta_1)$ *and* $\Re(x_1, y_1, \alpha_1'', \beta_1'') > \Re(m_1', n_1', \alpha_1', \beta_1')$.

5. Proposed method to find the fuzzy optimal solution of balanced fuzzy transportation problems with new representation of trapezoidal fuzzy numbers

In this section, a new representation of trapezoidal fuzzy numbers, named as JMD representation of trapezoidal fuzzy numbers, is proposed and the method, proposed in Section 4, is modified with JMD representation of trapezoidal fuzzy numbers. Also, it is shown that it is better to use the JMD representation of trapezoidal fuzzy numbers instead of existing representation of trapezoidal fuzzy numbers for finding the fuzzy optimal solution of fuzzy transportation problems.

5.1. JMD **representation of trapezoidal fuzzy numbers.** In this section, some new definitions are introduced:

Definition 8. *Let* (m, n, α, β) *be a trapezoidal fuzzy number then its JMD representation is* $(x, \alpha, \gamma, \beta)_{JMD}$, *where* $x = m - \alpha, \alpha = \alpha \geq 0, \gamma = n - m \geq 0, \beta = \beta \geq 0$.

Definition 9. *A trapezoidal fuzzy number* $\tilde{A} = (x, \alpha, \gamma, \beta)_{JMD}$ *is said to be zero trapezoidal fuzzy number if and only if* $x = 0, \alpha = 0, \gamma = 0, \beta = 0$.

Definition 10. *A trapezoidal fuzzy number* $\tilde{A} = (x, \alpha, \gamma, \beta)_{JMD}$ *is said to be non-negative trapezoidal fuzzy number if and only if* $x \geq 0$.

Definition 11. *Two trapezoidal fuzzy numbers* $\widetilde{A} = (x_1, \alpha_1, \gamma_1, \beta_1)_{JMD}$ *and* $\widetilde{B} = (x_2, \alpha_2, \gamma_1, \beta_2)_{JMD}$ *are said to be equal i.e.,* $\widetilde{A} = \widetilde{B}$ *if and only if* $x_1 = x_2$, $\alpha_1 = \alpha_2$, $\gamma_1 = \gamma_2$, $\beta_1 = \beta_2$.

Definition 12. *The ranking formula, presented in Definition 7, is converted into the following ranking formula*
Let $(x, \alpha, \gamma, \beta)_{JMD}$ *be a trapezoidal fuzzy number then* $\Re(x, \alpha, \gamma, \beta) = \frac{4x+3\alpha+2\gamma+\beta}{4}$.

5.1.1. Arithmetic operations of JMD type trapezoidal fuzzy numbers. The arithmetic operations of JMD type trapezoidal fuzzy numbers can be easily defined by using the following steps:
Step 1. Convert the given JMD type trapezoidal fuzzy numbers into (m, n, α, β) type trapezoidal fuzzy numbers by using the Definition 8.
Step 2. Now apply the arithmetic operations of (m, n, α, β) type trapezoidal fuzzy numbers, discussed in Section 2.2.
Step 3. Convert the (m, n, α, β) type trapezoidal fuzzy number, obtained from Step 2, into JMD type trapezoidal fuzzy number.

5.2. Proposed method. In this section the method, proposed in Section 4, to find the fuzzy optimal solution of FTP occurring in real life situations is modified by representing all the parameters as JMD type trapezoidal fuzzy numbers. The steps of the proposed method are as follows:
Step 1. Find the total fuzzy availability $\sum_{i=1}^{p} \tilde{a}_i$ and the total fuzzy demand $\sum_{j=1}^{q} \tilde{b}_j$. Let $\sum_{i=1}^{p} \tilde{a}_i = (x, \alpha, \gamma, \beta)_{JMD}$ and $\sum_{j=1}^{q} \tilde{b}_j = (y, \lambda, \delta, \mu)_{JMD}$. Examine that the problem is balanced or not, i.e., $\sum_{i=1}^{p} \tilde{a}_i = \sum_{j=1}^{q} \tilde{b}_j$ or $\sum_{i=1}^{p} \tilde{a}_i \neq \sum_{j=1}^{q} \tilde{b}_j$.
Case (i) If the problem is balanced, i.e., $\sum_{i=1}^{p} \tilde{a}_i = \sum_{j=1}^{q} \tilde{b}_j$, then Go to Step 2.
Case (ii) If $\sum_{i=1}^{p} \tilde{a}_i \neq \sum_{j=1}^{q} \tilde{b}_j$ then convert the unbalanced problem into balanced problem as follows:
Case (a) If $x \leq y$, $\alpha \leq \lambda$, $\gamma \leq \delta$ and $\beta \leq \mu$ then introduce a dummy source with fuzzy availability $(y-x, \lambda-\alpha, \delta-\gamma, \mu-\beta)_{JMD}$. Assume the fuzzy transportation cost for unit quantity of the product from the introduced dummy source to all destinations as zero trapezoidal fuzzy number then Go to Step 2.
Case (b) If $x \geq y$, $\alpha \geq \lambda$, $\gamma \geq \delta$ and $\beta \geq \mu$ then introduce a dummy destination with fuzzy demand $(x-y, \alpha-\lambda, \gamma-\delta, \beta-\mu)_{JMD}$. Assume the fuzzy transportation cost for unit quantity of the product from all sources to the introduced dummy destination as zero trapezoidal fuzzy number then Go to Step 2.
Case (c) If neither Case(a) nor Case(b) is satisfied then introduce a dummy source with fuzzy availability (maximum $\{0, y-x\}$, maximum $\{0, \lambda-\alpha\}$, maximum $\{0, \delta-\gamma\}$, maximum $\{0, \mu-\beta\})_{JMD}$ and (maximum $\{0, x-y\}$, maximum $\{0, \alpha-\lambda\}$, maximum $\{0, \gamma-\delta\}$, maximum $\{0, \beta-\mu\})_{JMD}$. Assume the fuzzy transportation cost for unit quantity of the product from the introduced dummy source to all destinations and from all sources to the introduced dummy destination as zero trapezoidal fuzzy number then Go to Step 2.
Step 2. Now the balanced fuzzy transportation problem may be formulated as follows:

$$\text{Minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$$

$$\text{subject to } \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, i = 1, 2, 3, ..., m; m = p \text{ or } p + 1$$
$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j, j = 1, 2, 3, ..., n;\ n = q \text{ or } q + 1$$
$$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$$
$\tilde{x}_{ij}$ is a non-negative trapezoidal fuzzy number

where, $\tilde{c}_{ij} = (x'_{ij}, \alpha'_{ij}, \gamma'_{ij}, \beta'_{ij})_{JMD}$; $\tilde{a}_i = (x_i, \alpha_i, \gamma_i, \beta_i)_{JMD}$;
$\tilde{b}_j = (x'{}_j, \alpha'_j, \gamma'_j, \beta'_j)_{JMD}$; $\tilde{x}_{ij} = (x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij})_{JMD}$.

Step 3. Now our objective is to find $\tilde{x}_{ij}$ such that

$$\text{Minimize } \Re(\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij})$$
$$\text{subject to } \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, i = 1, 2, 3, ..., m$$
$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j, j = 1, 2, 3, ..., n$$
$$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$$
$\tilde{x}_{ij}$ is a non-negative trapezoidal fuzzy number.

Step 4. Let $\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij} = (x_0, \alpha_0, \gamma_0, \beta_0)_{JMD}$, then the fuzzy linear programming problem (FLPP), obtained in Step 3, may be written as:

Minimize $\Re(x_0, \alpha_0, \gamma_0, \beta_0)$

subject to $(\sum_{j=1}^{n} x_{ij}, \sum_{j=1}^{n} \alpha_{ij}, \sum_{j=1}^{n} \gamma_{ij}, \sum_{j=1}^{n} \beta_{ij})_{JMD} = (x_i, \alpha_i, \gamma_i, \beta_i)_{JMD},\ 1 \le i \le m$

$$(\sum_{i=1}^{m} x_{ij}, \sum_{i=1}^{m} \alpha_{ij}, \sum_{i=1}^{m} \gamma_{ij}, \sum_{i=1}^{m} \beta_{ij})_{JMD} = (x'_j, \alpha'_j, \gamma'_j, \beta'_j)_{JMD},\ 1 \le j \le n$$
$$\sum_{i=1}^{m}(x_i, \alpha_i, \gamma_i, \beta_i)_{JMD} = \sum_{j=1}^{n}(x'_j, \alpha'_j, \gamma'_j, \beta'_j)_{JMD}$$
$(x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij})_{JMD}$ is a non-negative trapezoidal fuzzy number.

Step 5. The FLPP, obtained in Step 4, is converted into following crisp linear programming problem:

Minimize $\frac{1}{4}(4x_0 + 3\alpha_0 + 2\gamma_0 + \beta_0)$

subject to $\sum_{j=1}^{n} x_{ij} = x_i$, $i = 1, 2, 3, ..., m$; $\quad \sum_{j=1}^{n} \alpha_{ij} = \alpha_i,\ i = 1, 2, 3, ..., m$

$$\sum_{j=1}^{n} \gamma_{ij} = \gamma_i,\ i = 1, 2, 3, ..., m; \quad \sum_{j=1}^{n} \beta_{ij} = \beta_i,\ i = 1, 2, 3, ..., m$$
$$\sum_{i=1}^{m} x_{ij} = x'_j,\ j = 1, 2, 3, ..., n; \quad \sum_{i=1}^{m} \alpha_{ij} = \alpha'_j,\ j = 1, 2, 3, ..., n$$
$$\sum_{i=1}^{m} \gamma_{ij} = \gamma'_j,\ j = 1, 2, 3, ..., n; \quad \sum_{i=1}^{m} \beta_{ij} = \beta'_j,\ j = 1, 2, 3, ..., n$$
$x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij} \ge 0\ \forall\ i, j$.

Step 6. Find the optimal solution $x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij}$ by solving the crisp linear programming problem, obtained in Step 5.
Step 7. Find the fuzzy optimal solution $(\tilde{x}_{ij})$ by putting the values of $x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij}$ in $\tilde{x}_{ij} = (x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij})_{JMD}$.
Step 8. Find the minimum total fuzzy transportation cost by putting the values of $\tilde{x}_{ij}$ in $\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$.

Remark 3. *The proposed JMD representation of trapezoidal fuzzy number may be called as "JAI MATA DI" or "JAI MEHAR DI" and the proposed method may be called as Mehar's method for solving fuzzy transportation problems. Mehar is a lovely daughter of Parmpreet Kaur (Research scholar under the supervision of Dr. Amit Kumar).*

5.3. Advantages of JMD representation over existing representation of trapezoidal fuzzy numbers. In this section, advantages of JMD representation over existing representation of trapezoidal fuzzy numbers are discussed:

By comparing the methods, proposed in Section 4 and Section 5.2, the following results are obtained:
(i) If all the parameters are represented by (m, n, α, β) type trapezoidal fuzzy numbers and a balanced fuzzy transportation problem with m constraints and n variables is converted into a balanced crisp transportation problem, then the number of constraints in CTP = 4 × number of constraints in fuzzy transportation problem + 2 × number of fuzzy variables.

(ii) If all the parameters of fuzzy transportation problems are represented by JMD type trapezoidal fuzzy numbers and a balanced fuzzy transportation problem with m constraints and n variables is converted into a balanced crisp transportation problem, then the number of constraints in CTP = 4 × number of constraints in fuzzy transportation problem.

On the basis of above results, it can be easily seen that if all the parameters are represented by JMD type trapezoidal fuzzy numbers then the number of constraints in the converted CTP will be less then as compared to the number of constraints obtained by using the existing representation of trapezoidal fuzzy numbers.

Remark 4. *Non-negative restrictions on fuzzy variables i.e, $\tilde{x}_{ij}$ is a non-negative trapezoidal fuzzy number, is also considered as a constraint. Similarly, non-negative restrictions on crisp variables i.e, $x_{ij} \geq 0$ is also considered as a constraint.*

6. Advantages of the proposed method

By using the proposed method a decision maker has the following advantages:
(i) The final results are non-negative trapezoidal fuzzy numbers i.e., there is no negative part in the obtained trapezoidal fuzzy numbers.
(ii) Using the proposed method methods an unbalanced fuzzy transportation

problem can be easily converted into balanced fuzzy transportation problem without using α-cut technique.
(iii) It is easy to apply the proposed methods, compare to the existing methods, to find the fuzzy optimal solution of FTP occurring in real life situations.

7. Numerical example

To illustrate the method, proposed in Section 5.2, a FTP is solved by using the proposed method. Also the obtained results are compared with the results obtained by using the method, proposed in Section 4.

Example 7.1. *A company has two sources S_1 and S_2 and three destinations D_1, D_2 and D_3 ; the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination is $\tilde{c}_{ij}$, where*

$$[\tilde{c}_{ij}]_{2\times 3} = \begin{pmatrix} (10,10,10,10)_{JMD} & (50,10,10,20)_{JMD} & (80,10,20,10)_{JMD} \\ (60,10,10,10)_{JMD} & (70,10,20,20)_{JMD} & (20,10,20,10)_{JMD} \end{pmatrix}.$$

The fuzzy availability of the product at first and second sources are $(70,20,0,10)_{JMD}$ and $(40,20,10,10)_{JMD}$ and the fuzzy demand of the product at first, second and third destinations are $(30,10,10,20)_{JMD}$, $(20,10,10,10)_{JMD}$ and $(40,10,0,30)_{JMD}$ respectively. The company wants to determine the fuzzy quantity of the product that should be transported from each of the sources to each destination so that the total fuzzy transportation cost is minimum.

Solution: The fuzzy optimal solution of the chosen FTP by using the method based on FLPF, proposed in Section 5.2, can be obtained as follows:

Step 1. Total fuzzy availability $= (110,40,10,20)_{JMD}$ and total fuzzy demand $= (90,30,20,60)_{JMD}$.
Since total fuzzy availability $\neq$ total fuzzy demand, so it is an unbalanced fuzzy transportation problem.
Now as described in the proposed method (using Case(c) of Step 1 of the proposed method), the unbalanced fuzzy transportation problem can be converted into a balanced fuzzy transportation problem, by introducing a dummy source S_3 with fuzzy availability $(0,0,10,40)_{JMD}$ and a dummy destination D_4 with fuzzy demand $(20,10,0,0)_{JMD}$ so that total fuzzy availability = total fuzzy demand i.e., $(110,40,10,20)_{JMD} \oplus (0,0,10,40)_{JMD} = (90,30,20,60)_{JMD} \oplus (20,10,0,0)_{JMD}$.
Assuming the fuzzy transportation cost for unit quantity of the product from dummy source S_3 to all destinations and from all sources to dummy destination D_4 as zero trapezoidal fuzzy number i.e., $\tilde{c}_{14} = \tilde{c}_{24} = \tilde{c}_{31} = \tilde{c}_{32} = \tilde{c}_{33} = \tilde{c}_{34} = (0,0,0,0)_{JMD}$.
Step 2. Let $\tilde{x}_{ij} = (x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij})_{JMD}$ be the fuzzy quantity of the product that should be transported from the i^{th} source to the j^{th} destination so that the total fuzzy transportation cost is minimum. The obtained balanced fuzzy transportation problem may be formulated into the following FLPP:
Minimize $((10,10,10,10)_{JMD} \otimes \tilde{x}_{11} \oplus (50,10,10,20)_{JMD} \otimes \tilde{x}_{12} \oplus (80,10,20,10)_{JMD} \otimes \tilde{x}_{13} \oplus (0,0,0,0)_{JMD} \otimes \tilde{x}_{14} \oplus (60,10,10,10)_{JMD} \otimes \tilde{x}_{21} \oplus (70,10,20,20)_{JMD} \otimes \tilde{x}_{22} \oplus$

$(20,10,20,10)_{JMD} \otimes \tilde{x}_{23} \oplus (0,0,0,0)_{JMD} \otimes \tilde{x}_{24} \oplus (0,0,0,0)_{JMD} \otimes \tilde{x}_{31} \oplus (0,0,0,0)_{JMD} \otimes \tilde{x}_{32} \oplus (0,0,0,0)_{JMD} \otimes \tilde{x}_{33} \oplus (0,0,0,0)_{JMD} \otimes \tilde{x}_{34})$

subject to $\tilde{x}_{11} \oplus \tilde{x}_{12} \oplus \tilde{x}_{13} \oplus \tilde{x}_{14} = (70,20,0,10)_{JMD}$

$\tilde{x}_{21} \oplus \tilde{x}_{22} \oplus \tilde{x}_{23} \oplus \tilde{x}_{24} = (40,20,10,10)_{JMD}$

$\tilde{x}_{31} \oplus \tilde{x}_{32} \oplus \tilde{x}_{33} \oplus \tilde{x}_{34} = (0,0,10,40)_{JMD}$

$\tilde{x}_{11} \oplus \tilde{x}_{21} \oplus \tilde{x}_{31} = (30,10,10,20)_{JMD}$

$\tilde{x}_{12} \oplus \tilde{x}_{22} \oplus \tilde{x}_{32} = (20,10,10,10)_{JMD}$

$\tilde{x}_{13} \oplus \tilde{x}_{23} \oplus \tilde{x}_{33} = (40,10,0,30)_{JMD}$

$\tilde{x}_{14} \oplus \tilde{x}_{24} \oplus \tilde{x}_{34} = (20,10,0,0)_{JMD}$

$\tilde{x}_{11}, \tilde{x}_{12}, \tilde{x}_{13}, \tilde{x}_{14}, \tilde{x}_{21}, \tilde{x}_{22}, \tilde{x}_{23}, \tilde{x}_{24}, \tilde{x}_{31}, \tilde{x}_{32}, \tilde{x}_{33}, \tilde{x}_{34}$ are non-negative trapezoidal fuzzy numbers.

Step 3. Using Step 3 to Step 5 of the proposed method, the formulated FLPP is converted into the following crisp linear programming problem:

Minimize $\frac{1}{4}(100x_{11} + 90\alpha_{11} + 70\gamma_{11} + 40\beta_{11} + 270x_{12} + 220\alpha_{12} + 160\gamma_{12} + 90\beta_{12} + 400x_{13} + 320\alpha_{13} + 230\gamma_{13} + 120\beta_{13} + 300x_{21} + 240\alpha_{21} + 170\gamma_{21} + 90\beta_{21} + 370x_{22} + 300\alpha_{22} + 220\gamma_{22} + 120\beta_{22} + 160x_{23} + 140\alpha_{23} + 110\gamma_{23} + 60\beta_{23})$

subject to

$x_{11} + x_{12} + x_{13} + x_{14} = 70$, $\alpha_{11} + \alpha_{12} + \alpha_{13} + \alpha_{14} = 20$, $\gamma_{11} + \gamma_{12} + \gamma_{13} + \gamma_{14} = 0$, $\beta_{11} + \beta_{12} + \beta_{13} + \beta_{14} = 10$, $x_{21} + x_{22} + x_{23} + x_{24} = 40$, $\alpha_{21} + \alpha_{22} + \alpha_{23} + \alpha_{24} = 20$, $\gamma_{21} + \gamma_{22} + \gamma_{23} + \gamma_{24} = 10$, $\beta_{21} + \beta_{22} + \beta_{23} + \beta_{24} = 10$, $x_{31} + x_{32} + x_{33} + x_{34} = 0$, $\alpha_{31} + \alpha_{32} + \alpha_{33} + \alpha_{34} = 0$, $\gamma_{31} + \gamma_{32} + \gamma_{33} + \gamma_{34} = 10$, $\beta_{31} + \beta_{32} + \beta_{33} + \beta_{34} = 40$, $x_{11} + x_{21} + x_{31} = 30$, $\alpha_{11} + \alpha_{21} + \alpha_{31} = 10$, $\gamma_{11} + \gamma_{21} + \gamma_{31} = 10$, $\beta_{11} + \beta_{21} + \beta_{31} = 20$, $x_{12} + x_{22} + x_{32} = 20$, $\alpha_{12} + \alpha_{22} + \alpha_{32} = 10$, $\gamma_{12} + \gamma_{22} + \gamma_{32} = 10$, $\beta_{12} + \beta_{22} + \beta_{32} = 10$, $x_{13} + x_{23} + x_{33} = 40$, $\alpha_{13} + \alpha_{23} + \alpha_{33} = 10$, $\gamma_{13} + \gamma_{23} + \gamma_{33} = 0$, $\beta_{13} + \beta_{23} + \beta_{33} = 30$, $x_{14} + x_{24} + x_{34} = 20$, $\alpha_{14} + \alpha_{24} + \alpha_{34} = 10$, $\gamma_{14} + \gamma_{24} + \gamma_{34} = 0$, $\beta_{14} + \beta_{24} + \beta_{34} = 0$
$x_{11}, \alpha_{11}, \gamma_{11}, \beta_{11}, x_{12}, \alpha_{12}, \gamma_{12}, \beta_{12}, x_{13}, \alpha_{13}, \gamma_{13}, \beta_{13}, x_{14}, \alpha_{14}, \gamma_{14}, \beta_{14}, x_{21}, \alpha_{21}, \gamma_{21}, \beta_{21}, x_{22}, \alpha_{22}, \gamma_{22}, \beta_{22}, x_{23}, \alpha_{23}, \gamma_{23}, \beta_{23}, x_{24}, \alpha_{24}, \gamma_{24}, \beta_{24}, x_{31}, \alpha_{31}, \gamma_{31}, \beta_{31}, x_{32}, \alpha_{32}, \gamma_{32}, \beta_{32}, x_{33}, \alpha_{33}, \gamma_{33}, \beta_{33}, x_{34}, \alpha_{34}, \gamma_{34}, \beta_{34} \geq 0$.

Step 4. The optimal solution of the crisp linear programming problem, obtained in Step 3, is $x_{11} = 30, \alpha_{11} = 10, \gamma_{11} = 0, \beta_{11} = 10, x_{12} = 20, \alpha_{12} = 10, \gamma_{12} = 0, \beta_{12} = 0, x_{13} = 0, \alpha_{13} = 0, \gamma_{13} = 0, d_{13} = 0, x_{14} = 20, \alpha_{14} = 0, \gamma_{14} = 0, \beta_{14} = 0, x_{21} = 0, \alpha_{21} = 0, \gamma_{21} = 10, \beta_{21} = 0, x_{22} = 0, \alpha_{22} = 0, \gamma_{22} = 0, \beta_{22} = 0, x_{23} = 40, \alpha_{23} = 10, \gamma_{23} = 0, \beta_{23} = 10, x_{24} = 0, \alpha_{24} = 10, \gamma_{24} = 0, \beta_{24} = 0, x_{31} = 0, \alpha_{31} = 0, \gamma_{31} = 0, \beta_{31} = 10, x_{32} = 0, \alpha_{32} = 0, \gamma_{32} = 10, \beta_{32} = 10, x_{33} = 0, \alpha_{33} = 0, \gamma_{33} = 0, \beta_{33} = 20, x_{34} = 0, \alpha_{34} = 0, \gamma_{34} = 0, \beta_{34} = 0$.

Step 5. Putting the values of $x_{ij}, \alpha_{ij}, \gamma_{ij}$ and β_{ij} in $\tilde{x}_{ij} = (x_{ij}, \alpha_{ij}, \gamma_{ij}, \beta_{ij})_{JMD}$, the fuzzy optimal solution is $\tilde{x}_{11} = (30,10,0,10)_{JMD}$ $\tilde{x}_{12} = (20,10,0,0)_{JMD}$, $\tilde{x}_{13} = (0,0,0,0)_{JMD}$, $\tilde{x}_{14} = (20,0,0,0)_{JMD}$, $\tilde{x}_{21} = (0,0,10,0)_{JMD}$, $\tilde{x}_{22} = (0,0,0,0)_{JMD}$, $\tilde{x}_{23} = (40,10,0,10)_{JMD}$, $\tilde{x}_{24} = (0,10,0,0)_{JMD}$,
$\tilde{x}_{31} = (0,0,0,10)_{JMD}$, $\tilde{x}_{32} = (0,0,10,10)_{JMD}$, $\tilde{x}_{33} = (0,0,0,20)_{JMD}$, $\tilde{x}_{34} = (0,0,0,0)_{JMD}$.

Step 6. Putting the values of $\tilde{x}_{11}, \tilde{x}_{12}, \tilde{x}_{13}, \tilde{x}_{14}, \tilde{x}_{21}, \tilde{x}_{22}, \tilde{x}_{23}, \tilde{x}_{24}, \tilde{x}_{31}, \tilde{x}_{32}, \tilde{x}_{33}, \tilde{x}_{34}$ in $((10,10,10,10)_{JMD} \otimes \tilde{x}_{11} \oplus (50,10,10,20)_{JMD} \otimes \tilde{x}_{12} \oplus (80,10,20,10)_{JMD} \otimes$

$\tilde{x}_{13} \oplus (0,0,0,0)_{JMD} \otimes \tilde{x}_{14} \oplus (60,10,10,10)_{JMD} \otimes \tilde{x}_{21} \oplus (70,10,20,20)_{JMD} \otimes \tilde{x}_{22} \oplus (20,10,20,10)_{JMD} \otimes \tilde{x}_{23} \oplus (0,0,0,0)_{JMD} \otimes \tilde{x}_{24} \oplus (0,0,0,0)_{JMD} \otimes \tilde{x}_{31} \oplus (0,0,0,0)_{JMD} \otimes \tilde{x}_{32} \oplus (0,0,0,0)_{JMD} \otimes \tilde{x}_{33} \oplus (0,0,0,0)_{JMD} \otimes \tilde{x}_{34})$ the minimum total fuzzy transportation cost is $(2100, 2000, 2500, 2600)_{JMD}$.

Step 7. Using Definition 8, (m, n, α, β) representation of $(2100, 2000, 2500, 2600)_{JMD}$ is (4100,6600,2000,2600).

7.1. Results and discussion. The results of the numerical example, obtained by using the methods proposed in Section 4 and 5.2, are shown in Table 1.

Table 1. Results using existing and proposed representation of trapezoidal fuzzy numbers

Type of trapezoidal fuzzy number	Number of constraints in FTP	Number of fuzzy variables in FTP	Number of constraints in converted CTP	Minimum total fuzzy transportation cost
(m, n, α, β)	19	12	$(4 \times 19)+$ (2×12) = 76+24 =100	(4100,6600,2000,2600)
$(x, \alpha, \gamma, \beta)_{JMD}$	19	12	4×19 =76	$(2100, 2000,$ $2500, 2600)_{JMD}$ =(4100,6600,2000,2600)

On the basis of results, shown in Table 1, it can be easily seen that if all the parameters are represented by JMD type trapezoidal fuzzy numbers, instead of (m, n, α, β) type trapezoidal fuzzy numbers, and proposed method is applied to find the fuzzy optimal solution of fuzzy transportation problems then the minimum total fuzzy transportation cost is same but the total number of constraints, in converted CTP problem, are less than the number of constraints, obtained by using the (m, n, α, β) type trapezoidal fuzzy numbers. Hence, it is better to use JMD representation instead of (m, n, α, β) type trapezoidal fuzzy numbers to find the fuzzy optimal solution of fuzzy transportation problems.

8. Conclusions

A new method with a new representation of trapezoidal fuzzy numbers is proposed to find the fuzzy optimal solution of fuzzy transportation problems. Since the proposed method is based on the classical transportation method so it is easy to learn and to apply the proposed method to find the fuzzy optimal solution of fuzzy transportation problems occurring in real life situations. The main advantage of the proposed method is that the obtained fuzzy optimal solution and fuzzy optimal value both are non-negative fuzzy numbers.

Acknowledgements

The authors acknowledge the Editor-in-Chief of the journal, Professor Cheon Seoung Ryoo and the anonymous reviewers for their indispensable input that improved the paper significantly. Dr. Amit Kumar want to acknowledge the adolescent inner blessings of Mehar without which it was not possible to think the idea proposed in this manuscript. Mehar is a lovely daughter of Parmpreet Kaur (Research scholar under the supervision of Dr. Amit Kumar).

References

1. S. Chanas, W. Kolodziejczyk and A.A Machaj, *A fuzzy approach to the transportation problem*, Fuzzy Sets and Systems **13** (1984), 211-221.
2. S. Chanas and D. Kuchta, *A concept of the optimal solution of the transportation problem with fuzzy cost coefficients*, Fuzzy Sets and Systems **82** (1996), 299-305.
3. D.S. Dinagar, K. Palanivel, *The transportation problem in fuzzy environment*, International Journal of Algorithms, Computing and Mathematics **2** (2009), 65-71.
4. D. Dubois, and H. Prade, *Fuzzy Sets and Systems : Theory and Applications*, Academic Press, New York, 1980.
5. A. Gani and K.A. Razak, *Two stage fuzzy transportation problem*, Journal of Physical Sciences **10** (2006), 63-69.
6. P. Gupta, and M.K. Mehlawat, *An algorithm for a fuzzy transportation problem to select a new type of coal for a steel manufacturing unit*, TOP **15** (2007), 114-137.
7. F.L. Hitchcock, *The distribution of a product from several sources to numerous localities*, Journal of Mathematical Physics **20** (1941), 224-230.
8. T.S. Liou and M.J. Wang, *Ranking fuzzy number with integral value*, Fuzzy Sets and Systems **50** (1992), 247-255.
9. S.T. Liu and C. Kao, *Solving fuzzy transportation problems based on extension principle*, European Journal of Operational Research **153** (2004), 661-674.
10. M. Oheigeartaigh, *A fuzzy transportation algorithm*, Fuzzy Sets and Systems **8** (1982), 235-243.
11. P. Pandian, G. Natarajan, *A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems*, Applied Mathematical Sciences **4** (2010), 79-90.
12. O.M. Saad and S.A. Abbas, *A parametric study on transportation problem under fuzzy environment*, The Journal of Fuzzy Mathematics **11** (2003), 115-124.
13. L.A. Zadeh, *Fuzzy sets*, Information and Control **8** (1965), 338-353.
14. H.J. Zimmermann, *Fuzzy programming and linear programming with several objective functions*, Fuzzy Sets and Systems **1** (1978), 45-55.

Dr. Amit Kumar is Assistant Professor in School of Mathematics and Computer Applications, Thapar University, Patiala (Punjab), India. He has published sixteen papers in International Journals and six papers in proceedings of International Conferences and has attended ten workshops and conferences. His current area of research is Fuzzy Optimization, Fuzzy Reliability Analysis and Vague Set Theory.

School of Mathematics and Computer Applications, Thapar University, Patiala (Punjab), India.
e-mail:amit_rs_iitr@yahoo.com

Miss Amarpreet Kaur is a Ph.D student under the supervision of Dr. Amit Kumar. She has completed M.Sc (Maths) from Guru Nanak Dev University, Amritsar (Punjab), India. Her area of research is Fuzzy Transportation.

School of Mathematics and Computer Applications, Thapar University, Patiala (Punjab), India.
e-mail:amanpreettoor@gmail.com