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EXISTENCE FOR A NONLINEAR IMPULSIVE FUNCTIONAL INTEGRODIFFERENTIAL EQUATION WITH NONLOCAL CONDITIONS IN BANACH SPACES

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Abstract. In this paper, we consider the existence of mild solutions for a certain class of nonlinear impulsive functional evolution integrodifferential equation with nonlocal conditions in Banach spaces. A sufficient condition is established by using Schaefer's fixed point theorem combined with an evolution system. An example is also given to illustrate our result.

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1. Introduction

The purpose of this paper is to prove the existence of mild solutions for nonlinear impulsive functional evolution integrodifferential equation with nonlocal conditions of the form

$$x'(t) = A(t)x(t) + F\left(t, x(\sigma_1(t)), \dots, x(\sigma_n(t)), \int_0^t h(t,s,x(\sigma_{n+1}(s)))ds\right),$$
$$t \in J = [0,b], \quad t \neq t_k, \quad k = 1, \dots, m, \tag{1}$$

$$x(0) + g(x) = x_0, \tag{2}$$

$$\Delta x(t_k) = I_k(x(t_k)), \quad k = 1, \dots, m, \tag{3}$$

where the unknown $x(\cdot)$ takes values in the Banach space X, and the family $\{A(t) : 0 \leq t \leq b\}$ of unbounded linear operator generates a linear evolution systems. $F : J \times X^{n+1} \to X$, $h : J \times J \times X \to X$, $g : PC(J,X) \to X$, $\sigma_i : J \to J, i = 1, ..., n+1$, $I_k : X \to X, k = 1, 2, \dots, m$, are appropriate functions; $0 < t_1 < \dots < t_m < b$, are prefixed points and the symbol $\Delta x(t_k) =$

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$x(t_k^+) - x(t_k^-)$, where $x(t_k^-)$ and $x(t_k^+)$ represent the right and left limits of $x(t)$ at $t = t_k$, respectively.

The theory of impulsive differential and partial differential have become an important area of investigation in the past two decades because of their applications to various problems arising in communications, control technology, impact mechanics, electrical engineering, medicine, and biology, see the monographs of Bainov and Semeonov [4], Lakshmikantham et al. [20], and Samoilenko and Perestyuk [28], the papers [1,5,17,18,26,27] and the references therein. Recently, several authors [15,24,29] have investigated the impulsive integrodifferential equations in abstract spaces. We refer to the books of Erbe et al. [12], Hale [16], and Henderson [19], and the references cited therein.

The nonlocal Cauchy problem was initiated by Byszewski [7,8]. The importance of the problem consists in the fact that it is more general and has a better effect than the classical initial condition. Subsequently, it has been studied extensively under various conditions on A(or $A(t)$) and F, g by several authors [9,11,22,23,30]. Very recently, there have been extensive study of impulsive differential equations or inclusions with nonlocal conditions, and concerning this matter we cite the pioneer works Liang et al. [21], Benchohra et al. [6], Abada et al. [1], and Anguraj et al. [3] studied the existence, uniqueness and continuous dependence of mild solution of a nonlocal Cauchy problem for an impulsive neutral functional differential evolution equation. The purpose of this paper is to continue the study of these authors. We get the existence results for mild solutions of problem (1)-(3) when the nonlocal item g is only depends upon the continuous properties on $PC(J, X)$. Our results are based on the theory of evolution families, the Banach contraction principle and Schaefer's fixed point theorem.

The rest of this paper is organized as follows. In Section 2, we introduce some notations and necessary preliminaries. In Section 3, we prove the existence results of mild solutions of system (1)-(3). Finally, a concrete example is presented in Section 4 to show the application of our main results.

2. Preliminaries

In this section, we shall introduce some basic definitions, lemmas which are used throughout this paper.

Let $C(J, X)$ denote the Banach space of continuous functions from J into X with the norm

$$\| x \|_J = \sup\{\| x(t) \| : t \in J\}$$

and let $L(X)$ denote the Banach space of bounded linear operators from X to X.

A measurable function $x : J \to X$ is Bochner integrable if and only if $\| x \|$ is Lebesgue integrable (For properties of the Bochner integral see Yosida [31]). $L^1(J, X)$ denotes the Banach space of measurable functions $x : J \to X$ which

are Bochner integrable normed by

$$\| x \|_{L^1} = \int_0^b \| x(t) \| \, dt \quad \text{for all} \quad x \in L^1(J, X).$$

For the family $\{A(t) : 0 \leq t \leq b\}$ of linear operators, we need the following assumptions(see [13]).

(I) the domain $D(A)$ of $\{A(t) : 0 \leq t \leq b\}$ is dense in the Banach space X and independent of t, $A(t)$ is a closed linear operator.

(II) For each $t \in J$, the resolvent $R(\lambda, A(t))$ exists for all λ with $\mathrm{Re}\lambda \geq 0$ and there exists $K > 0$ such that

$$\| R(\lambda, A(t)) \| \leq \frac{K}{(|\lambda| + 1)}.$$

(III) For any $t, s, \tau \in J$, there exists a $0 < \delta < 1$ and $K > 0$ such that

$$\| (A(t) - A(\tau))A^{-1}(s) \| \leq K|t - \tau|^{\delta}.$$

And for each $t \in J$ and some $\lambda \in \rho(A(t))$, the resolvent $R(\lambda, A(t))$ set of $A(t)$ is a compact operator.

In order to define the solution of (1)- (3), we introduce the space $PC([0, b], X) = \{x : J \to X : x(t)$ is continuous at $t \neq t_k$ and left continuous at $t = t_k$ and the right limit $x(t_k^+)$ exists for $k = 1, 2..., m\}$, which is a Banach space with the norm

$$\| x \|_{PC} := \sup_{t \in J} \| x(t) \| .$$

Then $PC(J, X)$ is a Banach space.

To simplify the notations, we put $t_0 = 0, t_{m+1} = b$ and for $x \in PC([0, b], X)$ we denote by $\hat{x}_k \in C([t_k, t_{k+1}]; X), k = 0, 1, ..., m$, the function given by

$$\hat{x}_k(t) := \begin{cases} x(t) & \text{for } \ t \in (t_k, t_{k+1}], \\ x(t_k^+) & \text{for } \ t = t_k. \end{cases}$$

Moreover, for $B \subseteq PC([0, b], X)$ we denote by $\hat{B}_k, k = 0, 1, ..., m$, the set $\hat{B}_k = \{\hat{x}_k : x \in B\}$.

Next, to set the framework for our main result, we will make use of the following definitions and lemmas.

Definition 2.1. [25] *A family of linear operators $\{U(t, s) : 0 \leq s \leq t \leq b\}$ on X is called an evolution system if the following conditions hold:*

(a) *$U(t, s) \in B(X)$ the space of bounded linear transformations on X whenever $0 \leq s \leq t \leq b$ and for each $x \in X$ the mapping $(t, s) \to U(t, s)x$ is continuous;*

(b) *$U(t, s)U(s, \tau) = U(t, \tau)$ whenever $0 \leq \tau \leq s \leq t \leq b$.*

Definition 2.2. *A function $x(\cdot) \in PC(J, X)$ is said to be a mild solution to problem* (1)-(3) *if it satisfies the following integral equation*

$$x(t) = U(t,0)[x_0 - g(x)] + \sum_{0<t_k<t} U(t,t_k)I_k(x(t_k)) + \int_0^t U(t,s) \times F\Big(s, x(\sigma_1(s)), ..., x(\sigma_n(s)), \int_0^s h(s,\tau,x(\sigma_{n+1}(\tau)))d\tau\Big)ds, \ t \in J. \quad (4)$$

Lemma 2.1. A set $B \subseteq PC([0,b],X)$ is relatively compact in $PC([0,b],X)$ if, and only if, the set $\tilde{B}_k$ is relatively compact in $C([t_k,t_{k+1}];X)$, for every $k = 0, 1, ..., m$.

Lemma 2.2.[14] *If conditions* (I)-(III) *are satisfied, then the family* $\{A(t) : 0 \leq t \leq b\}$ *generates a unique linear evolution system* $\{U(t,s) : 0 \leq s \leq t \leq b\}$ *is a compact linear operator on* X *whenever* $t - s > 0 (0 \leq s < t \leq b)$.

Lemma 2.3. (Schaefer's fixed point theorem [10]) *Let E be a normed linear space. Let $Q : E \to E$ be a completely continuous operator, that is, it is continuous and the image of any bounded set is contained in a compact set and let*

$$\zeta(Q) = \{x \in E : x = \lambda Qx \text{ for some } 0 < \lambda < 1\}.$$

Then either $\zeta(Q)$ is unbounded or Q has a fixed point.

Further we assume the following hypotheses:

(H1) $U(t,s)$ is a compact linear operator on X whenever $t - s > 0$ and there exists a constant $M > 0$, such that $\| U(t,s) \| \leq M, \ \ 0 \leq s < t \leq b$.

(H2) The function $F : J \times X^{n+1} \to X$ is continuous and there exists constants $L > 0, L_1 > 0$, such that for all $x_i, y_i \in X, i = 1, ..., n+1$, we have

$$\| F(t,x_1,x_2,...,x_{n+1}) - F(t,y_1,y_2,...,y_{n+1}) \| \leq L\Big[\sum_{i=1}^{n+1} \| x_i - y_i \|\Big],$$

and

$$L_1 = \max_{t \in J} \| F(t,0,...,0) \| .$$

(H3) The function $h : J \times J \times X \to X$ is continuous and there exists constants $N > 0, N_1 > 0$, such that for all $x, y \in X$,

$$\| h(t,s,x) - h(t,s,y) \| \leq N \| x - y \|,$$

and

$$N_1 = \max_{0 \leq s \leq t \leq b} \| h(t,s,0) \| .$$

(H4) $\sigma_i : J \to J, i = 1, ..., n+1$, are continuous functions such that $\sigma_i(t) \leq t, i = 1, ..., n+1$.

(H5) $I_k \in C(X, X), k = 1, ..., m$ are all compact operators, and there exist constants $c_k > 0, k = 1, ..., m$ such that

$$0 \leq \limsup_{\|x\| \to \infty} \frac{\| I_k(x) \|}{\| x \|} \leq c_k, \qquad k = 1, ..., m, \ x \in X.$$

(H6) (a) The function $g(\cdot) : PC(J, X) \to X$ is continuous and there exists a $\delta \in (0, t_1)$ such that $g(\phi) = g(\psi)$ for any $\phi, \psi \in PC(J, X)$ with $\phi = \psi$ on $[\delta, b]$.

(b) There is a constant $c > 0$ such that

$$0 \leq \limsup_{\|\phi\|_{PC} \to \infty} \frac{\| g(\phi) \|}{\| \phi \|_{PC}} \leq c, \qquad \phi \in PC(J, X),$$

and

$$M\left(c + \sum_{k=1}^{m} c_k\right) e^{ML(n+Nb)} < 1. \tag{5}$$

3. Main results

Theorem 3.1. *Suppose that assumptions* (H1)-(H6) *are satisfied, then the impulsive nonlocal Cauchy problem* (1)-(3) *has at least one mild solution on J.*

Proof. Let $L_0 := 2ML(n + Nb) > 0$ and we introduce in the space $PC(J, X)$ the equivalent norm defined as

$$\| \phi \|_V := \sup_{t \in J} e^{-L_0 t} \| \phi(t) \| .$$

Then, it is easy to see that $V := (PC(J, X), \| \cdot \|_V)$ is a Banach space. Fix $v \in PC(J, X)$ and for $t \in J, \phi \in V$, we now define an operator

$$(Q_v\phi)(t) = U(t, 0)[x_0 - g(v)] + \sum_{0 < t_k < t} U(t, t_k) I_k(v(t_k)) + \int_0^t U(t, s) \times F\left(s, \phi(\sigma_1(s)), ..., \phi(\sigma_n(s)), \int_0^s h(s, \tau, \phi(\sigma_{n+1}(\tau)))d\tau\right) ds. \tag{6}$$

Since $U(\cdot, 0)(x_0 - g(v)) \in PC(J, X)$, it follows, from (H1)-(H4) that $(Q_v\phi)(t) \in V$ for all $\phi \in V$. Let $\phi, \psi \in V$. We have

$$e^{-L_0 t} \| (Q_v\phi)(t) - (Q_v\psi)(t) \| \leq e^{-L_0 t} \int_0^t \Bigg\| U(t, s) \Bigg[F\left(s, \phi(\sigma_1(s)), ..., \phi(\sigma_n(s)), \int_0^s h(s, \tau, \phi(\sigma_{n+1}(\tau)))d\tau\right)$$

$$
\begin{aligned}
&- F\Big(s, \psi(\sigma_1(s)), ..., \psi(\sigma_n(s)), \int_0^s h(s,\tau,\psi(\sigma_{n+1}(\tau)))d\tau\Big)\Big]\Big\| ds \\
&\leq ML\int_0^t e^{-L_0 t}\Big[\parallel \phi(\sigma_1(s)) - \psi(\sigma_1(s)) \parallel + \cdots + \parallel \phi(\sigma_n(s)) - \psi(\sigma_n(s)) \parallel \\
&+ \Big\| \int_0^s h(s,\tau,\phi(\sigma_{n+1}(\tau)))d\tau - \int_0^s h(s,\tau,\psi(\sigma_{n+1}(\tau)))d\tau \Big\|\Big] ds \\
&\leq ML\int_0^t e^{-L_0 t}\Big[e^{L_0\sigma_1(s)}\sup_{s\in J} e^{-L_0 s} \parallel \phi(s) - \psi(s) \parallel \\
&+ \cdots + e^{L_0\sigma_n(s)}\sup_{s\in J} e^{-L_0 s} \parallel \phi(s) - \psi(s) \parallel \\
&+ N\int_0^s \parallel \phi(\sigma_{n+1}(\tau)) - \psi(\sigma_{n+1}(\tau)) \parallel d\tau\Big] ds \\
&\leq ML\int_0^t e^{-L_0 t}\Big[ne^{L_0 s}\sup_{s\in J} e^{-L_0 s} \parallel \phi(s) - \psi(s) \parallel \\
&+ Nbe^{L_0\sigma_{n+1}(s)}\sup_{s\in J} e^{-L_0 s} \parallel \phi(s) - \psi(s) \parallel\Big] ds \\
&\leq ML\int_0^t e^{L_0(s-t)}\Big[n\sup_{s\in J} e^{-L_0 s} \parallel \phi(s) - \psi(s) \parallel \\
&+ Nb\sup_{s\in J} e^{-L_0 s} \parallel \phi(s) - \psi(s) \parallel\Big] ds \\
&\leq ML(n+Nb)\int_0^t e^{L_0(s-t)} ds \parallel \phi - \psi \parallel_V \leq \frac{ML(n+Nb)}{L_0} \parallel \phi - \psi \parallel_V, \quad t \in J,
\end{aligned}
$$

which implies that

$$
e^{-L_0 t} \parallel (Q_v\phi)(t) - (Q_v\psi)(t) \parallel \leq \frac{1}{2} \parallel \phi - \psi \parallel_V, \quad t \in J.
$$

Thus

$$
\parallel Q_v\phi - Q_v\psi \parallel_V \leq \frac{1}{2} \parallel \phi - \psi \parallel_V, \quad \phi, \psi \in V.
$$

Therefore, Q_v is a strict contraction. By the Banach contraction principle, we conclude that Q_v has a unique fixed point $\phi_v \in V$ and Eq. (6) has a unique mild solution on $[0, b]$. Set

$$
\tilde{v}(t) := \begin{cases} v(t) & \text{if } t \in (\delta, b], \\ v(\delta) & \text{if } t \in [0, \delta]. \end{cases}
$$

From (6), we have

$$
\phi_{\tilde{v}}(t) = U(t,0)[x_0 - g(\tilde{v})] + \sum_{0<t_k<t} U(t,t_k)I_k(v(t_k)) + \int_0^t U(t,s)
$$

$$\times F\bigg(s, \phi_{\tilde{v}}(\sigma_1(s)), ..., \phi_{\tilde{v}}(\sigma_n(s)), \int_0^s h(s, \tau, \phi_{\tilde{v}}(\sigma_{n+1}(\tau)))d\tau\bigg)ds. \tag{7}$$

Consider the map $\Gamma : PC_\delta = PC([\delta, b], X) \to PC_\delta$ defined by

$$(\Gamma v)(t) = \phi_{\tilde{v}}(t), \quad t \in [\delta, b]. \tag{8}$$

We shall show that Γ satisfies all conditions of Lemma 2.3. The proof will be given in several steps.

Step 1. The set $\Omega = \{v \in PC_\delta : \lambda \in (0,1), v = \lambda P(v)\}$ is bounded.

Indeed, let $\lambda \in (0,1)$ and let $v \in C_\delta$ be a possible solution of $v = \lambda P(v)$ for some $0 < \lambda < 1$. This implies, by (7) and (8), that for each $t \in (0, b]$ we have

$$v(t) = \lambda\phi_{\tilde{v}}(t) = \lambda U(t,0)[x_0 - g(\tilde{v})] + \lambda \sum_{0<t_k<t} U(t,t_k)I_k(v(t_k)) + \lambda \int_0^t U(t,s)$$
$$\times F\bigg(s, \phi_{\tilde{v}}(\sigma_1(s)), ..., \phi_{\tilde{v}}(\sigma_n(s)), \int_0^s h(s, \tau, \phi_{\tilde{v}}(\sigma_{n+1}(\tau)))d\tau\bigg)ds. \tag{9}$$

In view of (H5)and (H6), there exist positive constants $\epsilon_k (k = 1, ..., m), \epsilon, \gamma$ and $\bar{\gamma}$ such that, for all $\| v \| > \gamma$ and $\| \phi \|_{PC} > \bar{\gamma}$

$$\| I_k(v) \| \leq (c_k + \epsilon_k) \| v \|, \quad \| g(\phi) \| \leq (c + \epsilon) \| \phi \|_{PC}, \tag{10}$$

$$M(c + \epsilon + \sum_{k=1}^{m}(c_k + \epsilon_k))e^{ML(n+Nb)} < 1. \tag{11}$$

Let

$$E_1 = \{v : \| v \| \leq \gamma\}, \quad E_2 = \{v : \| v \| > \gamma\},$$
$$F_1 = \{\phi : \| \phi \|_{PC} \leq \bar{\gamma}\}, \quad F_2 = \{\phi : \| \phi \|_{PC} > \bar{\gamma}\},$$
$$C_1 = \max\{\| I_k(v) \|, v \in E_1\}, \quad C_2 = \max\{\| g(\phi) \|, \phi \in F_1\}.$$

Thus,

$$\| I_k(v) \| \leq C_1 + (c_k + \epsilon_k) \| v \|, \tag{12}$$

$$\| g(\phi) \| \leq C_2 + (c + \epsilon) \| \phi \|_{PC} . \tag{13}$$

By (H1)-(H4), (12) and (13), from (9) we have for each $t \in (0, b]$, $\| v(t) \| \leq \| \phi_{\tilde{v}}(t) \|$ and

$$\| \phi_{\tilde{v}}(t) \| \leq \| U(t,0)[x_0 - g(\tilde{v})] \| + \bigg\| \sum_{0<t_k<t} U(t,t_k)I_k(v(t_k)) \bigg\| + \int_0^t \bigg\| U(t,s)$$
$$\times F\bigg(s, \phi_{\tilde{v}}(\sigma_1(s)), ..., \phi_{\tilde{v}}(\sigma_n(s)), \int_0^s h(s, \tau, \phi_{\tilde{v}}(\sigma_{n+1}(\tau)))d\tau\bigg)\bigg\| ds$$

$$
\begin{aligned}
&\leq M[\| x_0 + g(\tilde{v}) \|] + M\sum_{k=1}^{m} \| I_k(v(t_k)) \| \\
&\quad + M\int_0^t \Bigg[\Bigg\| F\Bigg(s, \phi_{\tilde{v}}(\sigma_1(s)), ..., \phi_{\tilde{v}}(\sigma_n(s)), \int_0^s h(s,\tau,\phi_{\tilde{v}}(\sigma_{n+1}(\tau)))d\tau\Bigg) \\
&\quad - F(s,0,...,0)\Bigg\| + \| F(s,0,...,0) \|\Bigg] ds \\
&\leq M[\| x_0 \| + C_2 + (c+\epsilon) \| \tilde{v} \|_{PC}] + M\sum_{k=1}^{m}[C_1 + (c_k+\epsilon_k) \| v(t_k) \|] \\
&\quad + M\int_0^t \Bigg\{L\Bigg[\sup_{s\in(0,b]} \| \phi_{\tilde{v}}(s) \| + \cdots + \sup_{s\in(0,b]} \| \phi_{\tilde{v}}(s) \| \\
&\quad + N\int_0^s [\| h(s,\tau,\phi_{\tilde{v}}(\sigma_{n+1}(\tau))) - h(s,\tau,0) \| + \| h(s,\tau,0) \|]d\tau\Bigg] + L_1\Bigg\} ds \\
&\leq M[\| x_0 \| + C_2 + (c+\epsilon) \| \tilde{v} \|_{PC}] + M\sum_{k=1}^{m}[C_1 + (c_k+\epsilon_k) \| v(t_k) \|] \\
&\quad + M\int_0^t \Bigg\{L\Bigg[n \sup_{s\in(0,b]} \| \phi_{\tilde{v}}(s) \| + b(N \sup_{s\in(0,b]} \| \phi_{\tilde{v}}(s) \| + N_1)\Bigg] + L_1\Bigg\} ds \\
&\leq M^* + M(c+\epsilon) \| \tilde{v} \|_{PC} + M\sum_{k=1}^{m}(c_k+\epsilon_k) \| v(t_k) \| \\
&\quad + ML(n+Nb)\int_0^t \sup_{s\in(0,b]} \| \phi_{\tilde{v}}(s) \| ds,
\end{aligned}
$$

where $M^* = M[\| x_0 \| + C_2 + mC_1] + Mb(LbN_1 + L_1)$. Using the Gronwall's inequality, we get

$$
\sup_{s\in(0,b]} \| \phi_{\tilde{v}}(t) \| \leq [M^* + M(c+\epsilon) \| \tilde{v} \|_{PC} + M\sum_{k=1}^{m}(c_k+\epsilon_k) \| v(t_k) \|]e^{ML(n+Nb)},
$$

and the previous inequality holds. Consequently,

$$
\| v \|_{PC} \leq [M^* + M(c+\epsilon) \| \tilde{v} \|_{PC} + M\sum_{k=1}^{m}(c_k+\epsilon_k) \| v \|_{PC}]e^{ML(n+Nb)},
$$

and, therefore,

$$
\| v \|_{PC} \leq \frac{M^* e^{ML(n+Nb)}}{1 - M(c+\epsilon+\sum_{k=1}^{m}(c_k+\epsilon_k))e^{ML(n+Nb)}} < \infty.
$$

Thus the proof of boundedness of the set Ω is complete.

Step 2. Γ is a compact operator.

For each constant $r > 0$, let

$$B_r(\delta) := \left\{ \phi \in PC_\delta;\ \sup_{\delta \le t \le b} \parallel \phi(t) \parallel \le r \right\}.$$

Then $B_r(\delta)$ is a bounded closed convex set in PC_δ. To this end, we consider the decomposition $\Gamma = \Gamma_1 + \Gamma_2$, where Γ_1, Γ_2 are the operators on $B_r(\delta)$ defined respectively by

$$\begin{aligned}(\Gamma_1 v)(t) &= U(t,0)[x_0 - g(\tilde{v})] + \int_0^t U(t,s) \\ &\times F\left(s, \phi_{\tilde{v}}(\sigma_1(s)), ..., \phi_{\tilde{v}}(\sigma_n(s)), \int_0^s h(s,\tau,\phi_{\tilde{v}}(\sigma_{n+1}(\tau)))d\tau\right) ds, \quad t \in [\delta, b].\end{aligned}$$

$$(\Gamma_2 v)(t) = \sum_{0<t_k<t} U(t,t_k) I_k(v(t_k)), \qquad t \in [\delta, b].$$

We first show that Γ_1 is a compact operator.

(i). $\Gamma_1(B_r(\delta))$ is equicontinuous.

Let $\delta \le \tau_1 < \tau_2 \le b$, we have

$$\begin{aligned}&\parallel \Gamma_1 v(\tau_2) - \Gamma_1 v(\tau_1) \parallel \\ &\le \parallel U(\tau_2,0) - U(\tau_1,0)][x_0 - g(\tilde{v})] \parallel + \Bigg\| \int_0^{\tau_2} U(\tau_2,s) \\ &\quad \times F\left(s, \phi_{\tilde{v}}(\sigma_1(s)), ..., \phi_{\tilde{v}}(\sigma_n(s)), \int_0^s h(s,\tau,\phi_{\tilde{v}}(\sigma_{n+1}(\tau)))d\tau\right) ds \\ &\quad - \int_0^{\tau_1} U(\tau_1,s) \\ &\quad \times F\left(s, \phi_{\tilde{v}}(\sigma_1(s)), ..., \phi_{\tilde{v}}(\sigma_n(s)), \int_0^s h(s,\tau,\phi_{\tilde{v}}(\sigma_{n+1}(\tau)))d\tau\right) \Bigg\| ds \\ &\le \parallel U(\tau_2,0) - U(\tau_1,0) \parallel [\parallel x_0 \parallel + (c+\epsilon)r] + \int_0^{\tau_1} \parallel U(\tau_2,s) - U(\tau_1,s) \parallel \\ &\quad \times \left\| F\left(s, \phi_{\tilde{v}}(\sigma_1(s)), ..., \phi_{\tilde{v}}(\sigma_n(s)), \int_0^s h(s,\tau,\phi_{\tilde{v}}(\sigma_{n+1}(\tau)))d\tau\right) \right\| ds \\ &\quad + M \int_{\tau_1}^{\tau_2} \left\| F\left(s, \phi_{\tilde{v}}(\sigma_1(s)), ..., \phi_{\tilde{v}}(\sigma_n(s)), \int_0^s h(s,\tau,\phi_{\tilde{v}_2}(\sigma_{n+1}(\tau)))d\tau\right) \right\| ds.\end{aligned}$$

Noting that

$$\begin{aligned}&\left\| F\left(s, \phi_{\tilde{v}}(\sigma_1(s)), ..., \phi_{\tilde{v}}(\sigma_n(s)), \int_0^s h(s,\tau,\phi_{\tilde{v}}(\sigma_{n+1}(\tau)))d\tau\right) \right\| \\ &\quad \le \left\| F\left(s, \phi_{\tilde{v}}(\sigma_1(s)), ..., \phi_{\tilde{v}}(\sigma_n(s)), \int_0^s h(s,\tau,\phi_{\tilde{v}}(\sigma_{n+1}(\tau)))d\tau\right) \right.\end{aligned}$$

$$
\begin{aligned}
&- F(s,0,...,0)\Big\| + \| F(s,0,...,0) \| \\
&\le L\left[\| \phi_{\tilde{v}}(\sigma_1(s)) \| + \cdots + \| \phi_{\tilde{v}}(\sigma_n(s)) \| + \left\| \int_0^s h(s,\tau,\phi_{\tilde{v}}(\sigma_{n+1}(\tau)))d\tau \right\| \right] + L_1 \\
&\le L\Bigg[\sup_{s\in[\delta,b]} \| \phi_{\tilde{v}}(s) \| + \cdots + \sup_{s\in[\delta,b]} \| \phi_{\tilde{v}}(s) \| + \int_0^s [\| h(s,\tau,\phi_{\tilde{v}}(\tau)) - h(s,\tau,0) \| \\
&\quad + \| h(s,\tau,0) \|]d\tau \Bigg] + L_1 \\
&\le L\left[n \sup_{s\in[\delta,b]} \| \phi_{\tilde{v}}(s) \| + b[N \sup_{s\in[\delta,b]} \| \phi_{\tilde{v}}(s) \| + N_1] \right] + L_1 \\
&\le L\left[(n+Nb) \sup_{s\in[\delta,b]} \| \phi_{\tilde{v}}(s) \| + bN_1 \right] + L_1 \\
&\le L[(n+Nb)r + bN_1] + L_1.
\end{aligned}
$$

We see that $\| \Gamma_1 v(\tau_2) - \Gamma_1 v(\tau_1) \|$ tend to zero independently of $v \in B_r(\delta)$ as $\tau_2 - \tau_1 \to 0$, since the compactness of $U(t,s)$ for $t-s>0$, implies the continuity in the uniform operator topology. Thus the family of functions $\{(\Gamma_1 v) : v \in B_r(\delta)\}$ is equicontinuous on $[\delta, b]$.

(ii). The set $\Gamma_1(B_r(\delta))(t)$ is precompact compact in X.

Let $\delta < t \le s \le b$ be fixed and ε a real number satisfying $0 < \varepsilon < t$. For $v \in B_r(\delta)$, we define

$$
\begin{aligned}
(\Gamma_{1,\varepsilon} v)(t) &= U(t,0)[x_0 - g(\tilde{v})] + \int_0^{t-\varepsilon} U(t,s) \\
&\qquad \times F\left(s, \phi_{\tilde{v}}(\sigma_1(s)), ..., \phi_{\tilde{v}}(\sigma_n(s)), \int_0^s h(s,\tau,\phi_{\tilde{v}}(\sigma_{n+1}(\tau)))d\tau \right) ds \\
&= U(t,0)[x_0 - g(\tilde{v})] + U(t,t-\varepsilon) \int_0^{t-\varepsilon} U(t-\varepsilon,s) \\
&\qquad \times F\left(s, \phi_{\tilde{v}}(\sigma_1(s)), ..., \phi_{\tilde{v}}(\sigma_n(s)), \int_0^s h(s,\tau,\phi_{\tilde{v}}(\sigma_{n+1}(\tau)))d\tau \right) ds.
\end{aligned}
$$

Using the compactness of $U(t,s)$ for $t-s>0$, we deduce that the set $\{(\Gamma_{1,\varepsilon} v)(t) : v \in B_r(\delta)\}$ is precompact in X for every $\varepsilon, 0 < \varepsilon < t$. Moreover, for every $v \in B_r(\delta)$ we have

$$
\begin{aligned}
&\| (\Gamma_1 v)(t) - (\Gamma_{1,\varepsilon} v)(t) \| \\
&\le \int_{t-\varepsilon}^t \left\| U(t,s) F\left(s, \phi_{\tilde{v}}(\sigma_1(s)), ..., \phi_{\tilde{v}}(\sigma_n(s)), \int_0^s h(s,\tau,\phi_{\tilde{v}}(\sigma_{n+1}(\tau)))d\tau \right) \right\| ds \\
&\le M \int_{t-\varepsilon}^t [L(n+Nb)r + LbN_1 + L_1] ds.
\end{aligned}
$$

Therefore, there are precompact sets arbitrarily close to the set $\{\Gamma_1(v) : v \in B_r(\delta)\}$. Hence $\Gamma_1(B_r(\delta))(t)$ is precompact in X.

Next, it remains to verify that Γ_2 is also a compact operator.

We begin by showing $\Gamma_2(B_r(\delta))$ is equicontinuous. For any $\varepsilon > 0$ and $0 < t < b$. Since the functions $I_k, k = 1, 2, ..., m$, are compact in X, from the properties of the evolution family $U(t, s)$, we find that the set $W = \{U(t, t_k)I_k(v(t_k)) : v \in B_r(\delta)\}$ is precompact in X. We can choose $0 < \xi < b - t$ such that

$$\| U(t + h, t)x - x \| < \frac{\varepsilon}{m}, \quad x \in W,$$

when $|h| < \xi$. For each $v \in B_r(\delta), t \in (0, b)$ be fixed, $t \in [t_i, t_{i+1}]$, such that

$$\begin{aligned}
&\| \widehat{[(\Gamma_2 v)]}_i(t + h) - \widehat{[(\Gamma_2 v)]}_i(t) \| \\
&\leq \sum_{k=1}^{m} \| (U(t + h, t_k)I_k(v(t_k)) - U(t, t_k)I_k(v(t_k)) \| \\
&\leq \sum_{k=1}^{m} \| (U(t + h, t) - I)U(t, t_k))I_k(v(t_k) \| \\
&\leq \varepsilon.
\end{aligned}$$

As $h \to 0$ and ε sufficiently small, the right-hand side of the above inequality tends to zero independently of v, so $[\widehat{\Gamma_2(B_r(\delta))}]_i, i = 1, 2, ..., m$, are equicontinuous.

Now we prove that $[\widehat{\Gamma_2(B_r(\delta))}]_i, i = 1, 2, ..., m$, is precompact for every $t \in [\delta, b]$.

From the following relations

$$\widehat{[(\Gamma_2 v)]}_i(t) = \sum_{0<t_k<t} U(t, t_k)I_k(v(t_k)) \in \sum_{k=1}^{m} U(t, t_k)I_k(B_r(\delta)[0, X]).$$

We conclude that $[\widehat{\Gamma_2(B_r(\delta))}]_i, i = 1, 2, ..., m$, is precompact for every $t \in [t_i, t_{i+1}]$.

By Lemma 2.1, we infer that $\Gamma_2(B_r(\delta))$ is precompact. Now an application of the Arzelá-Ascoli theorem justifies the precompactness of $\Gamma_2(B_r(\delta))$. Therefore, Γ_2 is a compact operator, and hence Γ is a compact operator.

Step 3. Γ is continuous.

From (7) and (H1)-(H6), we deduce that for $v_1, v_2 \in B_r(\delta),\ t \in [0, b]$,

$$\begin{aligned}
&\| \phi_{\tilde{v}_1}(t) - \phi_{\tilde{v}_2}(t) \| \\
&\leq \| U(t, 0)[g(\tilde{v}_1) - g(\tilde{v}_2)] \| \\
&\quad + \left\| \sum_{0<t_k<t} U(t, t_k)I_k(v_1(t_k)) - \sum_{0<t_k<t} U(t, t_k)I_k(v_2(t_k)) \right\| \\
&\quad + \int_0^t \left\| U(t, s)\left[F\left(s, \phi_{\tilde{v}_1}(\sigma_1(s)), ..., \phi_{\tilde{v}_1}(\sigma_n(s)), \int_0^s h(s, \tau, \phi_{\tilde{v}_1}(\sigma_{n+1}(\tau)))d\tau\right)\right.\right.
\end{aligned}$$

$$
\begin{aligned}
&- F\Big(s, \phi_{\tilde{v}_2}(\sigma_1(s)), ..., \phi_{\tilde{v}_2}(\sigma_n(s)), \int_0^s h(s,\tau,\phi_{\tilde{v}_2}(\sigma_{n+1}(\tau)))d\tau\Big)\Big]\Big\| ds \\
&\leq M \parallel g(\tilde{v}_1) - g(\tilde{v}_2) \parallel + M\sum_{k=1}^{m} \parallel I_k(v_1(t_k)) - I_k(v_2(t_k)) \parallel \\
&\quad + M\int_0^t L\Big[\parallel \phi_{\tilde{v}_1}(\sigma_1(s)) - \phi_{\tilde{v}_2}(\sigma_1(s)) \parallel + \cdots + \parallel \phi_{\tilde{v}_1}(\sigma_n(s)) - \phi_{\tilde{v}_2}(\sigma_n(s)) \parallel \\
&\quad + \int_0^s \Big\| h(s,\tau,\phi_{\tilde{v}_1}(\sigma_{n+1}(\tau)))d\tau - \int_0^s h(s,\tau,\phi_{\tilde{v}_2}(\sigma_{n+1}(\tau)))d\tau \Big\|\Big] ds \\
&\leq M \parallel g(\tilde{v}_1) - g(\tilde{v}_2) \parallel + M\sum_{k=1}^{m} \parallel I_k(v_1(t_k)) - I_k(v_2(t_k)) \parallel \\
&\quad + ML\int_0^t \Big[\sup_{s\in[0,b]} \parallel \phi_{\tilde{v}_1}(s) - \phi_{\tilde{v}_2}(s) \parallel + \cdots + \sup_{s\in[0,b]} \parallel \phi_{\tilde{v}_1}(s) - \phi_{\tilde{v}_2}(s) \parallel \\
&\quad + N\int_0^s \parallel \phi_{\tilde{v}_1}(\sigma_{n+1}(s)) - \phi_{\tilde{v}_2}(\sigma_{n+1}(s)) \parallel \Big] ds \\
&\leq M \parallel g(\tilde{v}_1) - g(\tilde{v}_2) \parallel + M\sum_{k=1}^{m} \parallel I_k(v_1(t_k)) - I_k(v_2(t_k)) \parallel \\
&\quad + ML\int_0^t [n \sup_{s\in[0,b]} \parallel \phi_{\tilde{v}_1}(s) - \phi_{\tilde{v}_2}(s) \parallel + Nb \sup_{s\in[0,b]} \parallel \phi_{\tilde{v}_1}(s) - \phi_{\tilde{v}_2}(s) \parallel] ds \\
&\leq M \parallel g(\tilde{v}_1) - g(\tilde{v}_2) \parallel + M\sum_{k=1}^{m} \parallel I_k(v_1(t_k)) - I_k(v_2(t_k)) \parallel \\
&\quad + ML(n+Nb)\int_0^t \sup_{s\in[0,b]} \parallel \phi_{\tilde{v}_1}(s) - \phi_{\tilde{v}_2}(s) \parallel ds.
\end{aligned}
$$

Using again the Gronwall's inequality, we have that, for t, v_1, v_2 as above

$$
\begin{aligned}
\sup_{s\in[0,b]} \parallel \phi_{\tilde{v}_1}(t) - \phi_{\tilde{v}_2}(t) \parallel \leq M\Big[&\parallel g(\tilde{v}_1) - g(\tilde{v}_2) \parallel \\
&+ M\sum_{k=1}^{m} \parallel I_k(v_1(t_k)) - I_k(v_2(t_k)) \parallel \Big] e^{ML(n+Nb)b},
\end{aligned}
$$

which implies that

$$
\begin{aligned}
\parallel \Gamma v_1 - \Gamma v_2 \parallel_{PC} \leq M\Big[&\parallel g(\tilde{v}_1) - g(\tilde{v}_2) \parallel \\
&+ M\sum_{k=1}^{m} \parallel I_k(v_1(t_k)) - I_k(v_2(t_k)) \parallel \Big] e^{ML(n+Nb)b},
\end{aligned}
$$

for any $t \in [\delta, b], v_1, v_2 \in B_r(\delta)$. Therefore, Γ is continuous.

These arguments enable us to conclude that $\Gamma$is completely continuous. We can now apply Lemma 2.3 to conclude that Γ has at least fixed point $\tilde{v}_* \in C_\delta$. Let $x = \phi_{\tilde{v}_*}$. Then, we have

$$x(t) = U(t,0)[x_0 - g(\tilde{v}_*)] + \sum_{0<t_k<t} U(t,t_k)I_k(v(t_k)) + \int_0^t U(t,s) \times F\bigg(s, x(\sigma_1(s)), ..., x(\sigma_n(s)), \int_0^s h(s,\tau,x(\sigma_{n+1}(\tau)))d\tau\bigg)ds. \quad (14)$$

Note that $x = \phi_{\tilde{v}_*} = (\Gamma\tilde{v}_*)(t) = \tilde{v}_*$, $t \in [\delta, b]$. By (H6)(a), we obtain

$$g(x) = g(\tilde{v}_*) \quad \text{and} \quad v_*(t_k) = x(t_k).$$

This implies, combined with (14), that $x(t)$ is a mild solution of the problem (1)-(3) and the proof of Theorem 3.1 is complete.

From the above proof of Theorem 3.1, we immediately obtain the following corollaries.

Corollary 3.1 *Suppose that* (H1)-(H4),(H6)(a), *and the following conditions are satisfied:*

(H5)$'$ $I_k \in C(X,X), k = 1,...,m$ *are all compact operators, and there exist constants* $a_k > 0, b_k > 0, \alpha_k \in [0,1), k = 1,...,m,$ *such that*

$$\| I_k(x) \| \leq a_k + b_k \| x \|^{\alpha_k}, \qquad k = 1,...,m, \ x \in X.$$

(H6)$'$ *There exist constants* d_1 *and* $d_2, \mu \in [0,1)$ *such that*

$$\| g(\phi) \| \leq d_1 + d_2 \| \phi \|^{\mu}, \qquad \phi \in PC(J,X).$$

Then the impulsive nonlocal Cauchy problem (1)-(3) *has at least one mild solution on* J.

Corollary 3.2 *Suppose that* (H1)-(H4),(H6)(a) *and the following conditions are satisfied:*

(H5)$''$ $I_k \in C(X,X), k = 1,...,m$ *are all compact operators, and there exist constants* $\bar{a}_k > 0, \bar{b}_k > 0, k = 1,...,m,$ *such that*

$$\| I_k(x) \| \leq \bar{a}_k + \bar{b}_k \| x \|, \qquad k = 1,...,m, \ x \in X.$$

(H6)$''$ *There exist constants* $\bar{d}_1$ *and* $\bar{d}_2$ *such that*

$$\| g(\phi) \| \leq \bar{d}_1 + \bar{d}_2 \| \phi \|, \qquad \phi \in PC(J,X).$$

Then the impulsive nonlocal Cauchy problem (1)-(3) *has at least one mild solution on* J, *provided that*

$$M\bigg(\bar{d}_2 + \sum_{k=1}^{m} \bar{b}_k\bigg)e^{ML(n+Nb)} < 1. \quad (15)$$

4. Application

To illustrate the application of the obtained results of this paper, we study the following example in this section:

$$z_t(t,x) = \frac{\partial^2}{\partial x^2} a_0(t,x) z(t,x) + a_1(t) z(\sin t, x) + \sin z(t,x) + \frac{1}{1+t^2} \int_0^t a_2(s) z(\sin s, x) ds, \quad (16)$$

$$\Delta z(t_k, x) = \int_0^{t_k} c_k(t_k - s) z(s,x), \quad k = 1, ..., m, \quad (17)$$

$$z(t,0) = z(t,\pi) = 0, \quad (18)$$

$$z(0,x) + \int_\delta^1 [z(s,x) + \log(1 + |z(s,x)|)] ds = z_0(x), \ 0 \le t \le 1, 0 \le x \le \pi, \quad (19)$$

where $\delta > 0$, $z_0(x) \in X = L^2([0,\pi])$ and $z_0(0) = z_0(\pi) = 0$. Here, the functions $a_0(t,x)$ is continuous and is uniformly Hölder continuous in t.

Let $X = L^2([0,\pi])$ and the operators $A(t)$ be defined by

$$A(t)w = a_0(t,x) w''$$

with the domain $D(A) = \{w \in X : w, w''$ are absolutely continuous, $w'' \in X, w(0) = w(1) = 0\}$, then $A(t)$ generates an evolution system $U(t,s)$ satisfying assumptions (I)-(III)(see[13]).

We assume that

(a) The functions $a_i(\cdot), i = 1,2,3$, is continuous on $[0,1]$, and

$$l_i = \sup_{0 \le s \le 1} |a_i(s)| < 1, \ i = 1,2.$$

(b) The functions $c_k : \mathbb{R} \to \mathbb{R}$, $k = 1, 2, \dots, m$, are continuous, bounded and

$$\gamma_k = \left(\int_0^\pi (c_k(s))^2 ds \right)^{1/2} < \infty$$

for every $k = 1, 2, ..., m$.

Define respectively $F : [0,1] \times X \times X \to X, h : [0,1] \times [0,1] \times X \to X$, $I_k : X \to X$ and $g : PC([0,1], X) \to X$ by

$$F\left(t, z(\sigma(t)), \int_0^t h(t,s,z(\sigma(s))) ds\right)(x) = a_1(t) z(\sin t, x) + \sin z(t,x) + \frac{1}{1+t^2} \int_0^t a_2(s) z(\sin s, x) ds,$$

$$\int_0^t h(t,s,z(\sigma(s)))(x) ds = \frac{1}{1+t^2} \int_0^t a_2(s) z(\sin s, x) ds,$$

$$I_k(z)(x) = \int_0^{\pi} c_k(s)z(s,x), \quad k = 1, 2, ..., m$$

and

$$g(z)(x) = \int_{\delta}^{1} [z(s,x) + \log(1 + |z(s,x)|)]ds, \quad z \in PC([0,1], X).$$

It is easy to see that with these choices, the assumptions (H1)-(H6)(a) of Theorem 3.1 are satisfied. If we assume that

$$M\left[(1-\delta) + \sum_{k=1}^{m} \gamma_k\right] e^{M(1+l_1+l_2)(1+l_2)} < 1 \tag{20}$$

hold. Now the condition (H6)(b) in Section 2 holds and hence by Theorem 3.1, we deduce that the impulsive nonlocal Cauchy problem (16)-(19) has a mild solution on $[0, 1]$.

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