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THE NEW ALGORITHM FOR LDL^T DECOMPOSITION OF BLOCK HANKEL MATRICES†

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ABSTRACT. In this paper, with use of the displacement matrix, two special matrices are constructed. By these special matrices the block decompositions of the block symmetric Hankel matrix and the inverse of the Hankel matrix are derived. Hence, the algorithms according to these decompositions are given. Furthermore, the numerical tests show that the algorithms are feasible.

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1. Introduction

Hankel matrix is of special interest in view of various applications: communication, control engineering, filter design, identification, model reduction and broadband matching, and in different fields of mathematics, e.g., in systems theory, integral equations, and operator theory. In this paper, we will obtain the new algorithm for the block LDL^T decomposition of a block symmetric Hankel matrix H by developing the idea in [5] to the block Hankel matrix. Furthermore, by applying the similar techniques to the inverse of the block Hankel matrix, we will get the UDU^T decomposition of H^{-1}. There is an extensive literature on Hankel matrix; for some reference, see [1, 2, 3, 4].

This paper is organized as follows. Some elementary definitions and relative theorems of Hankel matrices are discussed in Section 2. The decompositions of symmetric Hankel matrix H and H^{-1} are derived in Section 3. Finally, we present two numerical examples to test the algorithms in Section 4.

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2. Main theorems

We will use the following notations. M^n denotes the square matrix of order n. Let $Z = z_n \otimes I_p$ where z_n is a displacement matrix and $I_p \in M^p$ is an identity matrix. $E_i = e_i \otimes I_p$ where e_i is a unit vector of order n. $\tilde{E}_i = \tilde{e}_i \otimes I_p$ where $\tilde{e}_i$ is a unit vector of order 2n.

Definition 1. A matrix $H \in M^{np}$ is called a block symmetric Hankel matrix, if H satisfies $H = (\Gamma_{i+j-1})_{i,j=1}^n$ where $\Gamma_k \in M^p (k = 1, \ldots, 2n-1)$ is a symmetric matrix.

Definition 2. A matrix $H \in M^{np}$ is called a block strong-nonsingular Hankel matrix if

$$H_{kp} = \begin{pmatrix} \Gamma_1 & \Gamma_2 & \ldots & \Gamma_k \\ \Gamma_2 & \Gamma_3 & \ldots & \Gamma_{k+1} \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_k & \Gamma_{k+1} & \ldots & \Gamma_{2k-1} \end{pmatrix}$$

for every $k = 1, \ldots, n,\ H_{kp}$ is nonsingular.

Theorem 1. *Let $H \in M^{np}$ be a block strong-nonsingular matrix. Introduce a series of matrices:*

$$H^{(0)} = H = B_0,\ H^{(i-1)} = L_i D_i U_i^T + H^{(i)},\ i = 1, \ldots, n. \tag{1}$$

where

$$L_i = H^{(i-1)} E_i,\ U_i = (H^{(i-1)})^T E_i,\ D_i = (L_i(i))^{-1}, \quad i = 1, \ldots, n, \tag{2}$$

and $L_i(i) \in M^p$ is the ith block of L_i, then $L_i,\ U_i,\ H^{(i)},\ i = 1, 2, \ldots, n$ have the following forms:

$$L_i = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ L_i(i) \\ \vdots \\ L_i(n) \end{pmatrix}, \quad U_i = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ U_i(i) \\ \vdots \\ U_i(n) \end{pmatrix},\ L_i(j) \in M^p,\ U_i(j) \in M^p, \tag{3}$$

$$H^{(i)} = \begin{pmatrix} 0 & 0 \\ 0 & B_i \end{pmatrix},\ B_i \in M^{(n-i)p},\ i = 1, 2, \ldots, n.$$

and

$$H = [L_1, \ldots, L_n] diag[D_1, \ldots, D_n][U_1^T, \ldots, U_n^T]^T. \tag{4}$$

Proof. Since H is a block strong-nonsingular matrix, the formulas (1), (2) are reasonable. Under the assumptions of Theorem 1, it is simple to verify that (3) is right. The proof of the block decomposition of H is following. Repeating the process (4) n times and recognizing that $H^{(n)} = 0$, we have

$$H = H^{(0)} = L_1 D_1 U_1^T + H^{(1)} = \sum_{i=1}^n L_i D_i U_i^T + H^{(n)}$$

$$= \quad [L_1, \ldots, L_n] diag[D_1, \ldots, D_n][U_1^T, \ldots, U_n^T]^T.$$

, i.e., the formula (4) makes sense. □

Remark 1. Let $H \in M^{np}$ be a block symmetric strong-nonsingular matrix, and $L = [L_1, \ldots, L_n]$, $D = diag(D_1, \ldots, D_n)$ in Theorem 1, then $H = LDL^T$ just is a block LDL^T decomposition of H.

Theorem 2. *Under the assumptions of Theorem 1, and suppose that*

$$\Delta H^{(i)} = VH^{(i)} - H^{(i)}V^T, \tag{5}$$

where V is a low triangular matrix, then B_i $(i = 0, \ldots, n-1)$ is nonsingular and $rank(\Delta H^{(i)}) \leq rank(\Delta H^{(i-1)})$ for every $i = 1, 2, \ldots, n$.

Proof. From (1)-(4), we have

$$B_{i-1} = (L_i^T(i), \ldots, L_i^T(n))^T D_i (U_i(i), \ldots, U_i(n)) + \begin{pmatrix} 0 & 0 \\ 0 & B_i \end{pmatrix}.$$

Since $D_i = (L_i(i))^{-1} = [(U_i(i))^T]^{-1}$, then

$$B_{i-1} = \begin{pmatrix} D_i^{-1} & S_i \\ t_i & B_i + t_i D_i S_i \end{pmatrix}.$$

where $t_i = \begin{pmatrix} L_i(i+1) \\ \vdots \\ L_i(n) \end{pmatrix}$, $S_i = (U_i(i+1), \ldots, U_i(n))$.

Recognizing that

$$\begin{pmatrix} I & 0 \\ -t_i D_i & I \end{pmatrix} \begin{pmatrix} D_i^{-1} & S_i \\ t_i & B_i + t_i D_i S_i \end{pmatrix} \begin{pmatrix} I & -D_i S_i \\ 0 & I \end{pmatrix} = \begin{pmatrix} D_i^{-1} & 0 \\ 0 & B_i \end{pmatrix}, \tag{6}$$

we obtain

$$det(B_{i-1}) = det(B_i) det(D_i^{-1}), \quad i = 1, 2, \ldots, n-1.$$

Since $B_0 = H$ is nonsingular and $det(D_i^{-1}) \neq 0$ $(i = 1, \ldots, n-1)$, then

$$det(B_i) \neq 0 \ (i = 0, 1, \ldots, n-1).$$

Denote $\Delta_i B_i = Z_3 B_i - B_i Z_3^T$, $\bar{\Delta}_i B_i^{-1} = B_i^{-1} Z_3 - Z_3^T B_i^{-1}$, $i = 1, 2, \ldots, n$, then $\Delta H^{(i)} = \Delta_i B_i$, and with the formula (6) we can get

$$\bar{\Delta}_{i-1} B_{i-1}^{-1} = \begin{pmatrix} * & * \\ * & \bar{\Delta}_i B_i^{-1} \end{pmatrix}, \ i = 1, 2, \ldots, n.$$

Since the rank of the matrix is greater than the rank of its submatrix, then

$$\begin{aligned} rank(\Delta H^{(i)}) &= rank(\Delta_i B^{(i)}) = rank[(\Delta_i B_i) B_i^{-1}] = rank[B_i(\bar{\Delta}_i B_i^{-1})] \\ &= rank(\bar{\Delta}_i B_i^{-1}) \leq rank(\bar{\Delta}_{i-1} B_{i-1}^{-1}) = rank(\Delta H^{(i-1)}). \end{aligned}$$

□

3. Two algorithms

3.1.The LDL^T decomposition for the block symmetric Hankel matrix. Let $H \in M^{np}$ be a block symmetric strong-nonsingular Hankel matrix. In order to get the $H = LDL^T$ decomposition, we only need obtain $L_i,\ D_i$. It is easy to verify that

$$ZH - HZ^T = mE_1^T - E_1 m^T,$$

where

$$m = \begin{pmatrix} 0 \\ \Gamma_1 \\ \vdots \\ \Gamma_{n-1} \end{pmatrix},$$

then $rank(\Delta H^0) \leq 2p$. Moreover, from Theorem 2, we know that

$$rank(\Delta H^{(i-1)}) \leq 2p,$$

and note that $\Delta H^{(i-1)}$ is a block antisymmetric matrix, thus there are two matrices

$$X_{i-1} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ X_{i-1}(i) \\ \vdots \\ X_{i-1}(n) \end{pmatrix}, \qquad Y_{i-1} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ Y_{i-1}(i) \\ \vdots \\ Y_{i-1}(n) \end{pmatrix},$$

such that

$$\Delta H^{(i-1)} = X_{i-1} Y_{i-1}^T - Y_{i-1} X_{i-1}^T. \tag{7}$$

On one hand, right multiplying (5) and (7) by E_i respectively, and noting that $H^{(i-1)} Z^T E_i = H^{(i-1)} E_{i-1} = 0$, we can get

$$ZH^{(i-1)} E_i - H^{(i-1)} Z_n^T E_i = (X_{i-1} Y_{i-1}^T - Y_{i-1} X_{i-1}^T) E_i.$$

Furthermore, from Theorem 1, we have $H^{(i-1)} E_i = L_i$, then

$$ZL_i = X_{i-1}(Y_{i-1}(i))^T - Y_{i-1}(X_{i-1}(i))^T. \tag{8}$$

With use of the formula (8), $L_i(i), \ldots, L_i(n-1)$ can be determined by the following formulas:

$$L_i(j) = X_i(j+1)(Y_{i-1}(i))^T - Y_{i-1}(j+1)(X_{i-1}(i))^T,\ j = i, \ldots, n-1.$$

and

$$X_{i-1}(i)(Y_{i-1}(i))^T = Y_{i-1}(i)(X_{i-1}(i))^T. \tag{9}$$

On the other hand, from the last row block of $H = LDL^T$, it holds

$$\sum_{j=1}^{i} L_j(n) D_j L_j^T(i) = \Gamma_{n+i-1} (i = 1, 2, \ldots, n),$$

then

$$L_i(n) = \begin{cases} \Gamma_n, & i = 1, \\ \Gamma_{n+i-1} - \sum_{j=1}^{i-1} L_j(n) D_j L_j^T(i), & i \geq 2. \end{cases}$$

Inserting $\Delta H^((i)) = ZH^{(i)} - H^{(i)}Z^T$ by $H^{(i)} = H^{(i-1)} - L_i D_i L_i^T$ and using (7), we can obtain

$$X_i(Y_i)^T - Y_i(X_i)^T = X_{i-1}(Y_{i-1})^T - Y_{i-1}(X_{i-1})^T - ZL_iD_i(L_i)^T + L_iD_i(L_i)^T Z^T.$$

with the formula (8), the above formula will become

$$\begin{aligned} X_i(Y_i)^T - Y_i(X_i)^T =& X_{i-1}(Y_{i-1})^T - Y_{i-1}(X_{i-1})^T \\ & - [X_{i-1}(Y_{i-1}(i))^T - Y_{i-1}(X_{i-1}(i))^T] D_i L_i^T \\ & + L_i D_i [X_{i-1}(Y_{i-1}(i))^T - Y_{i-1}(X_{i-1}(i))^T]^T. \end{aligned} \tag{10}$$

Noting that the formula (8) and $D_i = D_i^T$ $(i = 1, \ldots, n)$, we have the formula (10) by denoting

$$X_i = X_{i-1} - L_i D_i X_{i-1}(i), \; Y_i = Y_{i-1} - L_i D_i Y_{i-1}(i). \tag{11}$$

Hence, with the formulas (8)-(10), the algorithm for the LDL^T decomposition of the block symmetric strong-nonsingular Hankel matrix is given by Algorithm 1.

3.2.The UDU^T decomposition of H^{-1}. Constructing the two matrices of order 2np

$$Q = \begin{pmatrix} H & I_n \otimes I_p \\ I_n \otimes I_p & 0_n \otimes I_p \end{pmatrix}, \quad F = \begin{pmatrix} Z & 0 \\ 0 & Z^T \end{pmatrix},$$

and applying Theorem 1 to the matrix Q, we have

$$Q^{(0)} = \tilde{R}_0 = Q, \quad Q^{(i)} = \begin{pmatrix} 0_{ip \times ip} & 0 \\ 0 & \tilde{R}_i \end{pmatrix},$$

where $\tilde{R}_i \in M^{(2n-i)p}$.

Let

$$\tilde{L}_i = Q^{(i-1)} \tilde{E}_i, \; \tilde{D}_i = (\tilde{L}_i(i))^{-1} \in M^p,$$

thus

$$Q^{(i-1)} = \tilde{L}_i \tilde{D}_i \tilde{L}_i^T + Q^{(i)}. \tag{12}$$

Since Q is a band matrix, then $\tilde{L}_i$ has the following form

$$\tilde{L}_i = [0, \ldots, 0, (\tilde{L}_i(i))^T, \ldots, (\tilde{L}_i(i+n))^T, 0, \ldots, 0]^T,$$

Repeating the step (12) n times, we have the decomposition of Q

$$Q = (\tilde{L}_1, \tilde{L}_2, \ldots, \tilde{L}_n) \begin{pmatrix} \tilde{D}_1 & & & \\ & \tilde{D}_2 & & \\ & & \ddots & \\ & & & \tilde{D}_n \end{pmatrix} \begin{pmatrix} \tilde{L}_1^T \\ \tilde{L}_2^T \\ \vdots \\ \tilde{L}_n^T \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \tilde{R}_n \end{pmatrix}. \tag{13}$$

Algorithm 1 $H = LDL^T$

Require: A block symmetric strong-nonsingular Hankel matrix H.
Ensure: The matrices L, D.

$X_0(1) = 0 \otimes I_p \; Y_0(1) = 1 \otimes I_p$
for $k = 2$ to n **do**
  $X_0(k) = \Gamma_{k-1} \; Y_0(k) = 0 \otimes I_p$
end for
for $i = 1$ to n **do**
  for $k = i$ to $n-1$ **do**
    $L_i(k) = X_{i-1}(k+1)(Y_{i-1}(i))^T - Y_{i-1}(k+1)(X_{i+1}(i))^T$
  end for
  if $i = 1$ **then**
    $L_i(n) = \Gamma_n$
  else
    $L_i(n) = \Gamma_{n+i-1} - \sum\limits_{j=1}^{i-1} L_j(n) D_j L_j^T(i)$
  end if
  $D_i = [L_i(i)]^{-1}$
  for $k = i+1$ to n **do**
    $X_i(k) = X_{k-1}(k) - L_i(k) D_i X_{i-1}(i)$
    $Y_i(k) = Y_{k-1}(k) - L_i(k) D_i Y_{i-1}(i)$
  end for
end for
return

$$L = \begin{pmatrix} L_1(1) & & \\ \vdots & \ddots & \\ L_1(n) & \dots & L_n(n) \end{pmatrix}, \quad D = \begin{pmatrix} D_1 & & \\ & \ddots & \\ & & D_n \end{pmatrix}.$$

Partitioning

$$(\tilde{L}_1, \tilde{L}_2, \dots, \tilde{L}_n) = \begin{pmatrix} L_1 \\ U_1 \end{pmatrix}, \quad \tilde{D} = diag(\tilde{D}_1, \dots, \tilde{D}_n), \tag{14}$$

where $L_1 \in M^{np}$ is a block low triangular matrix, $U_1 \in M^{np}$ is a block up triangular matrix, substituting (13) by (14) and comparing the two sides of (13), we have

$$H = L_1 \tilde{D} L_1^T, \quad I_{np} = L_1 \tilde{D} U_1^T = U_1 \tilde{D} L_1^T.$$

Hence, $H^{-1} = U_1 \tilde{D} U_1^T$,this just is the block triangular decomposition of H^{-1}.

In the following process, $\tilde{L}_i$, $\tilde{D}_i$ are derived.

Let $\Delta Q^{(i)} = FQ^{(i)} - Q^{(i)} F^T (i = 1, \dots, n)$, observe that

$$FQ - QF^T = \tilde{h} \tilde{E}_1^T - \tilde{E}_1 \tilde{h}^T, \; \tilde{h} = (0, \Gamma_1^T, \dots, \Gamma_{n-1}^T, 0, \dots, 0)^T \tag{15}$$

we know $rank(\Delta Q^{(i-1)}) \leq 2p$ $(i = 1, 2, \ldots, n+1)$ from Theorem 2. Since $\Delta Q^{(i-1)}$ is a block antisymmetric matrix, there are two matrices of order $2np \times p$

$$\tilde{X}_{i-1} = (0, \ldots, 0, (\tilde{X}_{i-1}(i))^T, \ldots, (\tilde{X}_{i-1}(2n))^T)^T,$$
$$\tilde{Y}_{i-1} = (0, \ldots, 0, (\tilde{Y}_{i-1}(i))^T, \ldots, (\tilde{Y}_{i-1}(2n))^T)^T,$$

such that

$$\Delta Q^{(i-1)} = \tilde{X}_{i-1}\tilde{Y}_{i-1}^T - \tilde{Y}_{i-1}\tilde{X}_{i-1}^T. \tag{16}$$

Multiplying (16) by $\tilde{E}_i$ and noting that $Q^{(i-1)}F^T\tilde{E}_i = Q^{(i-1)}\tilde{E}_{i-1} = 0$, we can obtain

$$F\tilde{L}_i = \tilde{X}_{i-1}(\tilde{Y}_{i-1}(i))^T - \tilde{Y}_{i-1}(\tilde{X}_{i-1}(i))^T. \tag{17}$$

Hence $\tilde{L}_i(i), \ldots, \tilde{L}_i(n-1), \tilde{L}_i(n-2), \ldots, \tilde{L}_i(n+i)$ can be determined by formula (17), but $\tilde{L}_i(n)$, $\tilde{L}_i(n+1)$ can not.

Comparing the last row block of $H = L_1\tilde{D}L_1^T$, we yield

$$\tilde{L}_i(n) = \begin{cases} \Gamma_n, & i = 1, \\ \Gamma_{n+i-1} - \sum_{j=1}^{i-1} \tilde{L}_j(n)\tilde{D}_j\tilde{L}_j^T(i), & i \geq 2. \end{cases} \tag{18}$$

In the same way, with use of the first column block of $I = L_1\tilde{D}U_1^T$, there are

$$\tilde{L}_i(n+1) = \begin{cases} I_p, & i = 1, \\ -\sum_{j=1}^{i-1} \tilde{L}_j(n+1)\tilde{D}_j\tilde{L}_j^T(i), & i \geq 2. \end{cases} \tag{19}$$

Inserting $\Delta Q^{(i)} = FQ^{(i)} - Q^{(i)}F^T$ by $Q^{(i)} = Q^{(i-1)} - \tilde{L}_i\tilde{D}_i\tilde{L}_i^T$ and using (16), (17), we have

$$\begin{aligned}\tilde{X}_i\tilde{Y}_i^T - \tilde{Y}_i\tilde{X}_i^T =& \tilde{X}_{i-1}\tilde{Y}_{i-1}^T - \tilde{Y}_{i-1}\tilde{X}_{i-1}^T - [\tilde{X}_{i-1}(\tilde{Y}_{i-1}(i))^T - \tilde{Y}_{i-1}(\tilde{X}_{i-1}(i))^T] \\ & \tilde{D}_i\tilde{L}_i^T + \tilde{L}_i\tilde{D}_i[\tilde{X}_{i-1}(\tilde{Y}_{i-1}(i))^T - \tilde{Y}_{i-1}(\tilde{X}_{i-1}(i))^T]^T.\end{aligned}$$

Assume that

$$\tilde{X}_i = \tilde{X}_{i-1} - \tilde{L}_i\tilde{D}_i\tilde{X}_{i-1}(i), \;\; \tilde{Y}_i = \tilde{Y}_{i-1} - \tilde{L}_i\tilde{D}_i\tilde{Y}_{i-1}(i). \tag{20}$$

The algorithm for the UDU^T decomposition of H^{-1} are obtained by (16)-(20) and given in the Algorithm 2.

Algorithm 2 $H^{-1} = UDU^T$

Require: A block symmetric strong-nonsingular Hankel matrix H.
Ensure: The matrices U, D.

$X_0(1) = 0 \otimes I_p \quad Y_0(1) = 1 \otimes I_p$
for $k = 2$ to n **do**
    $X_0(k) = \Gamma_{k-1}$
end for
for $k = n+1$ to $2n$ **do**
    $X_0(k) = 0 \otimes I_p$
end for
for $k = 2$ to $2n$ **do**
    $Y_0(k) = 0 \otimes I_p$
end for
for $i = 1$ to n **do**
    for $k = i$ to $n-1$ **do**
        $L_i(k) = X_{i-1}(k+1)(Y_{i-1}(i))^T - Y_{i-1}(k+1)(X_{i+1}(i))^T$
    end for
    if $i = 1$ **then**
        $L_i(n) = \Gamma_n$
    else
        $L_i(n) = \Gamma_{n+i-1} - \sum_{j=1}^{i-1} L_j(n) D_j L_j^T(i)$
    end if
    if $i = 1$ **then**
        $L_i(n+1) = I_p$
    else
        $L_i(n+1) = -\sum_{j=1}^{i-1} L_j(n+1) D_j L_j^T(i)$
    end if
    $D_i = [L_i(i)]^{-1}$
    for $k = i+1$ to $n+i$ **do**
        $L_i(k) = X_{i-1}(k-1)(Y_{i-1}(k))^T - Y_{i-1}(k-1)(X_{i-1}(i))^T$
        $X_i(k) = X_{k-1}(k) - L_i(k) D_i X_{i-1}(i)$
        $Y_i(k) = Y_{k-1}(k) - L_i(k) D_i Y_{i-1}(i)$
    end for
end for
return

$$U = \begin{pmatrix} L_1(n+1) & \dots & L_n(n+1) \\ & \ddots & \vdots \\ & & L_n(2n) \end{pmatrix}, \quad D = \begin{pmatrix} D_1 & & \\ & \ddots & \\ & & D_n \end{pmatrix}.$$

4. Numerical examples

Example 1. Consider the block symmetric strong-nonsingular Hankel matrix:

$$H = \begin{pmatrix} 10 & 0 & 10 & 12 & 5 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 10 & 0 & 10 & 11 & 0 & 1 & 6 & 1 \\ 12 & 0 & 11 & 12 & 1 & 0 & 1 & 1 \\ 5 & 1 & 0 & 1 & 10 & 0 & 10 & 12 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 6 & 1 & 10 & 0 & 10 & 11 \\ 1 & 0 & 1 & 1 & 12 & 0 & 11 & 12 \end{pmatrix},$$

By Algorithm 1, we yield the matrices $D,\ L$ such that $H = LDL^T$

$$D = \begin{pmatrix} 0.1 & 0 & -1.2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1.2 & 0 & 2.4 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.7049 & -4.7295 & 3.2951 & -8.7459 \\ 0 & 0 & 0 & 0 & -4.7295 & 5.1770 & -2.8705 & 7.3754 \\ 0 & 0 & 0 & 0 & 3.2951 & -2.8705 & 1.9049 & -5.0541 \\ 0 & 0 & 0 & 0 & -8.7459 & 7.3754 & -5.0541 & 13.4951 \end{pmatrix},$$

$$L = \begin{pmatrix} 10 & 0 & 10 & 12 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 10 & 11 & 0 & 0 & 0 & 0 \\ 12 & 0 & 11 & 12 & 0 & 0 & 0 & 0 \\ 5 & 1 & 0 & 1 & -3.50 & 4.50 & 46.00 & 12.50 \\ 1 & 1 & 1 & 0 & 4.50 & -0.10 & -8.20 & -0.10 \\ 0 & 1 & 6 & 1 & 46.00 & -8.20 & -65.40 & 9.80 \\ 1 & 0 & 1.0 & 1 & 12.50 & -0.10 & 9.80 & 11.90 \end{pmatrix}.$$

Example 2. Consider the block symmetric strong-nonsingular Hankel matrix:

$$H = \begin{pmatrix} 110 & 1 & 0 & 0 & 100 & 0 \\ 1 & 19 & 0 & 0 & 0 & 110 \\ 0 & 0 & 100 & 0 & 110 & 1 \\ 0 & 0 & 0 & 110 & 1 & 19 \\ 100 & 0 & 110 & 1 & 0 & 0 \\ 0 & 110 & 1 & 19 & 0 & 0 \end{pmatrix},$$

By Algorithm 2, we can get the matrices $D,\ U$ such that $H^{-1} = UDU^T$

$$D = \begin{pmatrix} 0.0091 & -0.0005 & 0 & 0 & 0 & 0 \\ -0.0005 & 0.0527 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0100 & 0 & 0 & 0 \\ 0 & 0 & 0.0000 & 0.0091 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0047 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0016 \end{pmatrix},$$

$$U = \begin{pmatrix} 1.0000 & 0 & 0 & 0 & -0.9095 & 0.0527 \\ 0 & 1.0000 & 0 & 0 & 0.0479 & -5.7922 \\ 0 & 0 & 1.0000 & 0 & -1.1000 & -0.0100 \\ 0 & 0 & 0 & 1.0000 & -0.0091 & -0.1727 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 \end{pmatrix}.$$

The numerical examples show that Algorithm 1 and Algorithm 2 work well. Thus, the algorithms are available for block symmetric strong-nonsingular Hankel matrices and easy to realize. In future, we will consider the algorithms for block Hankel matrices under less limiting conditions.

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