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DYNAMIC BEHAVIOUR FOR A NONAUTONOMOUS SMOKING DYNAMICAL MODEL WITH DISTRIBUTED TIME DELAY

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Abstract. In this paper we have considered a dynamical mathematical model of the sub-populations of potential smokers (non-smokers), smokers, smokers who temporarily quit smoking, smokers who permanently quit smoking and a class of smoking associated illness by introducing time dependent parameters and distributed time delay to acquire smoking habit. Here, we have established some sufficient conditions on the permanence and extinction of the smoking class in the community by using inequality analytical technique. We have introduced some new threshold values R_0 and R^* and further obtained that the smoking class in the community will be permanent when $R_0 > 1$ and the smoking class in the community will be going to extinct when $R^* < 1$. By Lyapunov functional method, we have also obtained some sufficient conditions for global asymptotic stability of this model. Computer simulations are carried out to explain the analytical findings. The aim of the analysis of this model is to identify the parameters of interest for further study, with a view to informing and assisting policy-maker in targeting prevention and treatment resources for maximum effectiveness.

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1. Introduction

Cigarette smoking is the leading preventable cause of premature death and a leading cause of illness and mortality. Since the advent of tobacco in 6000 BC , smoking remains a major public health threat globally [16,21]. Cigarette smoking causes 87 percent of lung cancer deaths [14]. Lung cancer is the leading cause of cancer death in both men and women [1]. Smoking is also responsible for most cancers of the oral cavity, larynx, pharynx, esophagus and bladder. Moreover, it is a cause of kidney, pancreatic, cervical and stomach cancers, as

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well as acute myeloid leukemia [18]. It inflicts significant mortality, morbidity and continues to cost billions of dollars in health-care [16]. The health risks caused by cigarette smoking are not limited to smokers. Exposure to secondhand smoke (it is a combination of the smoke that is released from the end of a burning cigarette and the smoke exhaled from the lungs of smokers), or environmental tobacco smoke (ETS), significantly increases the risk of lung cancer and heart disease in nonsmokers, as well as several respiratory illnesses in young children [19]. Cigarette smoke contains about 4,000 chemical agents, including over 60 carcinogens. Moreover, many of these substances, such as carbon monoxide, tar, arsenic and lead are poisonous and toxic to the human body. Nicotine is a drug that is naturally present in the tobacco plant and is primarily responsible for a persons addiction to tobacco products, including cigarettes. During smoking, nicotine is absorbed quickly into the bloodstream and travels to the brain in a matter of seconds. Nicotine causes addiction to cigarettes and other tobacco products that is similar to the addiction produced by using heroin and cocaine [20].

Due to these facts, and the enormous public health burden related with smoking, it is of public health significance to study the dynamics of smoking in a community, intended at determining realistic methods for preventing this habit. Mathematical modellings have been used extensively to address questions of public health importance, dating back to the seminal works of Bernoulli (on modelling the dynamics of smallpox) in 1760 [4], Kermack and McKendrick [11-13] and those reported in more recent literatures (such as in [2,3,5-8,10] and the references there in), not much has been analyzed in terms of the mathematical modelling of human social behaviour [16]. Sharomi and Gumel [16] have done a qualitative study of the dynamics of smoking by using a system of autonomous ordinary differential equations.

It is well known that the spectrum of infectious disease is changing rapidly in conjunction with dramatic social and environmental changes. Worldwide, explosive population growth with increasing poverty and urban migration is going on, international travel and commerce are expanding, technology is changing rapidly-all of which are affecting the risk of exposure to infectious agents. Nonautonomous phenomenon often occurs in many realistic models. The nonautonomous phenomenon occurs mainly due to the seasonal variety, which makes the population to behave periodically. Since biological and environmental parameters are naturally subject to fluctuation in time, the effects of a periodically varying environment are considered as important selective forces on systems in a fluctuating environment. To investigate this kind of phenomenon, in the model, the coefficients should be periodic functions, then the system is called periodic system. The nonautonomous models can be regarded as an extension of the periodic models [22,23].

Motivated by the above facts, in this paper we have considered a dynamical mathematical model of the sub-populations of potential smokers (non-smokers), smokers, smokers who temporarily quit smoking, smokers who permanently quit smoking and a class of smoking associated illness by introducing time dependent parameters and distributed time delay to acquire smoking habit. In the proposed system all the coefficients are time-dependent, i.e., this system is nonautonomous. Usually, such systems have not any disease-free equilibrium and endemic equilibrium. There are many methods to deal with autonomous systems, but they may not be suitable to nonautonomous systems. Therefore, it is more difficult to study the dynamical behaviours in nonautonomous case [15]. Here, we have established some sufficient conditions on the permanence and extinction of the smoking class in the community by using inequality analytical technique. We have introduced some new threshold values R_0 and R^* and further obtained that the smoking class in the community will be permanent, this means that population of smokers will not vanish in time (i.e., the long-term survival), when $R_0 > 1$ and the smoking class in the community will be going to extinct when $R^* < 1$. By Lyapunov functional method, we have also obtained some sufficient conditions for global asymptotic stability of this model. We have observed that the time delay has no effect on the permanence of the system but it has an effect on the global asymptotic stability of this model. Our analytical results are validated through numerical simulations. The aim of the analysis of this model is to identify the parameters of interest for further study, with a view to informing and assisting policy-maker in targeting prevention and treatment resources for maximum effectiveness.

2. Model formulation

Our mathematical model is formulated as the following system of nonautonomous delay differential equations:

$$\begin{aligned}
\frac{dP(t)}{dt} &= \Lambda(t) - \beta(t)P(t)\int_0^h S(t-s)d\eta(s) - \mu(t)P(t),\\
\frac{dS(t)}{dt} &= \beta(t)P(t)\int_0^h S(t-s)d\eta(s) - \{\mu(t)+\gamma(t)+\xi(t)\}S(t) + p(t)X(t),\\
\frac{dX(t)}{dt} &= \gamma(t)(1-\sigma(t))S(t) - \{\mu(t)+p(t)\}X(t),\\
\frac{dY(t)}{dt} &= \gamma(t)\sigma(t)S(t) - \mu(t)Y(t),\\
\frac{dC(t)}{dt} &= \xi(t)S(t) - \{\mu(t)+\delta(t)\}C(t).
\end{aligned} \tag{2.1}$$

Here $N(t) = P(t) + S(t) + X(t) + Y(t) + C(t)$ denotes the total number of high-risk human population at time t; $P(t), S(t), X(t), Y(t), C(t)$ are the densities (or fractions) of potential smokers (non-smokers), smokers, smokers who temporarily quit smoking, smokers who permanently quit smoking and a class of smoking associated illness respectively at time t.

The quantities $\Lambda(t),\ \beta(t),\ \mu(t),\ \gamma(t),\ \sigma(t),\ \xi(t),\ p(t),\ \delta(t)$ are:

$\Lambda(t)$: The instantaneous recruitment rate function of the non-smoking (potential smoking) class from the larger embedding population.

$\beta(t)$: The transmission rate function to smoking class when non-smoker individuals contact with smokers and the rate of transmission is of the form:

$$\beta(t)P(t)\int_0^h S(t-s)d\eta(s).$$

Here we assume that the acquisition of smoking habit is analogous to acquiring disease infection; the mode 'contact' a non-smoker makes, the higher the likelihood of such an individual acquiring smoking habit. The nonnegative constant h is the time delay to acquire smoking habit. The function $\eta(s):[0,h]\to[0,\infty)$ is nondecreasing and has bounded variation such that:

$$\int_0^h d\eta(s)=\eta(h)-\eta(0)=1.$$

$\mu(t)$: The instantaneous natural death rate function.
$\gamma(t)(1-\sigma(t))$: The instantaneous rate function at which smokers temporarily quit smoking.
$\gamma(t)\sigma(t)$: The instantaneous rate function at which smokers permanently quit smoking. It is assumed that $0<\sigma(t)<1,\ \forall\ t\geq 0$.
$\xi(t)$: Developing rate function of smoking-related illness of the smokers.
$p(t)$: The instantaneous rate function at which temporarily quit smokers return back to smoking class.
$\delta(t)$: The instantaneous additional death rate function of the smoking-related illness class.

3. Permanence and extinction

In this section, we first introduce the following assumptions for system (2.1): functions $\Lambda(t),\ \beta(t),\ \mu(t),\ \gamma(t),\ \sigma(t),\ \xi(t),\ p(t),\ \delta(t)$ are positive continuous bounded and have positive lower bounds.

The initial conditions of (2.1) are given as

$$\begin{aligned}&P(\theta)=\varphi_1(\theta),\ S(\theta)=\varphi_2(\theta),\ X(\theta)=\varphi_3(\theta),\\&Y(\theta)=\varphi_4(\theta),\ C(\theta)=\varphi_5(\theta),\ -h\leq\theta\leq 0,\end{aligned}\tag{3.1}$$

where $\varphi=(\varphi_1,\varphi_2,\varphi_3,\varphi_4,\varphi_5)\in C$ such that $\varphi_i(\theta)\geq 0\ (i=1,2,3,4,5),\ \forall\ \theta\in[-h,0]$, and C denotes the Banach space $C([-h,0],\mathbb{R}^5)$ of continuous functions mapping the interval $[-h,0]$ into $\mathbb{R}^5$ and the norm of an element φ in C is designated by $\|\varphi\|=\sup_{-h\leq\theta\leq 0}\{|\varphi_1(\theta)|,|\varphi_2(\theta)|,|\varphi_3(\theta)|,|\varphi_4(\theta)|,|\varphi_5(\theta)|\}$. For a biological meaning, we further assume that $\varphi_i(0)>0,\ i=1,2,3,4,5$.

Let $C=C([-h,0],\mathbb{R}^n)$ is a Banach space of continuous functions mapping the interval $[-h,0]$ into $\mathbb{R}^n$ and designate the norm of an element φ in C by $\|\varphi\|=\sup_{-h\leq\theta\leq 0}|\varphi(\theta)|$. If $\sigma\in\mathbb{R},A\geq 0$, and $x\in C([\sigma-h,\sigma+A],\mathbb{R}^n)$, then for any $t\in[\sigma,\sigma+A]$, we let $x_t\in C$ be defined by $x_t(\theta)=x(t+\theta),-h\leq\theta\leq 0$.

Let us consider the following system of differential equations

$$\frac{dx(t)}{dt}=f(t,x_t)\tag{3.2}$$

on Ω, where Ω is a subset of $\mathbb{R} \times C$, and $f : \Omega \to \mathbb{R}^n$ is a given function.

A function x is said to be a solution of equation (3.2) on $[\sigma - h, \sigma + A)$ if there are $\sigma \in \mathbb{R}$ and $A > 0$ such that $x \in C([\sigma - h, \sigma + A), \mathbb{R}^n), (t, x_t) \in \Omega$ and $x(t)$ satisfies equation (3.2) for $t \in [\sigma, \sigma + A)$. For given $\sigma \in \mathbb{R}$, $\varphi \in C$, we say $x(\sigma, \varphi, f)$ is a solution of equation (3.2) with initial value φ at σ or simply a solution through (σ, φ) [9].

Theorem 3.1. [9] *Suppose Ω is an open subset in $\mathbb{R} \times C$, $f : \Omega \to \mathbb{R}^n$ is continuous and $f(t, \varphi)$ is Lipschitzian in φ in each compact set in Ω. If $(\sigma, \varphi) \in \Omega$, then there is a unique solution of* (3.2) *passing through (σ, φ).*

Corollary 3.1. *If the functions $\Lambda(t)$, $\beta(t)$, $\mu(t)$, $\gamma(t)$, $\sigma(t)$, $\xi(t)$, $p(t)$, $\delta(t)$ are continuous and bounded on $[0, +\infty)$ then there exists a unique solution of the system* (2.1) *with initial conditions* (3.1) *defined on $[0, +\infty)$.*

Here we wish to discuss the permanence of the system (2.1), this means that the long-term survival (i.e., will not vanish in time) of all components of the system (2.1), with initial conditions (3.1). It demonstrates how the smoking class in the community will be permanent (i.e., will not vanish in time) under some conditions. Also, we discuss how the smoking class in the community will be going to extinct under some conditions.

Let $f^l = \inf_{t \geq 0} f(t)$, $f^u = \sup_{t \geq 0} f(t)$, for a continuous and bounded function $f(t)$ defined on $[0, +\infty)$.

Definition : The system (2.1) is said to be permanent, i.e., the long-term survival (will not vanish in time) of all components of the system (2.1), if there are positive constants v_i and M_i $(i = 1, 2, 3, 4, 5)$ such that:

$$v_1 \leq \liminf_{t \to \infty} P(t) \leq \limsup_{t \to \infty} P(t) \leq M_1, \quad v_2 \leq \liminf_{t \to \infty} S(t) \leq \limsup_{t \to \infty} S(t) \leq M_2,$$

$$v_3 \leq \liminf_{t \to \infty} X(t) \leq \limsup_{t \to \infty} X(t) \leq M_3, \quad v_4 \leq \liminf_{t \to \infty} Y(t) \leq \limsup_{t \to \infty} Y(t) \leq M_4,$$

$$v_5 \leq \liminf_{t \to \infty} C(t) \leq \limsup_{t \to \infty} C(t) \leq M_5,$$

hold for any solution $(P(t), S(t), X(t), Y(t), C(t))$ of (2.1) with initial conditions (3.1). Here v_i and M_i $(i = 1, 2, 3, 4, 5)$ are independent of (3.1).

Theorem 3.2. *The system* (2.1) *with initial conditions* (3.1) *is permanent provided*

$$R_0 = \frac{\beta^l}{(\mu + \gamma + \xi)^u} \left(\frac{\Lambda^l}{\mu^u}\right) > 1. \tag{3.3}$$

Proof. We will give the following Propositions 3.1-3.6 to complete the proof of this theorem. □

Proposition 3.1. *The solution* $(P(t), S(t), X(t), Y(t), C(t))$ *of* (2.1) *with initial conditions* (3.1) *is positive for all* $t \geq 0$*, and*

$$\limsup_{t\to\infty} N(t) \leq \left(\frac{\Lambda}{\mu}\right)^u .$$

Proof. Since the functions $\Lambda(t),\ \beta(t),\ \mu(t),\ \gamma(t),\ \sigma(t),\ \xi(t),\ p(t),\ \delta(t)$ are continuous and bounded on $[0, +\infty)$, the solution $(P(t), S(t), X(t), Y(t), C(t))$ of (2.1) with initial conditions (3.1) exists and is unique on $[0, +\infty)$. Now,

$$\begin{aligned} P(t) = P(0)\exp&\left[-\int_0^t \{\beta(\omega)\int_0^h S(\omega - s)d\eta(s) + \mu(\omega)\}d\omega\right] \\ &+ \int_0^t \Lambda(u)\exp\left[\int_t^u \{\beta(\omega)\int_0^h S(\omega - s)d\eta(s) + \mu(\omega)\}d\omega\right] du > 0,\ \forall\ t \geq 0. \end{aligned}$$

Let us show that $S(t) > 0$ for all $t \in [0, \alpha)$, where $0 < \alpha \leq +\infty$. Otherwise, there exists a $t_1 \in (0, \alpha)$ such that $S(t_1) = 0,\ \dot{S}(t_1) \leq 0$ and $S(t) > 0$ for all $t \in [0, t_1)$. We claim that $X(t) > 0$ for all $t \in [0, t_1)$. Otherwise, there exists a $t_2 \in (0, t_1)$ such that $X(t_2) = 0,\ \dot{X}(t_2) \leq 0$ and $X(t) > 0$ for all $t \in [0, t_2)$. Now, from the third equation of system (2.1),

$$\begin{aligned} X(t_2) = X(0)\exp&\left\{-\int_0^{t_2} (\mu(s) + p(s))ds\right\} \\ &+ \int_0^{t_2} \gamma(u)(1 - \sigma(u))S(u)\exp\left\{\int_{t_2}^u (\mu(s) + p(s))ds\right\} du > 0, \end{aligned}$$

which is a contradiction with $X(t_2) = 0$. So $X(t) > 0$ for all $t \in [0, t_1)$.

Integrating the second equation of system (2.1) from 0 to t_1, we have:

$$\begin{aligned} S(t_1) = S(0)\exp&\left\{-\int_0^{t_1} (\mu(s) + \gamma(s) + \xi(s))ds\right\} \\ &+ \int_0^{t_1}\int_0^h \{\beta(u)P(u)S(u - s) + p(u)X(u)\} \\ &\qquad \times \exp\left\{\int_{t_1}^u (\mu(s) + \gamma(s) + \xi(s))ds\right\} d\eta(s)du > 0, \end{aligned}$$

which is a contradiction with $S(t_1) = 0$. So $S(t) > 0$ for all $t \geq 0$, and hence $X(t) > 0$ for all $t \geq 0$. From the fourth and fifth equations of (2.1), we have

$$Y(t) = Y(0)\exp\{-\int_0^t \mu(s)ds\} + \int_0^t \gamma(u)\sigma(u)S(u)\exp\{\int_t^u \mu(s)ds\}du > 0,\ \forall\ t \geq 0.$$

$$\begin{aligned} C(t) = C(0)\exp&\left\{-\int_0^t (\mu(s) + \delta(s))ds\right\} \\ &+ \int_0^t \xi(u)S(u)\exp\left\{\int_t^u (\mu(s) + \delta(s))ds\right\} du > 0,\ \forall\ t \geq 0. \end{aligned}$$

Thus $\forall\ t \in [0, +\infty)$,

$$\dot{N}(t) \leq \Lambda(t) - \mu(t)N(t) \ \Rightarrow \limsup_{t\to\infty} N(t) \leq \left(\frac{\Lambda}{\mu}\right)^u. \tag{3.4}$$

That is, $(P(t), S(t), X(t), Y(t), C(t))$ is uniformly bounded on $[0, +\infty)$.
This completes the proof. □

Proposition 3.2. *The solution* $(P(t), S(t), X(t), Y(t), C(t))$ *of* (2.1) *with initial conditions* (3.1) *satisfies*

$$\liminf_{t\to\infty} P(t) \geq \left\{ \frac{\Lambda}{\beta \left(\frac{\Lambda}{\mu}\right)^u + \mu} \right\}^l \equiv v_1 > 0. \tag{3.5}$$

Proof. By Proposition 3.1, for any $\epsilon > 0$, there exists a $t_1 > 0$ such that:

$$S(t) \leq \left(\frac{\Lambda}{\mu}\right)^u + \epsilon, \ \text{as } t \geq t_1.$$

Thus, from the first equation of system (2.1), when $t \geq t_1 + h$,

$$\dot{P}(t) \geq \Lambda(t) - \left\{\beta(t)\left(\left(\frac{\Lambda}{\mu}\right)^u + \epsilon\right) + \mu(t)\right\} P(t) \Rightarrow \liminf_{t\to\infty} P(t) \geq \left\{ \frac{\Lambda}{\beta\left(\left(\frac{\Lambda}{\mu}\right)^u + \epsilon\right) + \mu} \right\}^l.$$

Since $\epsilon > 0$ can be made arbitrarily small, the result of this proposition is valid.
This completes the proof. □

Proposition 3.3. *Assume that* $R_0 > 1$, *then for any solution* $(P(t), S(t), X(t), Y(t), C(t))$ *of* (2.1) *with initial conditions* (3.1) *we have*

$$\liminf_{t\to\infty} S(t) \geq \alpha e^{-(\mu+\gamma+\xi)^u (h+\rho)} \equiv v_2 > 0, \tag{3.6}$$

where $\alpha > 0$ *and* $\rho > 0$ *will be given in the proof.*

Proof. Since $R_0 > 1$, and it is obvious that

$$\frac{\Lambda^l}{G} \to \frac{\Lambda^l}{\mu^u} \text{ as } \alpha \to 0, \text{ where } G = \mu^u + \alpha\beta^u, \tag{3.7}$$

then there exists two positive constants α and ρ such that

$$\frac{\Lambda^l}{G} \{1 - \exp(-G\rho)\} \frac{\beta^l}{(\mu + \gamma + \xi)^u} > 1. \tag{3.8}$$

Let us consider the following differential function $V(t)$,

$$V(t) = S(t) + \int_0^h \int_{t-s}^t \beta(u+s)P(u+s)S(u)dud\eta(s). \tag{3.9}$$

The derivative of $V(t)$ along solution of (2.1) is

$$\begin{aligned}\dot{V}(t) &\geq \left[\int_0^h \beta(t+s)P(t+s)d\eta(s) - (\mu(t)+\gamma(t)+\xi(t))\right] S(t) \\ &\geq \left[\beta^l \int_0^h P(t+s)d\eta(s) - (\mu+\gamma+\xi)^u\right] S(t).\end{aligned} \tag{3.10}$$

We claim that it is impossible that $S(t) \leq \alpha$, $\forall\ t \geq t_1$ (t_1 is any nonnegative constant). Suppose the contrary, then as $t \geq t_1 + h$,

$$\dot{P}(t) \geq \Lambda(t) - (\mu(t) + \alpha\beta^u)P(t) \geq \Lambda^l - GP(t), \text{ where } G \text{ is given in (3.7).} \tag{3.11}$$

For $t > t_1 + h$, integrating the above inequality from $t_1 + h$ to t, we obtain

$$\begin{aligned}P(t) &\geq P(t_1+h)\exp\left(\int_t^{t_1+h} Gds\right) + \int_{t_1+h}^t \Lambda^l \exp\left(\int_t^s Gd\theta\right) ds \\ &\geq \left(\frac{\Lambda^l}{G}\right) \frac{\int_{t_1+h}^t G\exp\left(\int_0^s Gd\theta\right) ds}{\exp\left(\int_0^t Gd\theta\right)}.\end{aligned}$$

Hence,

$$P(t) \geq \left(\frac{\Lambda^l}{G}\right)[1 - \exp\{-G(t - t_1 - h)\}]. \tag{3.12}$$

Therefore

$$P(t) \geq \left(\frac{\Lambda^l}{G}\right)[1 - \exp\{-G\rho\}] \equiv P^\Delta,\ \forall\ t \geq t_1 + h + \rho \equiv t_2. \tag{3.13}$$

From (3.10) and (3.13), we have

$$\dot{V}(t) \geq (\mu+\gamma+\xi)^u\left[\frac{\beta^l P^\Delta}{(\mu+\gamma+\xi)^u} - 1\right]S(t),\ \forall\ t \geq t_2. \tag{3.14}$$

Let us take $\underline{s} = \min_{t_2 \leq t \leq t_2+h} S(t)$. Next we shall prove that $S(t) \geq \underline{s}$, $\forall\ t \geq t_2$. Suppose that it is not true, then $\exists T \geq 0$, such that $S(t) \geq \underline{s}$, for all $t_2 \leq t \leq t_2 + h + T$, $S(t_2 + h + T) = \underline{s}$ and $\dot{S}(t_2 + h + T) \leq 0$. On the other hand, by the second equation of (2.1), as $t = t_2 + h + T$,

$$\begin{aligned}\dot{S}(t) &\geq \beta(t)P(t)\int_0^h S(t-s)d\eta(s) - (\mu(t)+\gamma(t)+\xi(t))S(t) \\ &\geq \{\beta^l P^\Delta - (\mu+\gamma+\xi)^u\}\underline{s} = (\mu+\gamma+\xi)^u\left[\frac{\beta^l P^\Delta}{(\mu+\gamma+\xi)^u} - 1\right]\underline{s} > 0,\end{aligned}$$

since from (3.8), we have

$$\frac{\beta^l P^\Delta}{(\mu+\gamma+\xi)^u} > 1. \tag{3.15}$$

This is a contradiction. Hence, $S(t) \geq \underline{s}$, $\forall\ t \geq t_2$. Consequently, we have from (3.14),

$$\dot{V}(t) \geq (\mu+\gamma+\xi)^u \left[\frac{\beta^l P^{\Delta}}{(\mu+\gamma+\xi)^u} - 1\right] \underline{s} > 0,\ \forall\ t \geq t_2, \tag{3.16}$$

which implies $V(t) \to +\infty$ as $t \to +\infty$. From Proposition 3.1, $V(t)$ is bounded. This is a contradiction. Hence, the claim is proved. From this claim, we will discuss the following two possibilities:

(i) $S(t) \geq \alpha$ for all large t.

(ii) $S(t)$ oscillates about α for all large t.

Finally, we will show that $S(t) \geq \alpha e^{-(\mu+\gamma+\xi)^u(h+\rho)}$ for sufficiently large t. Evidently, we only need to consider the case (ii). Let t_1 and t_2 be sufficiently large times satisfying:

$S(t_1) = S(t_2) = \alpha$,

$S(t) < \alpha$ as $t \in (t_1, t_2)$.

If $t_2 - t_1 \leq h+\rho$, since $\dot{S}(t) \geq -(\mu+\gamma+\xi)^u S(t)$ and $S(t_1) = \alpha$ which implies $S(t) \geq \alpha e^{-(\mu+\gamma+\xi)^u(h+\rho)}$, $\forall\ t \in [t_1, t_2]$. If $t_2 - t_1 > h + \rho$, then it is obvious that $S(t) \geq \alpha e^{-(\mu+\gamma+\xi)^u(h+\rho)}$, for all t in $[t_1, t_1 + h + \rho]$. By (3.13), we have $P(t) \geq P^{\Delta}$, $\forall\ t \in [t_1 + h + \rho, t_2]$. Thus, proceeding exactly as the proof of the above claim, we see that $S(t) \geq \alpha e^{-(\mu+\gamma+\xi)^u(h+\rho)}$, $\forall\ t \in [t_1 + h + \rho, t_2]$. If it is not true, then there exists a $T^* \geq 0$ such that $S(t) \geq \alpha e^{-(\mu+\gamma+\xi)^u(h+\rho)}$, $\forall\ t \in [t_1, t_1+h+\rho+T^*]$, $S(t_1+h+\rho+T^*) = \alpha e^{-(\mu+\gamma+\xi)^u(h+\rho)}$ and $\dot{S}(t_1+h+\rho+T^*) \leq 0$. Using the second equation of system (2.1), as $t = t_1 + h + \rho + T^*$, we have

$$\begin{aligned}
\dot{S}(t) &\geq \beta(t)P(t)\int_0^h S(t-s)d\eta(s) - (\mu(t)+\gamma(t)+\xi(t))S(t) \\
&\geq \{\beta^l P^{\Delta} - (\mu+\gamma+\xi)^u\}\alpha e^{-(\mu+\gamma+\xi)^u(h+\rho)} \\
&= (\mu+\gamma+\xi)^u \left[\frac{\beta^l P^{\Delta}}{(\mu+\gamma+\xi)^u} - 1\right] \alpha e^{-(\mu+\gamma+\xi)^u(h+\rho)} > 0,
\end{aligned}$$

since from (3.8), we have

$$\frac{\beta^l P^{\Delta}}{(\mu+\gamma+\xi)^u} > 1. \tag{3.17}$$

This is a contradiction. Therefore, $S(t) \geq \alpha e^{-(\mu+\gamma+\xi)^u(h+\rho)}$, $\forall\ t \in [t_1, t_2]$. Hence,

$$\liminf_{t\to\infty} S(t) \geq \alpha e^{-(\mu+\gamma+\xi)^u(h+\rho)} \equiv v_2 > 0.$$

This completes the proof of Proposition 3.3. □

Proposition 3.4. *Assume that* $R_0 > 1$, *then for any solution* $(P(t), S(t), X(t), Y(t), C(t))$ *of* (2.1) *with initial conditions* (3.1) *we have*

$$\liminf_{t\to\infty} X(t) \geq \left\{\frac{\gamma(1-\sigma)}{\mu+p}\right\}^l v_2 \equiv v_3 > 0, \tag{3.18}$$

where $v_2 > 0$ *is given in the Proposition 3.3.*

Proof. From the third equation of (2.1) and by Propositions 3.3, the result follows. □

Proposition 3.5. *Assume that* $R_0 > 1$, *then for any solution* $(P(t), S(t), X(t), Y(t), C(t))$ *of* (2.1) *with initial conditions* (3.1) *we have*

$$\liminf_{t\to\infty} Y(t) \geq \left\{\frac{\gamma\sigma}{\mu}\right\}^l v_2 \equiv v_4 > 0, \tag{3.19}$$

where $v_2 > 0$ *is given in the Proposition 3.3.*

Proof. From the fourth equation of (2.1) and by Propositions 3.3, the result follows. □

Proposition 3.6. *Assume that* $R_0 > 1$, *then for any solution* $(P(t), S(t), X(t), Y(t), C(t))$ *of* (2.1) *with initial conditions* (3.1) *we have*

$$\liminf_{t\to\infty} C(t) \geq \left\{\frac{\xi}{\mu+\delta}\right\}^l v_2 \equiv v_5 > 0, \tag{3.20}$$

where $v_2 > 0$ *is given in the Proposition 3.3.*

Proof. From the fifth equation of (2.1) and by Propositions 3.3, the result follows. □

Thus, the system (2.1) with initial conditions (3.1) is permanent provided

$$R_0 = \frac{\beta^l}{(\mu+\gamma+\xi)^u}\left(\frac{\Lambda^l}{\mu^u}\right) > 1.$$

Next, we shall use the following lemma to discuss the extinction of the smoking class.

Lemma 3.1. *Consider an autonomous delay differential equation*

$$\dot{x}(t) = a_1 \int_0^h x(t-s)d\eta(s) - a_2 x(t), \tag{3.21}$$

where a_1, a_2 *are two constants. If* $0 \leq a_1 < a_2$, *then for any solution* $x(t)$ *with initial condition* $\varphi(\theta) \geq 0,\ \theta \in [-h, 0]$, *we have*

$$\lim_{t\to\infty} x(t) = 0.$$

Proof. Let us define the following Lyapunov functional:

$$V(t) = \frac{x^2(t)}{2} + \frac{a_1}{2}\int_0^h \int_{t-s}^t x^2(u)dud\eta(s).$$

Then the time derivative along system (3.21) is given by

$$\begin{aligned}\dot{V}(t) &= a_1\int_0^h x(t)x(t-s)d\eta(s) + \frac{a_1}{2}\int_0^h \{x^2(t)-x^2(t-s)\}d\eta(s) - a_2x^2(t)\\ &= -\frac{a_1}{2}\int_0^h \{x(t)-x(t-s)\}^2 d\eta(s) + a_1x^2(t) - a_2x^2(t) \le -(a_2-a_1)x^2(t)\\ &\Rightarrow \lim_{t\to\infty} x(t) = 0.\end{aligned}$$

□

Theorem 3.3. *If* $R^* = \left(\frac{\beta^u}{\mu^l}\right)\left(\frac{\Lambda}{\mu}\right)^u < 1$, *then*

$$\lim_{t\to\infty}\{S(t)+X(t)+Y(t)+C(t)\} = 0, \tag{3.22}$$

i.e., smokers, smokers who temporarily quit smoking, smokers who permanently quit smoking and a class of smoking associated illness in system (2.1) *will be going to extinction.*

Proof. Let $U(t) = S(t)+X(t)+Y(t)+C(t)$. Adding second, third, fourth and fifth equations of system (2.1) , we have

$$\frac{dU(t)}{dt} \le \beta(t)P(t)\int_0^h U(t-s)d\eta(s) - \mu(t)U(t). \tag{3.23}$$

By condition (3.22), there exist a small enough $\epsilon > 0$ such that:

$$\left(\frac{\beta^u}{\mu^l}\right)\left\{\left(\frac{\Lambda}{\mu}\right)^u + \epsilon\right\} < 1.$$

By Proposition 3.1, given $\epsilon > 0$ (no matter however small), there exists a $t_1 > 0$ such that:

$$P(t) \le \left(\frac{\Lambda}{\mu}\right)^u + \epsilon, \text{ as } t \ge t_1.$$

Therefore, from (3.23):

$$\frac{dU(t)}{dt} \le \mu^l\left[\left(\frac{\beta^u}{\mu^l}\right)\left\{(\frac{\Lambda}{\mu})^u + \epsilon\right\}\int_0^h U(t-s)d\eta(s) - U(t)\right], \ \forall\, t \ge t_1.$$

Using the comparison theorem of functional differential equations and Lemma 3.1, we have

$$\lim_{t\to\infty} U(t) = \lim_{t\to\infty}\{S(t)+X(t)+Y(t)+C(t)\} = 0.$$

□

From (3.22) we conclude that the spread of the smokers should be controlled by way of suitable protective measures of the society to reduce the values of $\beta(t)$ (transmission rate function to smoking class when non-smoker individuals contact with smokers), $\Lambda(t)$ (recruitment rate function of the non-smoking (potential smoking) class from the larger embedding population) and thereby to decrease R^*. This policy should be carried over and will complete when $R^* < 1$.

4. Global asymptotic stability

In this section, we derive sufficient conditions for global asymptotic stability of system (2.1) with initial conditions (3.1). We now state a definition of global asymptotic stability of solutions of system (2.1).

Definition : System (2.1) with initial conditions (3.1) is said to be globally asymptotically stable if

$$\lim_{t\to\infty} \mid P_1(t) - P_2(t) \mid = 0,\ \lim_{t\to\infty} \mid S_1(t) - S_2(t) \mid = 0,\ \lim_{t\to\infty} \mid X_1(t) - X_2(t) \mid = 0,$$

$$\lim_{t\to\infty} \mid Y_1(t) - Y_2(t) \mid = 0 \text{ and } \lim_{t\to\infty} \mid C_1(t) - C_2(t) \mid = 0,$$

hold for any two solutions $(P_1(t), S_1(t), X_1(t), Y_1(t), C_1(t))$ and $(P_2(t), S_2(t), X_2(t), Y_2(t), C_2(t))$ of (2.1) with initial conditions of type (3.1).

Assume that $(P(t), S(t), X(t), Y(t), C(t))$ is a solution of (2.1). By the uniform boundedness of solutions of (2.1), there is an $A > 0$ (in fact, $A = (\frac{\Lambda}{\mu})^u + \epsilon$ where $\epsilon > 0$ can be made arbitrarily small) independent of initial conditions (3.1) such that

$$0 \le P(t) \le A,\ 0 \le S(t) \le A,\ 0 \le X(t) \le A,\ 0 \le Y(t) \le A,\ 0 \le C(t) \le A,$$

for large enough t. Without loss of generality, we may assume that

$$0 \le P(t) \le A,\ 0 \le S(t) \le A,\ 0 \le X(t) \le A,\ 0 \le Y(t) \le A,\ 0 \le C(t) \le A, \forall\ t \ge 0.$$

Theorem 4.1. *If there exist $c_1 > 0, c_2 > 0, c_3 > 0, c_4 > 0$ and $c_5 > 0$ such that the functions $B_i(t)$ $(i = 1, 2, 3, 4, 5)$ are nonnegative on $[0, \infty)$ and for any interval sequence $\{[a_i, b_i]\}_1^\infty, [a_i, b_i] \cap [a_j, b_j] = \phi$ and $b_i - a_i = b_j - a_j > 0$, for all $i, j = 1, 2, \ldots$ and $i \ne j$, one has $\sum_{k=1}^{\infty} \int_{a_k}^{b_k} B_i(t)dt = \infty$, then system (2.1) with initial conditions (3.1) is globally asymptotically stable. Here,*

$$\begin{aligned}
B_1(t) &= c_1\mu(t) - c_2 A\beta(t),\\
B_2(t) &= c_2\mu(t) + (c_2 - c_3)\gamma(t) + (c_2 - c_5)\xi(t) + (c_3 - c_4)\gamma(t)\sigma(t)\\
&\qquad - (c_1 + c_2)A\int_0^h \beta(t+s)d\eta(s), \qquad (4.1)\\
B_3(t) &= c_3\mu(t) + (c_3 - c_2)p(t),\\
B_4(t) &= c_4\mu(t),\ B_5(t) = c_5(\mu(t) + \delta(t)).
\end{aligned}$$

Proof. Assume that $(P_1(t), S_1(t), X_1(t), Y_1(t), C_1(t))$ and $(P_2(t), S_2(t), X_2(t), Y_2(t), C_2(t))$ are any two solutions of system (2.1) with initial conditions of type (3.1).

Define $V_1(t) = \mid P_1(t) - P_2(t) \mid$. Then the right-upper derivative of $V_1(t)$ along the solution of system (2.1) and (3.1) is given by

$$\begin{aligned} D^+V_1(t) &= sgn(P_1(t)-P_2(t))\{-\beta(t)(P_1(t)-P_2(t))\int_0^h S_1(t-s)d\eta(s) \\ &\quad + \beta(t)P_2(t)\int_0^h (S_2(t-s)-S_1(t-s))d\eta(s) - \mu(t)(P_1(t)-P_2(t))\} \\ &\le -\beta(t) \mid P_1(t)-P_2(t) \mid \int_0^h S_1(t-s)d\eta(s) \\ &\quad + \beta(t)P_2(t)\int_0^h \mid S_1(t-s)-S_2(t-s) \mid d\eta(s) - \mu(t) \mid P_1(t)-P_2(t) \end{aligned}$$

$$\Rightarrow D^+V_1(t) \le -\mu(t) \mid P_1(t)-P_2(t) \mid +\beta(t)A\int_0^h \mid S_1(t-s)-S_2(t-s) \mid d\eta(s). \quad (4.2)$$

Define $V_2(t) =\mid S_1(t)-S_2(t) \mid$. Calculating the right-upper derivative of $V_2(t)$ along the solution of system (2.1) and (3.1), we have

$$\begin{aligned} D^+V_2(t) &= sgn(S_1(t)-S_2(t))\Big\{\beta(t)(P_1(t)-P_2(t))\int_0^h S_1(t-s)d\eta(s) \\ &\quad + \beta(t)P_2(t)\int_0^h (S_1(t-s)-S_2(t-s))d\eta(s) \\ &\quad - (\mu(t)+\gamma(t)+\xi(t))(S_1(t)-S_2(t)) + p(t)(X_1(t)-X_2(t))\Big\} \end{aligned}$$

$$\begin{aligned} \Rightarrow D^+V_2(t) &\le -(\mu(t)+\gamma(t)+\xi(t)) \mid S_1(t)-S_2(t) \mid +p(t) \mid X_1(t)-X_2(t) \mid \\ &\quad + \beta(t)A \mid P_1(t)-P_2(t) \mid +\beta(t)A\int_0^h \mid S_1(t-s)-S_2(t-s) \mid d\eta(s) \end{aligned} \quad (4.3)$$

Define $V_3(t) =\mid X_1(t)-X_2(t) \mid$, $V_4(t) =\mid Y_1(t)-Y_2(t) \mid$ and $V_5(t) =\mid C_1(t)-C_2(t) \mid$. Calculating the right-upper derivatives of $V_3(t), V_4(t)$ and $V_5(t)$ along the solution of system (2.1) and (3.1), we have

$$\begin{aligned} D^+V_3(t) &= sgn(X_1(t)-X_2(t))\{\gamma(t)(1-\sigma(t))(S_1(t)-S_2(t)) \\ &\quad - (\mu(t)+p(t))(X_1(t)-X_2(t))\} \\ \Rightarrow\ D^+V_3(t) &\le \gamma(t)(1-\sigma(t)) \mid S_1(t)-S_2(t) \mid -(\mu(t)+p(t)) \mid X_1(t)-X_2(t) \mid . \end{aligned} \quad (4.4)$$

$$\begin{aligned} D^+V_4(t) &= sgn(Y_1(t)-Y_2(t))\{\gamma(t)\sigma(t)(S_1(t)-S_2(t)) \\ &\quad - \mu(t)(Y_1(t)-Y_2(t))\} \\ \Rightarrow\ D^+V_4(t) &\le \gamma(t)\sigma(t) \mid S_1(t)-S_2(t) \mid -\mu(t) \mid Y_1(t)-Y_2(t) \mid . \end{aligned} \quad (4.5)$$

$$\begin{aligned} D^+V_5(t) &= sgn(C_1(t)-C_2(t))\{\xi(t)(S_1(t)-S_2(t)) \\ &\quad - (\mu(t)+\delta(t))(C_1(t)-C_2(t))\} \\ \Rightarrow\ D^+V_5(t) &\le \xi(t) \mid S_1(t)-S_2(t) \mid -(\mu(t)+\delta(t)) \mid C_1(t)-C_2(t) \mid . \end{aligned} \quad (4.6)$$

Define $V_6(t)$ as

$$V_6(t) = \int_0^h \int_{t-s}^t \beta(u+s)A \mid S_1(u) - S_2(u) \mid du d\eta(s).$$

The right-upper derivative of $V_6(t)$ along the solution of system (2.1) and (3.1) is given below:

$$D^+V_6(t) = A \mid S_1(t) - S_2(t) \mid \int_0^h \beta(t+s)d\eta(s) - \beta(t)A \int_0^h \mid S_1(t-s) - S_2(t-s) \mid d\eta(s). \quad (4.7)$$

Let $V(t) = c_1V_1(t) + c_2V_2(t) + c_3V_3(t) + c_4V_4(t) + c_5V_5(t) + (c_1 + c_2)V_6(t)$, then by using (4.2)-(4.7), we have

$$\begin{aligned} D^+V(t) \le & -B_1(t) \mid P_1(t) - P_2(t) \mid -B_2(t) \mid S_1(t) - S_2(t) \mid \\ & - B_3(t) \mid X_1(t) - X_2(t) \mid -B_4(t) \mid Y_1(t) - Y_2(t) \mid \\ & - B_5(t) \mid C_1(t) - C_2(t) \mid, \ \forall \ t \ge h, \end{aligned} \quad (4.8)$$

where $B_i(t)$, $(i = 1, 2, 3, 4, 5)$ are defined in (4.1).

Integrating (4.8) from h to t, we have

$$\begin{aligned} & \int_h^t \{B_1(t) \mid P_1(t) - P_2(t) \mid +B_2(t) \mid S_1(t) - S_2(t) \mid +B_3(t) \mid X_1(t) - X_2(t) \mid \\ & \quad + B_4(t) \mid Y_1(t) - Y_2(t) \mid +B_5(t) \mid C_1(t) - C_2(t) \mid\}dt \le V(h) - V(t) \end{aligned}$$

$$\begin{aligned} \Rightarrow \ & \int_h^t \{B_1(t) \mid P_1(t) - P_2(t) \mid +B_2(t) \mid S_1(t) - S_2(t) \mid +B_3(t) \mid X_1(t) - X_2(t) \mid \\ & \quad + B_4(t) \mid Y_1(t) - Y_2(t) \mid +B_5(t) \mid C_1(t) - C_2(t) \mid\}dt < \infty. \end{aligned} \quad (4.9)$$

By assumptions about $B_i(t)$, $(i = 1, 2, 3, 4, 5)$ and the boundedness of $(P_1(t), S_1(t), X_1(t), Y_1(t), C_1(t))$ and $(P_2(t), S_2(t), X_2(t), Y_2(t), C_2(t))$ on $[0, \infty)$, we obtain from system (2.1) that $\mid P_1(t) - P_2(t) \mid$, $\mid S_1(t) - S_2(t) \mid$, $\mid X_1(t) - X_2(t) \mid$, $\mid Y_1(t) - Y_2(t) \mid$ and $\mid C_1(t) - C_2(t) \mid$ are bounded and uniformly continuous on $[0, \infty)$. It follows from (4.9) that,

$$\begin{aligned} & \lim_{t\to\infty} \mid P_1(t) - P_2(t) \mid = 0, \ \lim_{t\to\infty} \mid S_1(t) - S_2(t) \mid = 0, \ \lim_{t\to\infty} \mid X_1(t) - X_2(t) \mid = 0, \\ & \lim_{t\to\infty} \mid Y_1(t) - Y_2(t) \mid = 0, \ \lim_{t\to\infty} \mid C_1(t) - C_2(t) \mid = 0. \end{aligned}$$

This shows that system (2.1) with initial conditions (3.1) is globally asymptotically stable. This completes the proof. □

Corollary 4.1. *If there exist $c_1 > 0, c_2 > 0, c_3 > 0, c_4 > 0$ and $c_5 > 0$ such that*

$$\begin{aligned} & \liminf_{t\to\infty} \{c_1\mu(t) - c_2A\beta(t)\} > 0, \\ & \liminf_{t\to\infty} \{c_2\mu(t) + (c_2 - c_3)\gamma(t) + (c_2 - c_5)\xi(t) + (c_3 - c_4)\gamma(t)\sigma(t) \\ & \qquad - (c_1 + c_2)A \int_0^h \beta(t+s)d\eta(s)\} > 0, \\ & \liminf_{t\to\infty} \{c_3\mu(t) + (c_3 - c_2)p(t)\} > 0, \end{aligned}$$

then system (2.1) *with initial conditions* (3.1) *is globally asymptotically stable.*

We observe that the lower values of $\beta(t)$ (transmission rate function to smoking class when non-smoker individuals contact with smokers) and $\Lambda(t)$ (recruitment rate function of the non-smoking (potential smoking) class from the larger embedding population) are leading to make $B_i(t) > 0$ $(i = 1, 2)$ which also keep the spread of the epidemic under control. The results of the Theorem 4.1 and Corollary 4.1 indicate that these parametric functions and also time delay have an effect on the global asymptotic stability, which may rule out any complicated behaviour (eg. limit cycles, chaos) of the proposed model.

From our everyday experience we know that the biological and environmental parameters are subject to fluctuation in time, the effects of a periodically varying environment have important selective forces on systems in a fluctuating environment. To investigate this kind of phenomenon, in the model, the coefficients should be periodic functions of time. Let us state a theorem related to this.

Theorem 4.2 ([17])**.** *If system* (2.1) *is $\psi-$periodic and there are positive constants v_i and M_i $(i = 1, 2, 3, 4, 5)$ such that:*

$$v_1 \le \liminf_{t\to\infty} P(t) \le \limsup_{t\to\infty} P(t) \le M_1, \ \ ; v_2 \le \liminf_{t\to\infty} S(t) \le \limsup_{t\to\infty} S(t) \le M_2,$$

$$v_3 \le \liminf_{t\to\infty} X(t) \le \limsup_{t\to\infty} X(t) \le M_3, \ \ v_4 \le \liminf_{t\to\infty} Y(t) \le \limsup_{t\to\infty} Y(t) \le M_4,$$

$$v_5 \le \liminf_{t\to\infty} C(t) \le \limsup_{t\to\infty} C(t) \le M_5,$$

hold for any solution $(P(t), S(t), X(t), Y(t), C(t))$ of (2.1) with initial conditions (3.1), then system (2.1) has positive periodic solution with period ψ.

Corollary 4.2. *If system* (2.1) *is $\psi-$periodic and conditions in Theorems 3.2 and 4.1 are valid, then there exists a unique positive $\psi-$periodic solution which is globally asymptotically stable.*

5. Numerical Simulation

In this section we present computer simulation of some solution of the system (2.1) using MATLAB.

Example 1. *Let* $\Lambda(t) = 24 + 2\sin t,\ \mu(t) = 12 + \sin t,\ \beta(t) = 2 + \sin t,\ \gamma(t) = 2 + \sin t,\ \sigma(t) = 0.5,\ p(t) = 2 + \sin t,\ \xi(t) = 2 + \sin t,\ \delta(t) = 2 + \cos t,\ \eta(s) = \frac{s}{h}, h = \frac{\pi}{2}$.

In this case $R_0 < 1$ and $R^* < 1$. Therefore, system (2.1) is not permanent and the disease in system (2.1) will be going to extinction. If you choose $c_1 = c_2 = 1, c_3 = c_4 = c_5 = 0.5$ then system (2.1) satisfies all the assumptions in Theorem 4.1 and Corollary 4.1 and hence system (2.1) with initial conditions of type (3.1) is globally asymptotically stable. Fig.1, Fig.2, Fig.3, Fig.4 and Fig.5 show trajectories of $P(t), S(t), X(t), Y(t)$ and $C(t)$ respectively for different initial conditions.

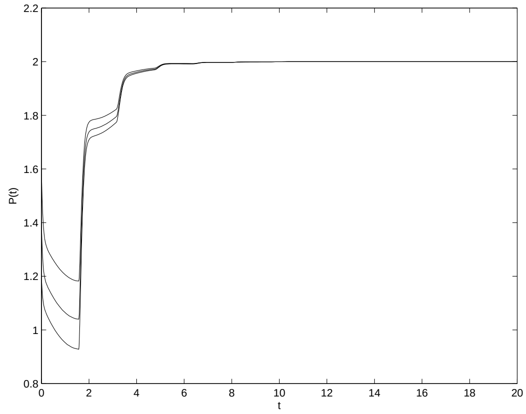

Fig.1. *Trajectories of P(t) for different initial conditions.*

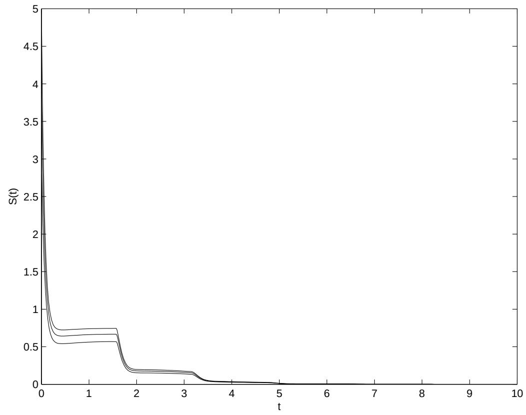

Fig.2. *Trajectories of S(t) for different initial conditions.*

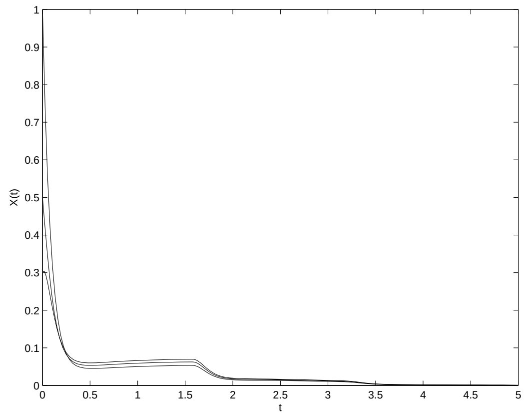

Fig.3. *Trajectories of X(t) for different initial conditions.*

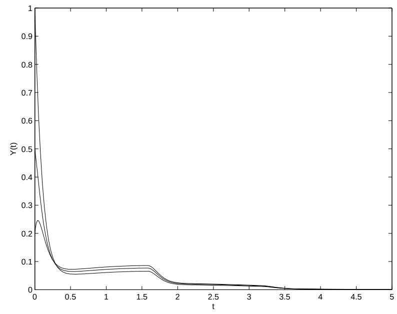

Fig.4. *Trajectories of Y(t) for different initial conditions.*

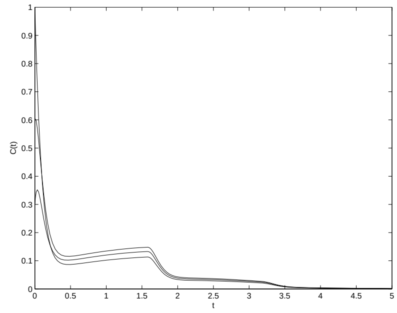

Fig.5. *Trajectories of C(t) for different initial conditions.*

Example 2. *Let* $\Lambda(t) = 48 + 6\sin t,\ \mu(t) = 8 + \sin t,\ \beta(t) = 6 + \sin t,\ \gamma(t) = 2 + \sin t,\ \sigma(t) = 0.5,\ p(t) = 2 + \sin t,\ \xi(t) = 2 + \sin t,\ \delta(t) = 2 + \cos t,\ \eta(s) = \frac{s}{h}, h = \frac{\pi}{2}$.

In this case $R_0 > 1$ and $R^* > 1$. Therefore, system (2.1) is permanent. In this case system (2.1) satisfies all the assumptions in Theorem 4.1 and Corollary 4.1 and hence system (2.1) with initial conditions of type (3.1) is globally asymptotically stable. Fig.6, Fig.7, Fig.8, Fig.9 and Fig.10 show trajectories of $P(t), S(t), X(t), Y(t)$ and $C(t)$ respectively for different initial conditions.

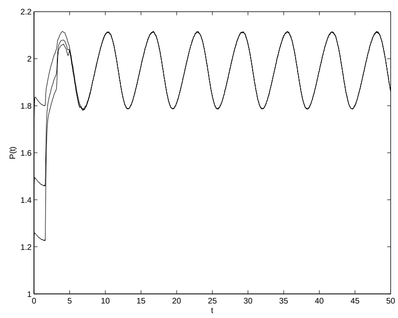

Fig.6. *Trajectories of P(t) for different initial conditions.*

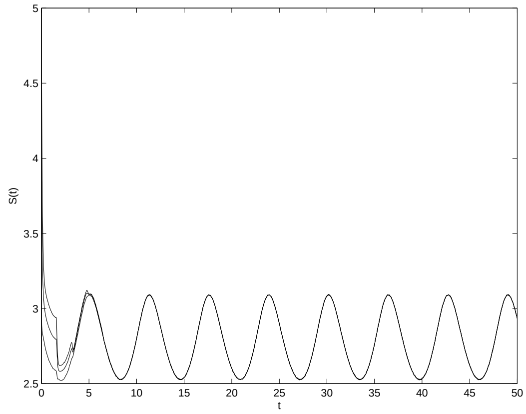

Fig.7. *Trajectories of S(t) for different initial conditions.*

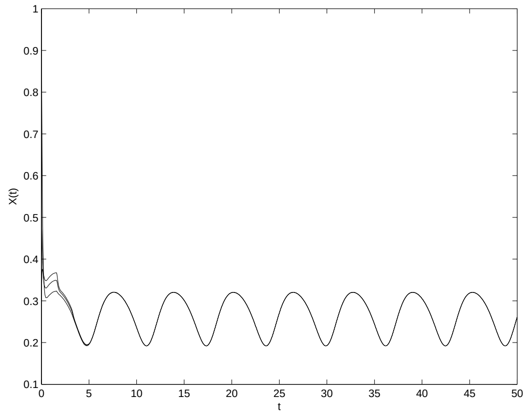

Fig.8. *Trajectories of X(t) for different initial conditions.*

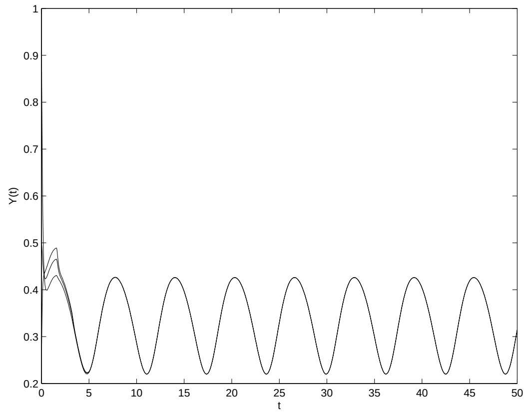

Fig.9. *Trajectories of Y(t) for different initial conditions.*

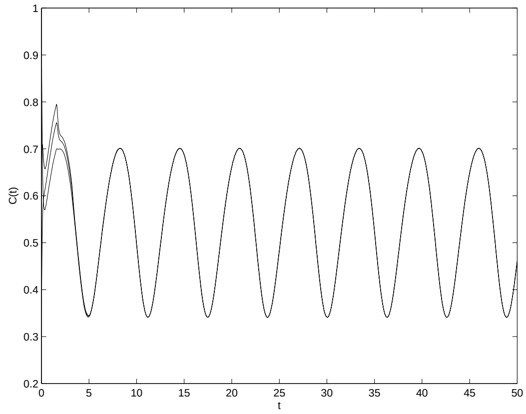

Fig.10. *Trajectories of C(t) for different initial conditions.*

6. Conclusions

Smoking cessation has major and immediate health benefits for men and women of all ages. Quitting smoking decreases the risk of lung and other cancers, heart attack, stroke and chronic lung disease. The earlier a person quits, the greater the health benefit. For example, research has shown that people who quit before age 50 reduce their risk of dying in the next 15 years by half compared with those who continue to smoke [1]. Smoking low-yield cigarettes, as compared to cigarettes with higher tar and nicotine, provides no clear benefit to health [18]. In this paper we have considered a dynamical mathematical model of the sub-populations of potential smokers (non-smokers), smokers, smokers who temporarily quit smoking, smokers who permanently quit smoking and a class of smoking associated illness by introducing time dependent parameters and distributed time delay to acquire smoking habit. Here, we have established some sufficient conditions on the permanence and extinction of the smoking class in the community by using inequality analytical technique. We have introduced some new threshold values $R_0 = \frac{\beta^l}{(\mu+\gamma+\xi)^u}(\frac{\Lambda^l}{\mu^u})$ and $R^* = (\frac{\beta^u}{\mu^l})(\frac{\Lambda}{\mu})^u$. We have obtained that the system (2.1) will be permanent, this means that the long-term survival (i.e., will not vanish in time) of all components of the system (2.1), with initial conditions (3.1) when $R_0 > 1$ and the smoking class in the community will be going to extinct when $R^* < 1$. By Lyapunov functional method, we have also obtained some sufficient conditions for global asymptotic stability of this model. The public health implication of the result of our mathematical analysis is that the spread of the smokers should be controlled by way of suitable protective measures of the society to reduce the values of $\beta(t)$ (transmission rate function to smoking class when non-smoker individuals contact with smokers), $\Lambda(t)$ (recruitment rate function of the non-smoking (potential smoking) class from the larger embedding population) and thereby to decrease R^*. This policy should be carried over and will complete when $R^* < 1$. We observe that the lower values of $\beta(t)$ and $\Lambda(t)$ are leading to make $B_i(t) > 0$ $(i = 1, 2)$ which also keep the spread of the epidemic under control. The results of the Theorem 4.1 and Corollary 4.1 indicate that these parametric functions and also time delay have an effect on the global asymptotic stability, which may rule out any complicated behaviour (eg. limit cycles, chaos) of the proposed model. We have also observed that the time delay has no effect on the permanence of the system but it has an effect on the global asymptotic stability of this model. Our analytical results are illustrated through computer simulations. The aim of the analysis of this model is to identify the parameters of interest for further study, with a view to informing and assisting policy-maker in targeting prevention and treatment resources for maximum effectiveness.

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